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Compensating the intensity fall-off effect in cone-beam tomography by an empirical weight formula

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Abstract

The Feldkamp–David–Kress (FDK) algorithm is widely adopted for cone-beam reconstruction due to its one-dimensional filtered backprojection structure and parallel implementation. In a reconstruction volume, the conspicuous cone-beam artifact manifests as intensity fall-off along the longitudinal direction (the gantry rotation axis). This effect is inherent to circular cone-beam tomography due to the fact that a cone-beam dataset acquired from circular scanning fails to meet the data sufficiency condition for volume reconstruction. Upon observations of the intensity fall-off phenomenon associated with the FDK reconstruction of a ball phantom, we propose an empirical weight formula to compensate for the fall-off degradation. Specifically, a reciprocal cosine can be used to compensate the voxel values along longitudinal direction during three-dimensional backprojection reconstruction, in particular for boosting the values of voxels at positions with large cone angles. The intensity degradation within the z plane, albeit insignificant, can also be compensated by using the same weight formula through a parameter for radial distance dependence. Computer simulations and phantom experiments are presented to demonstrate the compensation effectiveness of the fall-off effect inherent in circular cone-beam tomography.

1. Introduction

Cone-beam tomography maximally exploits the conical shape of x-ray radiation emanating from a point source. Due to divergent projection in three-dimensional (3D) space, x rays impinging upon a detector cell can be described by a pyrel (pixel pyramid) [1,2]. For a rectilinear flat-panel detector, there are as many pyrels as the number of detector cells. These pyrels are not identical (due to obliqueness) and are spatially packed within a projection pyramid. Pyrel-based iterative cone-beam tomographic reconstruction can reduce cone-beam artifacts, however, its implementation requires an enormous amount of computation [2,3]. Practical cone-beam tomographic reconstruction is always based on analytic forms, where cone-beam artifacts stemming from divergent cone-beam projections and an ill-posed inverse problem are unavoidable.

A circular scan orbit defines a plane at the middle of the object domain, hence the name midplane. The x-ray projection and the tomographic reconstruction at the midplane can be well

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handled by fan-beam tomography [4]. During circular cone-beam scanning with a flat panel detector, the x-ray projections across the off-midplane regions manifest as ray and fan-beam nutations [5]. By converting the cone-beam data into radon data, a doughnutlike region is filled in within a 3D radon domain [6–8]. Due to the presence of a null region (doughnut hole), the 3D inverse Radon transform suffers from data insufficiency; that is the inverse problem associated with a circular cone-beam tomography fails to meet the data sufficiency condition for volume reconstruction [9,10]. From the viewpoint of voxel value reproduction, the conspicuous cone-beam artifacts manifest as an intensity fall-off along the longitudinal direction. At first sight, a large flat panel detector can cover a large object domain for cone-beam scanning, which is desirable for the utilization of the conical x-ray beam emanating from a point source. However, a large object domain encounters severe cone-beam artifacts, especially in the peripheral regions with a large longitudinal distance to the midplane (i.e., this is the large cone angle problem). Therefore, in practice, circular cone-beam tomography is usually limited to reconstructing an object domain within a small cone angle (typically $<15^\circ$).

A circular orbit is a basic but important scan pattern in cone-beam tomography [11]. It is the most desirable scan scheme in dynamic and functional medical imaging, such as perfusion, cardiac imaging, vascular imaging, head imaging, and breast imaging. The well-known Feldkamp–Davis–Kress (FDK) algorithm [12] is widely used for cone-beam volume reconstruction. The FDK algorithm and its variations can be implemented efficiently due to its one-dimensional (1D) filtration and parallel implementation [11,12]. The FDK algorithm falls into the framework of the well-known filtered backprojection (FBP) [4]. In practice, the cone-beam data array acquired by a flat panel detector are row-wisely filtered with a reconstruction filter and followed by 3D backprojection for volume reconstruction [13,14]. To reduce cone-beam artifacts, a spatial variant filtering mechanism [15] has been proposed, at the cost of more complicated implementations. Other pursuits include manipulating the 3D Radon domain (Radon domain filling, Radon transform, and inverse transform) and various Fourier-based formulas [16–20]. All these techniques can reduce the fall-off effects to some extent at the cost of more computations and spatial blurring [7,16]. The cone-beam artifacts can also be reduced by weighting the conjugate rays (two rays arriving at a reconstruction point from two opposite sources points on the scan circle) [11]. In this paper, we will adopt the FDK algorithm for efficient cone-beam reconstruction and propose an empirical weight formula for compensating the notorious cone-beam artifact: intensity fall-off along longitudinal direction.

