A BPF-FBP tandem algorithm for image reconstruction in reverse helical cone-beam CT

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Purpose: Reverse helical cone-beam computed tomography (CBCT) is a scanning configuration for potential applications in image-guided radiation therapy in which an accurate anatomic image of the patient is needed for image-guidance procedures. The authors previously developed an algorithm for image reconstruction from nontruncated data of an object that is completely within the reverse helix. The purpose of this work is to develop an image reconstruction approach for reverse helical CBCT of a long object that extends out of the reverse helix and therefore constitutes data truncation.

Methods: The proposed approach comprises of two reconstruction steps. In the first step, a chord-based backprojection-filtration (BPF) algorithm reconstructs a volumetric image of an object from the original cone-beam data. Because there exists a chordless region in the middle of the reverse helix, the image obtained in the first step contains an unreconstructed central-gap region. In the second step, the gap region is reconstructed by use of a Park–Noo-formula-based filtered-back-projection (FBP) algorithm from the modified cone-beam data obtained by subtracting from the original cone-beam data the reprojection of the image reconstructed in the first step.

Results: The authors have performed numerical studies to validate the proposed approach in image reconstruction from reverse helical cone-beam data. The results confirm that the proposed approach can reconstruct accurate images of a long object without suffering from data-truncation artifacts or cone-angle artifacts.

Conclusions: They developed and validated a BPF-FBP tandem algorithm to reconstruct images of a long object from reverse helical cone-beam data. The chord-based BPF algorithm was utilized for converting the long-object problem into a short-object problem. The proposed approach is applicable to other scanning configurations such as reduced circular sinusoidal trajectories. © 2010 American Association of Physicists in Medicine. [DOI: [10.1118/1.3263618](https://doi.org/10.1118/1.3263618)]

Key words: cone-beam CT, image reconstruction, reverse helical, BPF, FBP

I. INTRODUCTION

Cone-beam computed tomography (CBCT) is one of the most popular modalities for image-guidance procedures such as patient setup and target localization in radiation therapy.^{1–3} Current technologies employed in clinical CBCT systems are based on a circular scanning geometry for data acquisition, and on algorithms such as the Feldkamp–Davis–Kress (FDK) algorithm⁴ for image reconstruction. Although images produced by these algorithms from circular cone-beam data are often acceptable for practical tasks, it is of interest to work toward enhanced image quality, thus leading to increased utility of CBCT in a variety of image-guided radiation therapy (IGRT) applications. Accurate images are par-

ticularly important in on-line adaptive radiation therapy, where CBCT images may be used for treatment replanning.^{5,6}

Although helical CBCT is well known to provide sufficient data for exact image reconstruction, a conventional helical CBCT with multiple turns would be difficult to achieve on an existing linear accelerator (LINAC) system because the gantry of a LINAC treatment system cannot rotate more than one turn in a single direction. Reverse helical CBCT, on the other hand, can be realized by reversing the gantry rotation after each turn while continuously translating the patient couch. Therefore, we have previously investigated reverse helical CBCT for potential use in IGRT and explored image-reconstruction algorithms for reverse helical CBCT.⁷

In the past decade, significant advances have been made in algorithm development for accurate image reconstruction in helical CBCT, including filtered-backprojection (FBP) algorithms^{8,9} and chord-based algorithms.^{10–12} These algorithms have also been extended to image reconstruction from general source trajectories.^{13,14} In a chord-based algorithm, the image is decomposed into chords each of which is a line segment connecting two points on a continuous source trajectory. The object image is obtained by reconstructing images on the chords intersecting the object. The chord-based algorithms can be applied to general source trajectories. One of the chord-based algorithms, which is referred to as the backprojection filtration (BPF) algorithm, can reconstruct accurate images within regions of interest (ROIs) from data containing truncations.¹⁵ However, because of the dependence of chord-based algorithms on the existence of chords, they cannot reconstruct images in chordless regions even though sufficient cone-beam data are provided.¹² For a reverse helical trajectory, it has been shown that there exists a chordless gap, which cannot be reconstructed by use of the chord-based BPF algorithm.⁷

