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Impact of Relative Residence Times in Highly Distinct Environments on the Distribution of Heavy Drinkers

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Abstract

Alcohol consumption is a function of social dynamics, environmental contexts, individuals' preferences and family history. Empirical surveys have focused primarily on identification of risk factors for high-level drinking but have done little to clarify the underlying mechanisms at work. Also, there have been few attempts to apply nonlinear dynamics to the study of these mechanisms and processes at the population level. A simple framework where drinking is modeled as a socially contagious process in low- and high-risk connected environments is introduced. Individuals are classified as light, moderate (assumed mobile), and heavy drinkers. Moderate drinkers provide the link between both environments, that is, they are assumed to be the only individuals drinking in both settings. The focus here is on the effect of moderate drinkers, measured by the proportion of their time spent in “low-” versus “high-” risk drinking environments, on the distribution of drinkers.

A simple model within our contact framework predicts that if the relative *residence times* of moderate drinkers is distributed randomly between low- and high-risk environments then the proportion of heavy drinkers is likely to be higher than expected. However, the full story even in a highly simplified setting is not so simple because “strong” local social mixing tends to increase high-risk drinking on its own. *High* levels of social interaction between light and moderate drinkers in low-risk environments can diminish the importance of the distribution of relative drinking times on the prevalence of heavy drinking.

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Keywords

Social Influence; Mathematical Models; Drinking Environments; Drinking Patterns; College Students; Residence Times

1 Introduction

Approximately 40% of US college students engage in “binge” drinking, defined as five or more drinks in a row for males and four or more for females [1]. Close to one third meet the Diagnostic and Statistical Manual-IV (DSM-IV) diagnostic criteria for abuse and 6% for dependence [2]. The availability of alcohol around college campuses seems strongly linked to drinking risks [3]. Certain living arrangements seem to protect college students from heavy drinking (e.g., living in parents’ homes) [4], while high-risk freshmen tend to group themselves into high-risk drinking environments upon their enrollment in college [5]. Increasing the time spent in high-risk environments reinforces the use of alcohol and increases the likelihood of engaging in risky drinking.

The preponderance of problems reported by students occur among “moderate drinkers” (i.e., those consuming 2–3 drinks per occasion) [6]. Thus, attention must be paid to the context in which less extreme forms of drinking occurs. Special emphasis has to be placed on the contextual conditions under which the drinking of moderate drinkers becomes “risky” (increasing the probability of the occurrence of alcohol-related problems and/or increased levels of drinking). Within the micro-environments in which college drinkers use alcohol, the factors most often identified as primary influences upon drinking are cognitive variables related to social norms [7] and alcohol expectancies [8,9]. The influence of family diminishes dramatically during this time. Drinking levels of college students are most affected by the drinking patterns of their peers and strongly impacted by positive and negative expectancies about drinking outcomes [7,10,11].

Social influences and expectancy effects are context dependent, that is, they differ between settings [9]. For college students, these settings are extremely heterogeneous. For example, student-living arrangements range from those in which alcohol is nominally banned (e.g., university dormitories) through partially regulated environments (e.g., fraternities and apartment complexes) to entirely unregulated settings (e.g., cooperatives and private apartments). Since drinking patterns segregate across the residences of drinkers, new residents will be exposed to different influences and pressures depending on where they live (e.g., a fraternity versus a university dormitory) [6]. Drinking patterns also segregate across different venues for use, with, for example, off-campus parties being the preferred venue for becoming intoxicated [12]. Thus, different places for drinking will also expose users to different influences and pressures for high-risk drinking, resulting in both levels of intoxication and alcohol-related problems being context specific [13,14]. The influence of such settings may also vary according to age. Underage students are more likely to drink in private than public settings (as well as to drink less often, but more per occasion) than students who have reached the legal drinking age [15]. Prior research has clearly identified shifting patterns of alcohol use that include high peak drinking levels among freshman and sophomore students, especially white males, that moderate as students progress through their junior and senior years as they reach the legal drinking age [6,16].

These observations lead us to consider two questions. First, how does one conceptualize the ecological dynamics relating choices of drinking places, and social influences in operation at those places, to drinking levels and their evolution over time? Second, what are the implications of these dynamics for high-risk drinking among various groups including college students?

While the current paper does not purport to provide a comprehensive answer to either of these questions it does provide a *framework*, a way of thinking, in which they can begin to be addressed. As befits the study of any ecological problem that exhibits a large degree of complexity, a great deal of initial conceptual work is required before appropriate models can be constructed that effectively inform subsequent empirical inquiries [17].