2. Methodology

A. One-Dimensional Filtered Backprojection Structure of the FDK Algorithm

The conventional cone-beam scan adopts a circular orbit, as diagramed in Fig. 1. In the description of the cone-beam projection geometry, the following parameters are needed: gantry radius, cone angle (formed by detector height and point source), fan-beam angle (formed by detector width and the point source), and source-to-detector distance. The gantry rotation axis (z axis in Fig. 1) is defined as the longitudinal direction. As the gantry rotates on a circle, the cone-beam projections circumscribe a common region around the gantry center, referred to as the object domain. The plane passing the circular orbit is defined as the midplane (corresponding to $z = 0$) of the object domain. The planes crossing the object domain and parallel to the midplane are called off-midplanes, transverse planes, or z planes with $z \neq 0$. With a flat-panel detector of $M \times N$ cells, the cone-beam scan captures a sequence of projection images, as denoted by $\{P_\beta(p, q), \beta \in [0, 2\pi], p \in [1, 2, \dots, M], q \in [1, 2, \dots, N]\}$. The projections are labeled by the projection angle β , as illustrated in Fig. 1. With the cone-beam dataset $\{P_\beta(p, q)\}$, the FDK algorithm for cone-beam reconstruction can be expressed by [4]

$$\begin{aligned}
f(x, y, z) &= \frac{1}{2} \int_0^{2\pi} \frac{1}{U^2} \int_{-\infty}^{\infty} \frac{D}{\sqrt{D^2 + p^2 + \xi^2}} \\
&\times P_\beta(p, \xi) h\left(\frac{1}{U} - p\right) dp d\beta \\
t &= x \cos \beta + y \sin \beta, \\
s &= -x \sin \beta + y \cos \beta \\
\xi &= \frac{Dz}{D-s}, \quad U = \frac{D-s}{D} \\
h(t) &= \int_{-\infty}^{\infty} |\omega| \exp(i2\pi\omega t) d\omega,
\end{aligned} \tag{1}$$

where D denotes the gantry radius. The 1D filtration structure is reflected in the convolution with the 1D ramp filter $h(t)$, and the 3D backprojection is expressed by the integral with respect to the projection angle β . Due to the 1D filtration on detector rows (indexed by p in $P_\beta(p, q)$), the FDK algorithm is often interpreted in terms of tilted fan-beam tomography [4,5]. (As an illustration in Fig. 1, the tilted fan-beam plane is defined by a detector row and the point source, nutating along the scan circle.) In implementation, a projection image is filtered along detector rows, and the filtered data entries are then uniformly distributed over the ray paths connecting the detector cells and the point source, as described by 3D backprojection. With the filtered projection data, a voxel value is calculated by integrating over the entries conveyed by 3D backprojection rays. By regrid-ding the object domain, we can reconstruct the object domain (or a subvolume therein) by an arbitrary grid resolution [21].

B. Longitudinal Fall-Off Effect in Cone-Beam Tomography

Conceptually, cone-beam tomography is a technique of reproducing a 3D object in terms of a physical property (such as the x-ray attenuation coefficient). The reproduction exactness associated with the FDK algorithm in Eq. (1), in terms of voxel value, is non-uniformly distributed over the object domain. Specifically, only the midplane (at $z = 0$ in Fig. 1) can be exactly reproduced by the fan-beam tomography. The z planes ($|z| > 0$) suffer from position-dependent reproduction errors. In appearance, the voxel values fall off consistently as the voxel-to-midplane distance, z , increases, hence the name fall-off effect. Figure 2 shows the fall-off phenomenon of a ball object reconstruction by the FDK algorithm [12]. The central cross section ($y = 0$) of the ball is displayed in Fig. 2(a), and its image reconstructed by the FDK algorithm is shown in Fig. 2(b). The numerical profiles of the verticals are plotted in Figs. 2(a1) and (b1). Clearly, the intensity fall-off effect is pronounced in Figs. 2(b) and (b1).