The Pack–Noo formula relates a 3D inverse Radon transform to a FBP of the conebeam data and can be used for image reconstruction from data that satisfy Tuy’s data sufficiency condition for exact image reconstruction in CBCT.^{12,15} Based on the Pack–Noo formula, a useful approach was developed for image reconstruction in CBCT with an umbrella trajectory¹⁶ and with general, closed trajectories.¹⁷ In that approach, the volumetric image is decomposed into polygon planes, which are defined by the intersection of planes with the source trajectory, and the images within the polygon planes are reconstructed through backprojecting the filtered data onto the polygons. The direction of a polygon side, which is by definition a chord, is used as the filtering direction in the reconstruction. We have applied this approach to image reconstruction in CBCT with general, open source trajectories.^{7,18} Because the Pack–Noo-formula-based algorithm is not dependent on the existence of chords, it can reconstruct an image within the chordless region where chord-based algorithms cannot. However, this algorithm can accurately reconstruct images only from non-truncated data of a short object that is completely enclosed by the polygons for a reverse helical trajectory.

In practice, the imaged object, e.g., a patient, is usually too long to be confined within the trajectory in a reverse helical CBCT, thus constituting the data truncation problem of a long object. In an attempt to address this long object problem in the reverse helical scan, we propose a BPF-FBP tandem algorithm that utilizes advantages of each of the algorithms for image reconstruction in reverse helical CBCT. The key of the proposed tandem algorithm lies in the observation that the chord-based BPF algorithm can be utilized for converting a long-object problem into a short-object problem confined to the chordless middle gap, which can then be reconstructed by the use of the Pack–Noo-formula-based algorithm. Specifically, the chord-based BPF algorithm reconstructs parts of the object volume from the original cone-beam data first, and the Pack–Noo-formula-based FBP

algorithm reconstructs the middle gap region from the residual cone-beam data obtained by subtracting the reprojected image of the BPF-reconstructed image from the original data. We perform numerical studies in which the NCAT phantom¹⁹ is used to validate and demonstrate the proposed approach.

II. BPF-FBP TANDEM ALGORITHM

The proposed algorithm is composed of two reconstruction steps in sequence. The first reconstruction is conducted by the use of the chord-based BPF algorithm. The chord-based BPF algorithm can deal with the longitudinal data truncation that occurs in a patient scan, and an exact volumetric image can be obtained, although the image contains a middle gap where no chords are available. The reconstructed volume image is then reprojected to produce cone-beam data corresponding to the scanning configuration that was used for acquiring the original cone-beam data. Subtracting the reprojected cone-beam data from the original cone-beam data yields the cone-beam data for the parts that are not reconstructible by the chord-based BPF algorithm. The resulting cone-beam projection contains three regions: Top, middle, and bottom parts. The top and bottom parts of each cone-beam projection, which correspond to inherently unreconstructible regions of the object at the ends of the trajectory, are removed, and the middle portion is used for the second step reconstruction with the Pack–Noo-formula-based FBP algorithm. After the second step reconstruction, the final image is obtained by combining the two reconstructed images. The algorithm is illustrated in this section after introducing the reverse helical trajectory.

II.A. Reverse helical trajectory

A reverse helical trajectory $\vec{r}_0(\lambda)$ can be expressed as

$$\vec{r}_0(\lambda) = \left(R \cos((-1)^n \lambda), R \sin((-1)^n \lambda), \frac{h}{2\pi} \lambda \right),$$

$$\lambda \in [\lambda_s, \lambda_e], \quad (1)$$

where R denotes the distance from the source to the rotation axis, h is the helical pitch, and λ_s and λ_e are the starting and ending angles of the trajectory, respectively. Integer n indicates the n th turn and can be written in terms of a floor function “ $\lfloor \cdot \rfloor$ ” as

$$n = \left\lfloor \frac{\lambda}{2\pi} \right\rfloor + 1. \quad (2)$$

Without loss of generality, we have assumed that the turn with $n=0$ in the reverse helical trajectory is a turn in the conventional helical trajectory. For a two-turn reverse helical trajectory specified by $\lambda \in [\lambda_s, \lambda_e] = [-2\pi, 2\pi]$, as shown in Fig. 1(a), we have

$$\vec{r}_0(\lambda) = \left(R \cos \lambda, R \sin \lambda, \frac{h}{2\pi} \lambda \right), \quad \lambda \in [-2\pi, 0),$$

$$\vec{r}_0(\lambda) = \left(R \cos(-\lambda), R \sin(-\lambda), \frac{h}{2\pi} \lambda \right), \quad \lambda \in [0, 2\pi]. \quad (3)$$

FIG. 1. (a) A two-turn reverse helical trajectory in the coordinate system $\{x, y, z\}$ that is fixed to the imaged object, (b) one-turn and two-turn chords (dotted lines), and (c) a polygon (dotted trapezoid) that is parallel to the yz plane are shown.