A Mathematical Approach

There are numerous statistical examinations of the development of and change in alcohol use among college students. However, there are few mathematical studies of the dynamics of drinking and consequently little insight into the impact of specific dynamic interventions on problem drinking. The model presented herein builds upon prior work in mathematical epidemiology [18,19]. The framework provides a starting point for the development of *specific population level* models whose mathematical analysis may provide partial answers or additional insights to questions of the type posed earlier within a restrictive, but clearly identified, set of assumptions. Derived from classical disease models, these contemporary models of contagion and transmission have been used to study patterns of social influence and their effects upon problem behaviors such as violence [20], eating disorders [21], alcohol use [22,23] and use of illicit drugs [24,25,26].

The dynamics of drinking are the result of a set of interacting factors that take place at various levels of social organization and over distinct temporal scales. In the present analysis, a simple compartmental model with two distinct drinking environments and three classes of drinkers is employed to examine the effects of residence time upon the differential development and persistence of heavy drinking. College-student drinking provides the context (reference frame) in which the model and its analysis is discussed. We focus on the effect of the proportion of time spent by moderate drinkers in low-risk (e.g., alcohol-free dormitories) versus high-risk (e.g. bars, nightclubs, sports events) drinking environments. The underlying assumption is that in the former, alcohol is not being actively promoted, whereas in the latter it is (through happy hours, targeted marketing, special pricing, etc.). That is, we presume that drinking in active drinking environments will have long term effects on the drinking behaviors of participants in these venues, since contacts between drinkers are known to promote heavier drinking [27, 28]. In other words, the social influences operating in drinking places can catalyze problem drinking to different extents. Our primary goal is to illustrate the importance of heterogeneous drinking environments on the prevalence of high-risk drinking using a simple model of drinking.

The focus is not on the mathematics or on generality (detailed and highly technical analyses of similar or related complex models can be found in our earlier work [25,29]), but rather on highlighting the added dimension that simple models bring to the discussion of the role of environmental context on drinking dynamics. Of course, the potential usefulness of our results (just like those of any field study) is a function of the assumptions made. The experiences collected through over a century of model-generated theories in ecology and epidemiology support the view that model results tend to do one or more of the following things. They confirm the results of equally limited field studies, generate novel hypotheses, raise new questions, and/or increase our attention to missed factors.

The remainder of the paper is organized as follows: Section 2 (Model Description) introduces a simple compartmental model with two drinking environments and *two highly stylized* forms of social mixing; Section 3 (Results) focuses on the partial analysis of the model, computes the conditions necessary for the establishment and persistence of a drinking culture, and provides summaries of the results of extensive simulations; Section 4 discusses the results and their implications, and outlines possible extensions. The Appendices provide details of the mathematical analysis of the model.

2 Model Description

We consider a stratified population of drinkers that interact in “low-risk” (E_1), and “high-risk” (E_2) drinking environments. Drinkers are classified according to their levels of drinking as susceptible (light or occasional drinkers, S), moderate drinkers (or M -drinkers; divided into two types M_1 and M_2), and heavy drinkers (H). The classes S , M , and H are not defined precisely here, albeit such definitions do exist (see [30], [31] and [32]). It is assumed that individuals in the S - and H -drinking states socialize primarily with individuals in a single environmental class, that is, only within low- and high-risk drinking environments, respectively. S and H individuals, within our model, are prisoners of their preferred drinking environments, a situation that changes only when their drinking status changes (here only from S to M or M to H). A *minimal degree* of “drinking environmental” heterogeneity is achieved through the inclusion of two classes of moderate drinkers: M_1 and M_2 . M_i -individuals socialize, in the context of drinking, *exclusively* in environment E_i . Hence, M_1 and M_2 are referred to as low- and high-risk moderate drinkers, respectively. M -drinkers are allowed to transition between both M classes at the rates γ_1 and γ_2 as shown in Figure 1. Epidemiological frameworks [18, 19] can handle high and detailed levels of heterogeneity [33,29,34,35]. However, the goal here is not to introduce a general model within a contagion framework (as has been done for other problems, for example, [33,34]), but rather to show that this approach can generate (under clearly specified assumptions) insights or hypotheses of the kind that are not evident from the direct statistical analyses of the data. This framework allows for the possibility of systematically exploring the impact of hypothetically effective intervention measures in a “virtual” setting under rather general assumptions.