Based upon the observations in Fig. 2(b1), the reconstruction exactness along a longitudinal direction can be described by a hatlike function [22], which assumes the maximum at the center and falls off towards both directions. Given a function describing the degradation behavior, the reconstruction exactness of the object domain can be described by a reconstruction exactness volume [22]. For simplicity, we suggest a cosine function of z distance to describe the longitudinal degradation as expressed by $\cos(cz/R)$ with $0 \leq cz/R < \pi/2$, where R is the gantry radius, thus z/R is a dimensionless quantity. The parameter c is reserved for adjusting the hatlike degradation shape. In practice, when scrutinizing a z plane at $|z| > 0$, we find that the reconstruction exactness therein suffers from somewhat intraplane nonuniformity as well.

C. Volume Reconstruction by Weighted Backprojection

For FBP-based cone-beam reconstruction in Eq. (1), the 3D backprojection assumes the same ray path as traveled by the forward projection. By introducing a weight factor for each backprojection, we can modify the FDK formula by

$$f(x, y, z) = \frac{1}{2} \int_0^{2\pi} \frac{W(x, y, z)}{U^2} \int_{-\infty}^{\infty} \frac{D}{\sqrt{D^2 + p^2 + \xi^2}} \times P(p, \xi; \beta) h\left(\frac{t}{U} - p\right) dp d\beta, \quad (2)$$

where $W(x, y, z)$ represents the weight factor for compensating the fall-off degradation; other notations are interpreted in the same way as in Eq. (1). Since the fall-off degradation can be modeled by a cosine function [22], we suggest a weight formula by a reciprocal cosine. For the fall-off effect along the longitudinal direction (across the z planes), the weight function should involve the variable z . Although the nonuniformity within a z plane is usually insignificant, we may account for it by another variable of radial distance (defined as the voxel-to-origin distance). As a result, we propose the weight formula by

$$W(x, y, z; c_1, c_2) = \frac{1}{\cos[c_1 z / (R - c_2 r)]} \quad \text{with} \quad r = \sqrt{x^2 + y^2 + z^2}, \quad (3)$$

where the parameters c_1 and c_2 are provided for accelerating or decelerating the z distance variable and the radial distance variable r , respectively, thereby adjusting the shape of the weight function. For hatlike description, the selection of parameters c_1 and c_2 should satisfy the following condition:

$$0 \leq c_1 z / (R - c_2 r) < \pi/2. \quad (4)$$

By setting the parameters c_1 and c_2 , some special cases are included as follows:

$$W(x, y, z; c_1, 0) = W(z; c_1) = \frac{1}{\cos(c_1 z / R)}, \quad (5a)$$

$$W(x, y, 0; c_1, c_2) \equiv 1, \quad (5b)$$

$$W(x, y, z; 0, c_2) \equiv 1. \quad (5c)$$

Equation (5a) represents the weight for the longitudinal fall-off effect only, Eq. (5b) represents no compensation for the midplane, and Eq. (5c) represents no compensation at all for the whole volume.

3. Simulations

We have suggested an empirical weight formula in Eq. (3) for the intensity loss compensation along the longitudinal direction (across z planes) as well as the radial direction (via the radial distance r). The two parameters, z and r , allow us to adjust the compensation efficacy, which can be evaluated in terms of the numerical errors between the compensated image and the original. In our computer simulations, we adopt the cone-beam tomography geometry with the parameter settings in Table 1.

By fixing $c_2 = 1$, we show the influence of the parameter $c_1 = \{0, 1, 1.3, 1.5\}$ in Fig. 3, which reveals the longitudinal fall-off effect (with respect to $|z|$). The vertical scanline profiles are plotted in Fig. 4, where the compensation efficacy is numerically characterized by the mean voxel value difference. It is noted that the parameter setting $\{c_1 = 1.5, c_2 = 1\}$ overcompensates the two peripheral regions.