We focus only on two-turn reverse helical trajectory in this work, but extension to multiple turns is straightforward. Let $f(\vec{r})$ be an object function with a cylindrical support that is transversely contained within the reverse helical trajectory but extends beyond the reverse helix longitudinally. The cone-beam projection from the source point $\vec{r}_0(\lambda)$ can be written as

$$g(\lambda, \hat{\theta}) = \int_0^\infty dt f(\vec{r}_0(\lambda) + t\hat{\theta}), \quad (4)$$

where $\hat{\theta}$ denotes the direction of x-ray transform of $f(\vec{r})$ from $\vec{r}_0(\lambda)$.

II.B. First-step (BPF) reconstruction

We first reconstruct the image from the acquired cone-beam data by the use of the chord-based BPF algorithm. Because the reconstruction formula has been explained in detail elsewhere,^{10,13} we summarize only briefly the essential steps of the BPF algorithm. A chord can be specified by λ_1 and λ_2 on a given source trajectory $\vec{r}_0(\lambda)$. Note that λ_1 and λ_2 can be selected from either $[-2\pi, 0]$ or $[0, 2\pi]$, or from both, where the former chord is referred to as a one-turn chord and the latter chord as a two-turn chord. Example chords are illustrated in Fig. 1(b) as dotted lines. Let $\hat{e}_c$ denote the direction of the chord, and let x_c indicate the coordinate of a point on the chord. The fixed coordinate and the chord coordinate of the point are related through

$$\vec{r} = \frac{1}{2}[\vec{r}_0(\lambda_1) + \vec{r}_0(\lambda_2)] + x_c \hat{e}_c, \quad x_c \in [-l, l], \quad (5)$$

where $l = \frac{1}{2}\|\vec{r}_0(\lambda_2) - \vec{r}_0(\lambda_1)\|$ denotes one-half of the chord length. Using $f_c(x_c, \lambda_1, \lambda_2)$ to denote the object function on the chord, we have

$$f(\vec{r}) = f_c(x_c, \lambda_1, \lambda_2), \quad (6)$$

where $\vec{r}$ and x_c are related through Eq. (5).

From cone-beam data $g(\lambda, \hat{\theta})$, one can compute a back-projection image onto the chord as

$$g_B(x_c, \lambda_1, \lambda_2) = \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\|\vec{r} - \vec{r}_0(\lambda)\|} \left[\frac{\partial}{\partial \lambda} g(\lambda, \hat{\theta}) \right]_{\hat{\theta}}. \quad (7)$$

Based on the backprojection image g_B on the chord, the object function $f_c(x_c, \lambda_1, \lambda_2)$ on the chord is reconstructed by

FIG. 2. Schematic of the projection of the unreconstructed portions of the object is shown. g_m represents the projection of the middle portion (f_m) of the unreconstructed object.

$$\begin{aligned} \bar{f}_c(x_c, \lambda_1, \lambda_2) &= \frac{1}{2\pi^2} \frac{1}{\sqrt{(x_{c2} - x_c)(x_c - x_{c1})}} \\ &\times \left[\int_{x_{c1}}^{x_{c2}} \frac{dx'_c}{x_c - x'_c} \sqrt{(x_{c2} - x'_c)(x'_c - x_{c1})} \right. \\ &\quad \left. \times g_B(x'_c, \lambda_1, \lambda_2) + C \right], \end{aligned} \quad (8)$$

with $x_c \in [x_{c1}, x_{c2}]$ where x_{c1} and x_{c2} are chosen to cover the object, and the constant C is given by

$$C = 2\pi \int_{x_{c1}}^{x_{c2}} f_c(x_c, \lambda_1, \lambda_2) dx_c = 2\pi g(\lambda_1, \hat{e}_c). \quad (9)$$

Throughout this paper, we will use an upper bar to represent the reconstructed image function. When the detector is large enough to accommodate the projections of all the chords that intersect with the object for a given source position, the BPF algorithm can reconstruct exactly images of the object on the chords. The reconstructed chord images are then converted into Cartesian images by means of interpolation. This volumetric image will contain a middle gap because no chords pass through the gap region.