Our model incorporates four mechanisms or drivers associated with drinking dynamics and progression. The first driver comes from the local social interactions between drinking types in low-risk drinking environments. The strength of this effect is incorporated through the parameter β_0 . The second driver is a function of the residence time spent by moderate drinkers while in high-risk environments. This effect is incorporated via the parameter α . The third driver is a function of environment-specific M -residence times (functions of γ_1 and γ_2). That is, this mechanism captures the relative time spent by an a typical M -drinker in low- versus high-risk environments. The fourth driver (modeled via the parameter β_2) is the result of the social interactions between S - and M_2 - individuals outside both drinking environments E_1 and E_2 .

Alternate ways of modeling relevant interactions may include a non-drinking environment where drinking and non-drinking classes coexist. The $S - M_2$ interactions in the flow chart diagram are drawn within the environment type E_1 , but their nature is differentiated by the use of a dotted line (Figure 1). Transitions which might have lesser effect in the college time-frame, such as regression of heavy drinkers back to the M or S classes, are not included in this model.

The denominators in the non-linear rate expressions in Figure 1 express the total population relevant to the interaction being modeled. The larger the denominator, the less likely that a social encounter between the selected types may occur. In the rate involving β_0 the total population involved is assumed to be $S + M_1$, whereas in the rate involving β_2 the total population involved is larger, e.g., $S + M_1 + M_2$. There are alternative ways of modeling such influences [33,34,35,36], but the goal here is to address, in a simple setting, the impact of environment-dependent influences on drinking dynamics.

The total recruitment rate into the system is assumed to match the total per-capita departure rate. That is, the size of the population (or community) under consideration (N) is constant. All new recruits are assumed to be susceptible. The total recruitment rate into the susceptible class is μN with a per-capita average departure rate μ within each drinking class. Thus the average

time spent in the community is $\frac{1}{\mu}$ units of time. In the college context the parameter μ measures student turnover rate (in our simulations we take $1/\mu$ to be 4 years). The assumed progression through the drinking classes is: $S \rightarrow M_1 \sim M_2 \rightarrow H$, with the total population at risk being $N = S + M_1 + M_2 + H$. The system (A.5) of equations for the model are stated in Appendix A. 1. Definitions of model variables and parameters are collected in Table 1. The term *high-level drinking* individuals is used to refer collectively to moderate and heavy drinkers. The community comprised of light, moderate, and heavy drinkers is called the *mixed drinking community*. The parameters $\beta_0, \beta_2, \gamma_1$, and γ_2 are non-negative, whereas the parameters μ , and α are positive. The variables S, M_1, M_2 , and H only take on non-negative values (i.e., zero or positive).

3 Results

3.1 Model Equilibria and Stability

System (A.5) has an high-level drinking “free” (or trivial) equilibrium (i.e., $S = N, M_1 = 0, M_2 = 0, H = 0$), which represents a population with only S-individuals. The possibility of the propagation of high levels of drinking within the S-community is analyzed through the examination of the impact generated by the introduction of a small number of “typical” moderate drinkers (an “invasion” process) in a large population of susceptibles. The theoretical characterization of “typical” moderate drinkers must include their “residence” habits, that is, the proportion of time such an individual spends in E_1 , and E_2 . A “typical” individual may not exist in our population (the mean of a sample is not necessarily represented by any member of the population). A “typical” M-drinker is a useful construct [37]. Following the approach outlined in [38], we compute the average number of conversions ($S \rightarrow M_1$), R_d , generated by a “typical” moderate drinker in a population where the proportion of S-individuals is approximately one. In fact,

$$R_d(\gamma) = \frac{\beta_0}{\mu(1-\gamma) + (\mu+\alpha)\gamma} (1 - \gamma) + \frac{\beta_2}{\mu(1-\gamma) + (\mu+\alpha)\gamma} \gamma, \quad (1)$$

where $\gamma \equiv \frac{\gamma_1}{\mu + \gamma_1 + \gamma_2 + \alpha}$. R_d , the drinking reproductive number, gives the average number of moderate drinkers generated by a typical moderate drinker during his/her time as moderate drinker, where the proportion of S-individuals is approximately one. If $\gamma \approx 0$ then

$$R_d \approx \frac{\beta_0}{\mu},$$

that is, individuals primarily drink in E_1 while most moderate drinkers do not move to E_2 . If $\gamma \approx 1$ then

$$R_d \approx \frac{\beta_2}{\mu + \alpha},$$

that is, most moderate drinkers spend their drinking time in E_2 . The basic drinking reproductive number can be rewritten as the weighted sum