In a similar way, by fixing $c_1 = 1$, we show the intensity loss compensations within a z plane with respect to the radial distance r , with $c_2 = \{0, 0.5, 1, 1.5\}$, in Fig. 5. The numerical profiles are provided in Fig. 6. It is seen that the fall-off effect can be compensated by adjusting parameter c_2 to some extent, but far from completeness and perfection.

Figure 7 shows the simulation results with the Shepp–Logan phantom. Specifically, Figs. 7(a) through 7(c) show a slice image of the original phantom, the FDK reconstruction by Eq. (1), and the weighted FDK reconstruction by Eq. (2). For closeup inspection and comparison, the profiles of the vertical scanlines in Figs. 7(a) through 7(d) are plotted in Fig. 7(d). It is seen that the image in Fig. 7(c) has a more uniform background than that in Fig. 7(b). The three small objects at the upper regions, which were dimly embedded in Fig. 7(b), now clearly emerge in Fig. 7(c).

4. Experiments

Cone-beam tomography can be used for breast volume imaging because the cone-beam volume scanning with a flat panel detector (e.g., $40 \times 30 \text{ cm}^2$ Paxscan4030CB, Varian Medical System Inc.) can totally cover a breast object. Figure 8 is a diagram of the geometry a cone-beam breast CT scanner. With a woman patient in prone poise, the pendant breast is scanned by a cone beam projection that assumes a half cone-angle of the conventional cone-beam geometry (see Fig. 1). The half cone-angle and half fan-angle values of the cone-beam projection geometry in Fig. 8 are $12^\circ (= \tan^{-1}(200/950))$ and $9^\circ (= \tan^{-1}(250/950))$, respectively. We conducted an experiment with a breast gel phantom on a cone-beam CT scanner [14]. With 300 projections, we reconstructed the phantom volume using Eq. (1), with the sagittal plane ($x = 0, z < 0$) shown in Fig. 9(a). By the reconstruction using Eq. (2), where the fall-off intensity was compensated with the formula in Eq. (3) with $c_1 = 1$ and $c_2 = 1$, we generated the sagittal image as shown in Fig. 9(b). The numerical profiles of the scanlines (marked by vertical white lines) in Figs. 9(a) and 9(b) are plotted in Fig. 9(c). This experiment demonstrates that the fall-off compensation formula is good for cone-beam volume imaging on low-contrast objects such as breast soft tissue.

5. Discussion

The longitudinal intensity fall-off phenomenon is the most conspicuous cone-beam artifact in circular cone-beam tomography. The compensation of the intensity fall-off effect boosts the voxel values and recovers signal from the suppressed objects (due to intensity loss) in the off-midplanes, thus facilitating volume segmentation, volume rendering, and slice image scrutiny over a longitudinal cross section by improving the intensity uniformity therein. However, this compensation cannot correct the longitudinal distortion, which manifests itself as a shape dilation towards both z directions [7]. From the viewpoint of image energy, the uniformity resulting from the fall-off compensation can be interpreted as adding more energy to the reconstructed image through the use of weights $W \geq 1$. Consequently, the image energy is no longer conserved after the intensity compensation.

6. Summary and Conclusion

Cone-beam tomographic reconstruction can be efficiently accomplished by the FDK algorithm. Due to the data insufficiency of a cone-beam dataset, the reconstructed volume suffers from