II.C. Reprojection and subtraction

Let $\bar{f}(\vec{r})$ denote the volume image reconstructed by the use of the BPF algorithm in the first step. Reprojection of the reconstructed image can be conducted by calculating projections of $\bar{f}(\vec{r})$ for the given scanning geometry as

$$g_r(\lambda, \hat{\theta}) = \int_0^\infty dt \bar{f}(\vec{r}_0(\lambda) + t\hat{\theta}). \quad (10)$$

Subtracting the reprojections from the original projections, one can obtain the projections of the unreconstructed portions of the object as

$$\begin{aligned} g_s(\lambda, \hat{\theta}) &= g(\lambda, \hat{\theta}) - g_r(\lambda, \hat{\theta}) \\ &= \int_0^\infty dt \{f(\vec{r}_0(\lambda) + t\hat{\theta}) - \bar{f}(\vec{r}_0(\lambda) + t\hat{\theta})\}. \end{aligned} \quad (11)$$

The unreconstructed portions of the object are composed of three distinct portions: Top, middle, and bottom, as shown in Fig. 2. Under the condition that there is no ray passing

through two or more of these portions from any source position, it is feasible to separate the projection of the middle portion from the remaining parts because the supports of the three unreconstructed portions of the object are disconnected in the projection at each view. For given scanning parameters such as the detector size and the source-to-axis distance, the helical pitch can be chosen to satisfy the above condition. Let $f_m(\vec{r})$ denote the middle part of the unreconstructed portions of the object. Its projection $g_m(\lambda, \hat{\theta})$ in Fig. 2 can be written as

$$g_m(\lambda, \hat{\theta}) = \int_0^\infty dt f_m(\vec{r}_0(\lambda) + t\hat{\theta}). \quad (12)$$

II.D. Second-step (FBP) reconstruction

The second step reconstructs the middle portion of the object, which was not reconstructible by use of the chord-based BPF algorithm, from data $g_m(\lambda, \hat{\theta})$. Consider a polygon plane in Fig. 1(c) that is parallel to the yz plane and that intersects the reverse helical trajectory at four distinct points $\vec{r}_0(\lambda_i)$, where $i=1, \dots, 4$ and $-2\pi \leq \lambda_1 < \lambda_2 < \dots < \lambda_4 \leq 2\pi$. An exact image reconstruction within the polygon for a two-turn reverse helical trajectory can be obtained as⁷

$$\begin{aligned} \bar{f}_m(\vec{r}) &= \frac{1}{2} \sum_{i=1}^4 \mathcal{K}(\vec{r}, \hat{e}_i, \lambda_i, \lambda_{i+1}) \\ &= \frac{1}{2} \sum_{i=1}^3 \mathcal{K}(\vec{r}, \hat{e}_i, \lambda_i, \lambda_{i+1}) - \frac{1}{2} \mathcal{K}(\vec{r}, \hat{e}_4, \lambda_1, \lambda_4), \end{aligned} \quad (13)$$

where $\hat{e}_i$ denotes the direction of a polygon side, i.e.,

$$\hat{e}_i = \frac{\vec{r}_0(\lambda_{i+1}) - \vec{r}_0(\lambda_i)}{\|\vec{r}_0(\lambda_{i+1}) - \vec{r}_0(\lambda_i)\|}, \quad (14)$$

where λ_i and λ_{i+1} denote the starting and ending points of the i th subsegment trajectory corresponding to the i th side of the polygon, and $i=1, 2, 3$. For $i=4$, we introduce λ_5 such that $\vec{r}_0(\lambda_5) = \vec{r}_0(\lambda_1)$. For each of the subsegment trajectories, the Pack–Noo formula provides a reconstruction as

$$\mathcal{K}(\vec{r}, \hat{e}_i, \lambda_i, \lambda_{i+1}) = -\frac{1}{2\pi^2} \int_{\lambda_i}^{\lambda_{i+1}} d\lambda \frac{1}{\|\vec{r} - \vec{r}_0(\lambda)\|} g_F(\lambda, \vec{r}, \hat{e}_i), \quad (15)$$