$$R_d(\gamma) = \beta_0(1 - \gamma)D(\gamma) + \beta_2\gamma D(\gamma),$$

where the γ -average period of M-influence is

$$D(\gamma) = \frac{1}{\mu(1 - \gamma) + (\mu + \alpha)\gamma}.$$

If $R_d \leq 1$ high-level drinking dies out in this community. In contrast, if $R_d > 1$ a drinking “outbreak” takes place, that is, the population of moderate (and consequently heavy) drinkers begins to grow (for analogous situations in epidemiology see [19]). Numerical simulations support the existence of a unique (“stable”) mixed drinking community, that is, a community with S , M_1 , M_2 , and H positive when $R_d > 1$. The mixed drinking state will also be referred to as the endemic drinking equilibrium, D_1 .

Numerical solutions (Figure 2) and experience with similar models strongly support the hypothesis that if $R_d > 1$ then the unique endemic drinking state, D_1 , is *locally asymptotically stable*¹. Whether or not it is *globally stable*² is not obvious, albeit extensive simulations using parameters derived from college data suggest that for such populations it is likely to be. This would mean that a mixed culture of drinking is likely to become established regardless of the size (> 0) of the initial population of drinkers. Furthermore, such a community would persist (re-organize itself) even after major perturbations, that is, large increases or reductions in the number of drinkers. We have proved local asymptomatic stability of D_1 only when R_d is slightly larger than one. The *Theorem* in Appendix A.2 states some mathematical results.

3.2 Computing the Effect of Residence Times

We derive the endemic drinking equilibrium state D_1 by setting the derivatives in the System A.5 equal to zero. At the equilibrium, the number of individuals in the classes (or compartments) of the model is *not changing with time*, but individuals continue to circulate among the compartments, that is, they enter and leave the classes at the prescribed rates.

Because the system is at equilibrium the class sizes are constant. Let $M^* \equiv M_1^* + M_2^*$. A constant γ , $0 < \gamma < 1$, can be found such that $M_1^* \equiv (1 - \gamma)M^*$, and $M_2^* \equiv \gamma M^*$. That is,

$$\frac{M_1^*}{M_2^*} = \frac{1 - \gamma}{\gamma}. \quad (2)$$

But from the steady state expressions of M_1 , and M_2 (see Appendix B.3), it follows that

$$\frac{M_1^*}{M_2^*} = \frac{\mu + \gamma_2 + \alpha}{\gamma_1} \quad (3)$$

¹A drinking equilibrium point or steady state is *locally asymptotically stable* if solutions starting from *nearby* initial conditions (values of state variables at time 0) remain nearby.

²A drinking equilibrium point or steady state is *globally stable* if starting from *any* initial condition that includes moderate drinkers the solution approaches this equilibrium as time becomes large.

and consequently (from Equation (2)) we see that

$$\frac{1-\gamma}{\gamma} = \left(\frac{\mu + \gamma_2 + \alpha}{\gamma_1} \right)$$

or

$$\gamma = \frac{\gamma_1}{\mu + \gamma_1 + \gamma_2 + \alpha}. \quad (4)$$

If $\gamma_1 > 0$ there is movement of moderate drinking individuals from E_1 to E_2 . Since at any time, in equilibrium, the fraction of members of M who are in E_2 is γ , we see that members of M spend a proportion γ of their time in E_2 . Hence, γ denotes the proportion of time, the relative *residence time*, spent by a generic (M^*) moderate drinking individual in E_2 (high-risk drinking environments), while $1 - \gamma$ denotes the proportion of time spent by the generic M^* individual in E_1 (low-risk drinking environments). We see that γ captures, on a 0 to 1 scale, the effect of the rates of movement between environments, γ_i ($i = 1, 2$). We say that the drinking environments are connected if $0 < \gamma < 1$ and highly connected when γ is near 0.5. The relative residence times in distinct drinking environments as parameterized by $\gamma \in (0, 1)$ have therefore been computed. We observe that R_d captures the essence of the “typical” moderate (M^*) drinker who must be part of both worlds (i.e., high- and low-risk environments) or who must have an average (i.e., weighted by two environments) preference.