intensity fall-off degradation along the longitudinal direction (the gantry rotation axis). Instead of pursuing reduction of this artifact by developing sophisticated cone-beam algorithms, we suggest compensating the intensity fall-off effect through the use of an empirical formula. Upon observation, the fall-off degradation associated with circular cone-beam tomography can be described by a cosine function, so we select a reciprocal of the cosine, defined over the domain $[0, \pi/2)$, as the weighting function. In addition to considerable intensity fall-off in the longitudinal direction, we also observe a slight intensity fall-off within the z planes. Considering these two problems, we construct a weight formula that involves two parameters: longitudinal distance (voxel-to-midplane distance) and radial distance (voxel-to-origin distance). For specific cases, the two parameters can be fine tuned to optimize the compensation. As a result of the compensation, the objects embedded in the peripheral region (with large longitudinal distance) can be visualized in longitudinal cross sectional images (sagittal or coronal) in the same display window as used for the midplane. Nevertheless, the geometrical distortion (dilation along the z direction) associated with the cone-beam tomography persists. In conclusion, by compensating the intensity fall-off effect, we expediently solve the longitudinal intensity loss problem associated with cone-beam tomography. This technique facilitates global segmentation, visualization, and analysis of the reconstructed volume data due to the uneven background compensation, thus extending the applications of the FDK algorithm to cone-beam tomography with large cone angles. It should be pointed out that the image energy (summation of image intensity values) of the compensated reconstruction is no longer conserved due to the intensity boosting along the longitudinal direction. Although the uneven background is compensated, the cone-beam artifacts of geometric distortions at large cone angles persist, which may be handled by computer graphic techniques.

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References

1. Chen Z, Ning R, Conover D. Accurate perspective projection calculation using a pixel-pyramid model for iterative cone-beam reconstruction. *Proc SPIE* 2003;5030:728–39.
2. Chen Z, Ning R. Pixel-pyramid model for divergent projection geometry. *Opt Eng* 2005;44:027002.
3. Mueller K, Yagel R, Wheller JJ. Anti-aliased three-dimensional cone-beam reconstruction of low-contrast objects with algebraic methods. *IEEE Trans Med Imaging* 1999;18:519–537. [PubMed: 10463130]
4. Kak, AC.; Slaney, M. *Principles of Computerized Tomographic Imaging*. IEEE; 1999.
5. Turbell, H. Ph.D. thesis. Linköping University; 2001. Cone-beam reconstruction using filtered backprojection.
6. Wang X, Ning R. A cone-beam reconstruction algorithm for circle-plus-arc data-acquisition geometry. *IEEE Trans Med Imag* 1999;18:815–824.
7. Chen Z, Ning R, Yu Y, Conover D. 3D PSF characterization of circle-plus-arc cone-beam tomography. *Proc SPIE* 2005;5745:664–675.
8. Grangeat, P. Mathematical framework of cone beam 3D reconstruction via the first derivative of the Radon transform. In: Herman, GT.; Louis, AK.; Natterer, F., editors. *Mathematical Methods in Tomography*, Vol. 1497 of *Lecture Notes in Mathematics*. Springer-Verlag; 1991. p. 66-97.
9. Tuy HK. An inversion formula for cone-beam reconstruction. *SIAM J Appl Math* 1983;43:546–552.
10. Zeng GL, Clack R, Gullberg GT. Implementation of Tuy's cone-beam inversion formula. *Phys Med Biol* 1994;39:493–507. [PubMed: 15551594]
11. Tang X, Hsieh J, Hagiwara A, Nilsen R, Thibault J, Drapkin E. A three-dimensional weighted cone beam filtered backprojection (CB-FBP) algorithm for image reconstruction in volumetric CT under a circular source trajectory. *Phys Med Biol* 2005;50:3889–905. [PubMed: 16077234]