where $g_F(\lambda, \vec{r}, \hat{e}_i)$ represents the filtered backprojection of the projections of the middle portion of the object, given by

$$g_F(\lambda, \vec{r}, \hat{e}_i) = \int_{-\pi}^{\pi} d\gamma \frac{1}{\sin \gamma} \frac{\partial g_m(\lambda, \hat{\theta}(\lambda, \vec{r}, \hat{e}_i, \gamma))}{\partial \lambda}, \quad (16)$$

$$\hat{\theta}(\lambda, \vec{r}, \hat{e}_i, \gamma) = \cos \gamma \hat{\alpha}(\lambda, \vec{r}) + \sin \gamma \hat{\beta}(\lambda, \vec{r}, \hat{e}_i), \quad (17)$$

$$\hat{\beta}(\lambda, \vec{r}, \hat{e}_i) = \frac{\hat{e}_i - (\hat{e}_i \cdot \hat{\alpha}(\lambda, \vec{r})) \hat{\alpha}(\lambda, \vec{r})}{\|\hat{e}_i - (\hat{e}_i \cdot \hat{\alpha}(\lambda, \vec{r})) \hat{\alpha}(\lambda, \vec{r})\|}, \quad (18)$$

and

$$\hat{\alpha}(\lambda, \vec{r}) = \frac{\vec{r} - \vec{r}_0(\lambda)}{\|\vec{r} - \vec{r}_0(\lambda)\|}. \quad (19)$$

The integration over γ in Eq. (16) indicates a filtering process over the data derivative, while the filtering direction is determined by a preselected unit vector $\hat{e}_i$ specified by Eq. (14) for the corresponding subsegment trajectory. For the imaging problem of practical interest, we show in the Appendix that the intersection of the middle portion of the object $f_m(\vec{r})$ and a polygon plane is enclosed completely within the polygon.

II.E. Image combination

Finally, combining $\bar{f}(\vec{r})$ and $\bar{f}_m(\vec{r})$ yields the image of the object without a middle gap and without suffering from the long-object data-truncation artifacts. A 3D mask matrix W_c , which has digital elements—either 0 or 1—and the same size as the reconstructed images, can be generated so that the voxels intersected by chords have the value of 1 and the others 0. The image combination can then be done by calculating $W_c \cdot \bar{f}(\vec{r}) + (\bar{I} - W_c) \cdot \bar{f}_m(\vec{r})$, where matrix multiplication is defined as element by element multiplication and $\bar{I}$ represents a 3D matrix of all 1's with the same array size of W_c .

II.F. Use of reduced $\bar{f}(\vec{r})$

Because of the discrete nature of image reconstruction, a seam line artifact at the boundaries of the reconstructed volume images by the two algorithms may occur in the combined images. Let the reconstructed volume image by the chord-based BPF algorithm $\bar{f}(\vec{r})$ be renamed as $\bar{f}_0(\vec{r})$. One can easily create $\bar{f}_1(\vec{r})$ by removing some portion of $\bar{f}_0(\vec{r})$ to ensure that the image reconstructed by the second algorithm, $\bar{f}_m(\vec{r})$, and $\bar{f}_0(\vec{r})$ have overlap regions near the boundaries. Since we know that the polygons always cover more than the middle gap portion of the object image, we can easily find such a reduced volume image $\bar{f}_1(\vec{r})$. Therefore, in the second-step reconstruction, $\bar{f}_1(\vec{r})$ will be used for $\bar{f}(\vec{r})$, and the resulting $\bar{f}_m(\vec{r})$ will be the middle gap portion of $\bar{f}_1(\vec{r})$. Image combination accordingly is given by $W_c \cdot \bar{f}_0(\vec{r}) + (\bar{I} - W_c) \cdot \bar{f}_m(\vec{r})$. The work flow of the proposed tandem approach is summarized in Fig. 3.