3.3 Numerical Results

This subsection has two parts. The first part (Figure 3) explores the impact of variations of model parameters γ_1 , γ_2 , and α on the basic drinking reproduction number, R_d . The second part (Figures 4, 5, and 6) focuses on the impact that relative residence times, γ , have on endemic drinking levels. Parameter estimates using available and regional national college data give $\beta_0 \sim 0.71$, $\beta_2 \sim 2.20$, $\alpha \sim 0.51$, $\mu \sim 0.27$, and $\gamma \sim 0.28$ [39]. However, the parameter values used to illustrate the results of numerical studies were selected purposefully. The community size is fixed at 1000 individuals. The social influence of moderate drinkers from low-risk and high-risk environments (β_0 and β_2) are assumed to be equal ($\beta_0 = \beta_2 = 2$). The average residence time in the system is assumed to be 4 years ($\mu = 0.25$). The per capita progression rate from moderate to heavy drinking, α , is taken to be 1.5. We fix the values of three of the above four parameters in order to focus on the effect of changing γ and either α , μ , or β_0 (Figures 4–6).

3.3.1 Effect of Movement and Progression Parameters on the Reproduction

Number— γ_i is the per capita transfer rate of moderate drinkers from environments E_i to E_j ($i \neq j$, $i = 1, 2$). α is the per capita progression rate of moderate drinkers (in E_2) to the heavy drinking class. The graph of the surface $\gamma_2 = f(\gamma_1, \alpha)$, obtained by equating R_d to 1 and fixing the values of β_0 , β_2 , and μ is shown in Figure 3. This figure captures the effect of γ_1 , γ_2 , and α on R_d . The parameter regions where $R_d > 1$ and $R_d < 1$ are highlighted also in Figure 3. The graph of f is the boundary on which $R_d = 1$. Figure 3 shows in what region (either side of the surface $\gamma_2 = f(\gamma_1, \alpha)$) the combination of the parameters (γ_1 , γ_2 , and α) lie. For drinking communities with $(\gamma_1, \alpha, \gamma_2) = (5, 4.5, 0.5)$, we have $\gamma = 0.4878$ and $1 - \gamma = 0.5122$. For this last example, even though the values of γ and $1 - \gamma$ are close to each other, high-level drinking will eventually die out since $R_d < 1$. For the community given by $(\gamma_1, \alpha, \gamma_2) = (4, 4.5, 8)$, we have $\gamma = 0.24$ and $1 - \gamma = 0.76$. This value of γ suggests that on average a typical moderate drinker spent 24% and 76% of his/her drinking time in high- and low-risk environments,

respectively. In this last community mixed drinking will grow (and most likely persist) since $R_d > 1$.

3.3.2 Drinking Patterns at Equilibrium—Endemic drinking equilibrium values for all classes of drinkers as a function of γ are plotted in Figures 4–6. In Figure 4 we fix the values of β_0, β_2, μ and vary γ and α . If the rate of progression from moderate to heavy drinking is low (i.e., $\alpha = 0.01$), the endemic equilibrium is nearly constant in γ (the proportion of time a moderate drinker spends in high-risk environments, E_2) and the M^* is large (Figure 4). Larger values of α (0.2 and 0.6) increase the sensitivity of equilibrium values to γ , with H^* increasing at the expense of M^* as γ increases. For the selected values of α , S^* remains small. For all $\alpha < 1$, increases in γ result in an increase in the number of heavy drinkers. That is, the more time moderate drinkers spend in high-risk environments, the larger the number of heavy drinkers in the community even when the progression rates from moderate to heavy drinking are low. Further increases in α (from 1.2 to 1.7), with γ values near 1, result in sharp increases in S^* and decreases in H^* . In this last situation M^* changes slowly becoming almost extinct. For example, with low movement between environments, decreases in the average moderate-drinking-period of a drinker from 10 months ($\alpha = 1.2$) to 7 months ($\alpha = 1.7$) result in decreases in the number of heavy drinkers from 60% to 20%. When $1 < \alpha < 1.8$ heavy drinkers “dominate” for most γ values, becoming about 70% of the total population for some intermediate value of γ . High values of α correspond to fast progression from M to H, which among college students, for example, may be defined as moving from a moderate to a recurrent high-risk (binge) drinking state. In summary, fast progression of drinkers from a moderate to a heavy drinking level increases the proportion of heavy drinkers in communities with high “connectivity” (i.e., γ near 0.5) between environments.