12. Feldkamp LA, Davis LC, Kress JW. Practical cone-beam algorithm. *J Opt Soc Am A* 1984;1:612–619.
13. Ning R, Tang X, Conover D, Yu R. Flat panel detector-based cone beam computed tomography with a circle-plus-two arcs data acquisition orbit: preliminary phantom study. *Med Phys* 2003;30:1694–1705. [PubMed: 12906186]
14. Ning R, Chen B, Yu R, Conover D, Tang X, Ning Y. Flat panel detector-based cone-beam volume CT angiography imaging: system evaluation. *IEEE Trans Med Imag* 2000;19:9494–963.
15. Defrise M, Clack R. A cone-beam reconstruction algorithm using shift-variant filtering and cone-beam backprojection. *IEEE Trans Med Imag* 1994;13:186–195.
16. Chen Z, Ning R. Filling the Radon domain of computed tomography by local convex combination. *Appl Opt* 2003;42:7043–7051. [PubMed: 14674649]
17. Axelsson C, Danielsson P. Three-dimensional reconstruction from cone-beam data in $O(N^3 \log N)$ time. *Phys Med Biol* 1994;39:477–491. [PubMed: 15551593]
18. Schaller S, Flohr T, Steffen P. An efficient Fourier method for 3D Radon inversion in exact cone-beam CT reconstruction. *IEEE Trans Med Imag* 1998;17:244–250.
19. Dusaussoy NJ. Voir: A volumetric image reconstruction algorithm based on Fourier techniques for inversion of the 3-D Radon transform. *IEEE Trans Image Process* 1996;5:121–131. [PubMed: 18285095]
20. Gottlieb D, Gustafsson B, Forssen P. On the direct Fourier method for computer tomography. *IEEE Trans Med Imag* 2000;19:223–232.
21. Chen Z, Ning R. Super-gridded cone-beam reconstruction and its application to point-spread function calculation. *Appl Opt* 2005;44:4615–4624. [PubMed: 16075872]
22. Chen Z, Ning R. Volume fusion of two-circular-orbits cone-beam tomography. *Appl Opt* 2006;45:5960–5966. [PubMed: 16926884]

Fig. 1. Diagram of the cone-beam scanner. The gantry orbit plane defines the midplane of the object domain, and the gantry rotation axis (z axis) defines the longitudinal direction. The cone-beam projections are indexed by the projection angle β . Each detector row and the point source (S) define a fan-beam plane.

Fig. 2.

Illustration of the longitudinal intensity fall-off effect in cone-beam tomography. (a) Cross-sectional image of a ball object, (b) the FDK-reconstructed image. The profiles of the longitudinal scanlines in (a) and (b) are plotted in (a1) and (b1), respectively.

Fig. 3.

Demonstration of the intensity fall-off compensation with different longitudinal degradations. The ball cross section ($y = 0$) was reconstructed by Eq. (2) with different parameter settings: (a) $c_1 = 0$, (b) $c_1 = 1$, (c) $c_1 = 1.3$, (d) $c_1 = 1.5$. All the images are displayed by window [0.95, 1.05]. The vertical lines are added to extract profiles (as shown in Fig. 5).

Fig. 4. Profiles of the scanlines in Fig. 4. The numbers in the figure legend show the mean voxel errors with respect to the original profile.

Fig. 5.

Influence of parameter c_2 in Eq. (3) on the intensity fall-off compensation. (a) $c_2 = 0$, (b) $c_2 = 0.5$, (c) $c_2 = 1$, (d) $c_2 = 1.5$. All the images are displayed by a window $[0.95, 1.05]$. The scanline profiles from these images are displayed in Fig. 7.

Fig. 6. Plots of the scanline profiles in Fig. 6. The numbers in the figure legend show the mean voxel errors with respect to the original profile.

Fig. 7. Simulation with a 3D Shepp–Logan phantom. (a) Slice image of the phantom, (b) image reconstructed by Eq. (1), (c) image reconstructed by Eq. (2), and (d) profiles of the vertical scanlines.

Fig. 8.
Geometry of a cone-beam CT scanner for breast imaging. The numbers are given in millimeters.

Fig. 9.

Experiment results with a gel breast phantom. (a) Sagittal plane reconstructed with FDK reconstruction, (b) the same sagittal plane with compensation reconstruction, (c) plots of the scanlines. Two bright spots are embedded objects in the phantom. The vertical white lines in (a) and (b) are added for marking the scanlines.

Table 1

Parameter Settings for Simulations

Parameter	Value
Gantry radius	50 cm
Source–detector distance	80 cm
Detector size (active area)	$52 \times 52 \text{ cm}^2$
Detector array (matrix)	256×256
Number of projections	360
Half cone-angle, half fan-angle	$18^\circ (= \text{atan}(25/80))$
Reconstruction volume (grid)	$256 \times 256 \times 256$
Voxel size	$1 \times 1 \times 1 \text{ mm}^3$