III. NUMERICAL EXPERIMENTS

We perform below numerical studies to demonstrate and validate the proposed method for image reconstruction of a long object from its reverse helical cone-beam data. In the studies, we used the abdomen portion of the NCAT phantom as the imaged object. Currently, the circular trajectory is used for acquiring practical cone-beam data, and the FDK algorithm is used for image reconstruction. Therefore, for comparison, a circular cone-beam scan has also been simulated for which images were reconstructed by use of the FDK algorithm. A virtual, flat panel detector at the isocenter was used in the numerical study. The scanning parameters are

FIG. 3. Flowchart of the proposed tandem algorithm. The chord-based algorithm and the Pack–Noo-formula-based algorithm are used in tandem to reconstruct distinct portions of the object image, and the reconstructed images are combined at the end.

summarized in Table I. Images were reconstructed in 256^3 cubic voxels with a total side length of 40 cm.

IV. RESULTS

The simulation results are presented below according to the steps described in Sec. II.

IV.A. First-step BPF reconstruction

In Fig. 4, we show images $\bar{f}_0(\vec{r})$ reconstructed by using the chord-based BPF algorithm (top row) within 2D slices at (a) $y=0$, (b) $x=0$, and (c) $z=4.375$ cm. Display window of $[0.14, 0.18] \text{ cm}^{-1}$ is used for all the images throughout this paper unless specified. An off-midplane image is shown in (c) because the midplane belongs to the chordless region. The chordless gap is clearly visualized in the images. For the purpose of preventing a seam line artifact as discussed earlier, rectangular boxes parallel to the yz plane along the x axis have been chosen to extend outside the chordless region by at least one voxel, and the image within each box has been deleted to yield $\bar{f}_1(\vec{r})$ as shown in the bottom row of Fig. 4. For convenience, the heights of the rectangular boxes have been chosen in this example from a linear function of x for $x < 0$ and a constant for $x \geq 0$ considering the shape of $\bar{f}_0(\vec{r})$.

TABLE I. Scanning parameters used in the numerical study.

Parameter	Value
Source-to-axis distance	100 cm
Source-to-detector (virtual) distance	100 cm
Helical pitch	20 cm
No. of turns	2
Detector width	40 cm
Detector height	40 cm
Detector array size	512×512
No. of angular sampling/turn	1000

FIG. 4. Slice images of the NCAT phantom at (a) $y=0$, (b) $x=0$, and (c) $z=4.375$ cm reconstructed by the chord-based BPF algorithm (top row). Middle portion bounded by the dotted box was removed to yield the volume image $\bar{f}_1(\vec{r})$ for reprojection (bottom row).

IV.B. Reprojection and subtraction

In Fig. 5(a), we show the projection of the NCAT phantom at a particular view. Based on the scanning geometry, we reprojected the BPF-reconstructed image $\bar{f}_1(\vec{r})$ to generate g_r shown in Fig. 5(b). A ray-driven line integral has been used for computation of projection and reprojection in this work.²⁰ The subtracted projection $g_s = g - g_r$ is shown in Fig. 5(c) at the same view as that in Fig. 5(a), and finally the middle portion g_m is obtained by removing the top and bottom data, as shown in Fig. 5(d).

IV.C. Second-step FBP reconstruction

From the subtracted cone-beam data g_m , we use the Pack–Noo-formula-based FBP algorithm to reconstruct the image of the middle-gap region. In Fig. 6, we display the reconstructed images within 2D slices at (a) $y=0$, (b) $x=0$, and (c) $z=0$ cm.

Figure 7 shows the combined images in Figs. 4 and 6. To avoid the seam line artifact, we used the volumetric image $\bar{f}_0(\vec{r})$ reconstructed by the chord-based algorithm in the combination, and the combined images are shown in Fig. 7. To demonstrate the seam line artifact, the combined images of $\bar{f}_1(\vec{r})$ and $\bar{f}_m(\vec{r})$ are presented in Fig. 8.

IV.D. Noise addition

The two algorithms used in the proposed approach may have different numerical and noise transfer properties, and the reconstructed images from noisy data consequently can

FIG. 5. Projections of (a) the original phantom, (b) the BPF-reconstructed phantom (c) the subtracted projection, and (d) the midportion-only projection.

FIG. 6. Slice images of the NCAT phantom at (a) $y=0$, (b) $x=0$, and (c) $z=0$ cm reconstructed by the Pack–Noo-formula-based FBP algorithm.

appear differently. We conducted a preliminary simulation study using about 0.25% Gaussian noise in the projection. As can be seen in Fig. 9, the image-noise level in the region reconstructed by the Pack–Noo-formula-based algorithm appears to be higher than that in the region reconstructed by the chord-based algorithm. Although more data are used for the FBP image reconstruction at a point on polygon than those for the BPF at a point on chord, data from the segments of the trajectory constituting sides of the polygon are not redundant with respect to the reconstruction point. Therefore, the data noise propagate additively onto the image noise at the reconstruction point. Note the image noise will be reduced through a weighted averaging when redundant data are summed. Similar noise property and explanation to that can be found in the Ref. 12 for $n\pi$ -helical cone-beam CT reconstruction.

IV.E. Comparison studies

We present below additional images reconstructed by use of the Pack–Noo-formula-based FBP algorithm only and the FDK algorithm for comparison.

IV.E.1. Pack–Noo-formula-based algorithm alone

Because data filtering occurs in the detector plane, and because the projections of an imaged object may be truncated along some filtering directions, the Pack–Noo-formula-based algorithm can cause truncation artifacts in the reconstructed images if the original projections are directly used. Figure 10 shows the reconstructed images of the NCAT phantom by the use of the Pack–Noo-formula-based algorithm from the original projections. Shading, deformation, and streaks are observed as truncation image artifacts.

IV.E.2. FDK algorithm

As the motivation of the work was to avoid cone-beam artifacts present in the circular CBCT images, we display for

FIG. 7. Slice images of the combination of the BPF-reconstructed image $\tilde{f}_0(\vec{r})$ and the FBP-reconstructed image $\tilde{f}_m(\vec{r})$ at (a) $x=0$, (b) $y=0$, and (c) $z=4.375$ cm.

FIG. 8. Slice images of the combination of the reduced, BPF-reconstructed image $\tilde{f}_1(\vec{r})$ and the FBP-reconstructed image $\tilde{f}_m(\vec{r})$ at (a) $x=0$, (b) $y=0$, and (c) $z=4.375$ cm. The seam line artifacts are observed.

comparison the true phantom images, circular CBCT images reconstructed by the FDK algorithm, and the reverse helical CBCT images reconstructed by the proposed approach in Fig. 11. Line profiles are shown in Fig. 12 for quantitative demonstration of the difference in the images. Intensity drop along the longitudinal direction, streaks, and image deformation are typical circular cone-beam image artifacts,^{21–23} streaks and image deformations appear relatively strong, but the intensity drop is not as noticeable partly because of the cylindrical shape, which is relatively uniform along the z axis, of the imaged object.

V. DISCUSSION

We previously developed an algorithm for image reconstruction from nontruncated data of a short object that is completely within the reverse helix.⁷ However, the algorithm cannot be directly used for reconstructing accurate images of a long object that extends out of the polygons for a reverse helical trajectory thereby causing data-truncation problem. Here, we have proposed an image-reconstruction approach for reverse helical CBCT, which is composed of chord-based BPF reconstruction and Pack–Noo-formula-based reconstruction steps in tandem, to address this long object problem. Through numerical studies, the proposed algorithm was validated for accurate image reconstruction of a long object from reverse helical cone-beam data.

In addition to data noise, additional factors can affect the accuracy of the images reconstructed by the proposed approach. In the reprojection step particularly, inconsistency of the data compared to the original projection, which is based on a continuous imaged object, can result from the discretized image and a method for calculation of the line integral such as the ray-driven method. This inconsistency can be minimized by increasing the image-matrix size and improving the line integral computation by averaging more rays

FIG. 9. Slice images of the combined, reconstructed NCAT phantom from 0.25% Gaussian noise in the projection at (a) $y=0$, (b) $x=0$, and (c) $z=4.375$ cm. The display window of $[0.1, 0.256] \text{ cm}^{-1}$ was used for better visualization.

FIG. 10. Slice images of the combined, reconstructed NCAT phantom at (a) $x=0$, (b) $y=0$, and (c) $z=4.375$ cm from the original projections g 's.

per detector pixel. Moreover, the data affected by this inconsistency are limited to the projection data corresponding to rays that pass through both the reconstructible and unreconstructible regions of the chord-based BPF algorithm. Physical factors such as scatter and beam hardening can also affect the accuracy of the reprojected data. However, the physical factors can be corrected, we believe, by the use of the existing correction methods.

Although only a two-turn reverse helical trajectory has been studied in this work, it is straightforward to extend this approach for multiple-turn reverse helical CBCT to increase scan volume along the longitudinal direction. The proposed algorithm allows accurate image reconstruction of a long object from reverse helical cone-beam data, and it may be applicable to other scanning configurations such as reduced circular sinusoidal trajectories.¹⁸

There may be several challenges in the proposed approach for practical applications in IGRT. Imaging radiation dose, although it should be discussed in conjunction with the image quality, can be higher than that from conventional circular scans. Computation time, although we believe is not a real challenge considering recent progress in the accelerated

FIG. 11. Slice images at $y=0$ (first column), $x=0$ (second column), and $z=12.8$ (third column) cm of the original NCAT phantom (top row), of the FDK-reconstructed phantom (middle row) and of the tandem algorithm reconstructed phantom (bottom row).

FIG. 12. Line profiles at (a) $x=0$, $y=0$ and at (b) $y=0$, $z=12.8$ cm of the true phantom (solid) of the FDK-reconstructed phantom (dotted) and of the BPF-FBP-reconstructed phantom (dashed).

computation techniques such as the graphic-processor-unit acceleration technique,²⁴ can be relatively more demanding compared to the existing one-shot algorithms for circular CBCT. More investigation is needed on the noise property of the two algorithms in the proposed approach for improving noise uniformity. Methods can be developed to improve the noise characteristics at the algorithm levels such as using data redundancy,²⁵ and denoising methods may be devised either in a prereconstruction or a postreconstruction fashion to achieve a uniform image noise level between the regions reconstructed by the two algorithms.

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APPENDIX: POLYGON AND CHORDLESS REGION

We show below that the chordless middle-gap portion of the object is always bounded by polygons that are parallel to the yz plane. Specifically, we assume that the size of the field of view (FOV) is smaller than the polygon widths, which is valid for potential clinical scanning configurations. Considering the symmetry of the trajectory with respect to the xy plane, it is sufficient to show that the reconstructible volume with the top helix (bottom helix) by the chord-based algorithm encloses the top sides (bottom sides) of the polygons. Without loss of generality, we focus on the top helix. For a given polygon parallel to yz plane, we can always find the two vertices on the trajectory, A and B , that define the top-side of the polygon, as shown in Fig. 13(a).

We consider a source point Q_1 at $\lambda=0$ on the trajectory and possible chords from that point. Because vertices A and B also lie on the trajectory, Q_1A and Q_1B are also chords. The top helix in Fig. 13(a) is projected to make a concave

FIG. 13. (a) Schematic of the top helix, the top side of a polygon (dotted line connecting A and B), and some chords branching from the point of $\lambda=0$ are shown. (b) Projection of the helix, the top side of the polygon, and the chords on to the detector plane from the source position Q_1 and Q_2 are shown. We assumed a virtual detector that lies in the polygon plane for illustration here. The points represented by primed letters depict the projection (open circles from the source position Q_2 , and solid squares from Q_1) on to the detector of the unprimed ones in space. The shaded region within the FOV in (b) corresponds to the projection of the reconstructible volume by the chord-based algorithm. The projection of the top side of the polygon is completely enclosed by the shaded region within the FOV. Considering the bottom helix similarly, we can claim the unreconstructible volume by the chord-based algorithm is bounded by the polygon.

curve, as shown in Fig. 13(b). Because of the curvature of the trajectory projection on the detector, the chord-surface is placed beneath the polygon top side.

Now let us consider a source point Q_2 at $\lambda=2\pi$ on the trajectory. Similarly, the top helix in Fig. 13(a) is projected to make a convex curve as shown in Fig. 13(b). Because of the curvature of the trajectory projection on the detector, the chord surface is now placed above the polygon top-side. Since the volume between these chord surfaces is completely filled by the additional available chords of the top helix,¹⁰ the top side of the polygon lies within the reconstructible volume by the chord-based algorithm. In other words, the reconstructible region by the chord-based algorithm always extends below the top side of the polygon within the FOV.

By the same token, the bottom side of a polygon lies within the reconstructible volume with the bottom helix by the chord-based algorithm. Therefore, considering both the top and bottom helices, one can claim the chordless middle-gap portion of the reconstructed image by the use of the chord-based BPF algorithm is always bounded by the polygons.

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