Figure 5 highlights the impact of β_0 . The endemic solution is sensitive to this parameter especially when $\beta_0 < 2$, that is, for values congruent with those estimated from college data [39]. At $\beta_0 = 0.2$ there is no endemic drinking equilibrium solution for small γ , but as β_0 increases from 0.2 the endemic drinking equilibrium (D_1) comes into existence for all γ values. For $\beta_0 \geq 0.6$ there is an intermediate value of γ where H^* reaches its maximum (the value estimated for β_0 from our college data was 0.71). For example, a large number of social contacts between S and M_1 produces a large number (more than 60%) of heavy drinkers when environments are highly connected (i.e., when γ is near 0.5). This maximum value of H^* increases and shifts to larger values of γ with increases in β_0 . Increases in β_0 beyond 0.6 lead to a sharp decline in the number of susceptibles and a large proportion of heavy drinkers is possible even when there is low connectivity between environments. This is because the progression rate to heavy drinkers’ class rises as M_2 class slowly increases at the expense of M_1 class. At $\beta_0 = 7$, probably an “unreasonably” high value, the susceptible curve falls below the moderate- and the heavy-drinkers curves, for most γ values. Hence, if there is a high connectivity between environments, decreases in social contacts of low-risk moderate drinkers with light drinkers will reduce the number of heavy drinkers and increase the number of light drinkers in the community. However, increases in the effectiveness of social contacts in E_1 will increase the size of the heavy drinking class. Furthermore, strong social interactions between S and M individuals in E_1 will lead to a disproportionate number of heavy drinkers (and, for the most part, this is unaffected by the time spent by moderate drinkers in high-risk drinking environments).

In Figure 6 we fix the values of α, β_0 , and β_2 and vary μ from 0.15 to 1, which corresponds (in the case of college students) to a situation in which individuals remain in the community between 1 and $6\frac{2}{3}$ years. The peak level of heavy drinkers, which takes place when $\gamma = 0.5$, changes from 80% to 20% as μ increases (see Figure 6). That is, if the environments are highly connected (γ near 0.5), increasing the average number of years ($1/\mu$) in this community from 1 year to $6\frac{2}{3}$ years, results in an increase in heavy drinkers from 20% to 80%. Extremely small

or very large values of μ may be unrealistic in some settings. For $\mu = 0.25$, high connectivity (i.e., $0.25 < \gamma < 0.75$) results in a large proportion (45% to 75%) of heavy drinkers even when the per capita environment-dependent social influences, β_0 and β_2 , are identical. In college populations a disproportionate number of students at risk of becoming excessive drinkers will be freshmen and sophomores, with possibly low expected college residence times. The computation of the average residence time in specific drinking environments must consider the natural community turnover rates.

4 Discussion

This study examined the effects of social influence, social context, and residence time on the initiation and maintenance of moderate and heavy drinking among individuals from a dynamic population that experiences high turnover rates over time. A simple contagion model of social influence that incorporates three different drinking levels and two contrasting drinking environments is examined. A threshold parameter, R_d (referred to as the drinking reproduction number), which indicates under what circumstances a drinking outbreak will occur, is computed. The parameter R_d gives the average number of the secondary conversions from light drinker to moderate drinker generated by a “typical” moderate drinker in a population of light or occasional drinkers. If R_d is less than 1, the population of moderate and heavy drinkers will not “survive.” That is, a stable population of high-level drinking individuals will not become established. However, R_d values above 1 will result in the creation of a mixed population of drinkers, comprised of light or occasional, moderate, and heavy drinkers.

The model is used to investigate how the steady state numbers (for $R_d > 1$) in the three drinking level classes change with the fraction of time that moderate drinkers spend in two types of drinking environments. The study assumes that such an equilibrium is at least locally stable, as suggested by simulations and some analysis. Our model predicts that the proportion of heavy drinkers will be lower if the residence time is heavily weighted toward either of the two distinct risk-level environments unless the local social interactions are significantly different or extremely high. Restriction in movement of drinkers between low-risk and high-risk environments may decrease the prevalence of heavy drinkers, but only under “weak” mixing regimes. A dramatic increase in the frequency and effectiveness of local interactions in environment E_1 (increase in β_0) may result in significant increase in the proportion of heavy drinkers “regardless” of the connectivity between the two types of environment. In our setting, high individual turnover rates (μ) and fast progression to heavy drinking (α) can impact drinking dynamics significantly. Increases on individuals residence times in the community (i.e., decreases in turnover rate) or increases in the rate of progression to heavy drinking can lead to increases in the proportion of heavy drinkers.

This application of a dynamic nonlinear model to the study of drinking behavior illustrates the complexities inherent in socially-driven processes [40,41]. The mathematical explication of these processes has clear implications for prevention programs and policies. It calls attention to the need to focus on reducing the risk of progression in high-risk environments and/or in placing or creating structures that limit the “contagion” (i.e., secondary conversions) facilitated in these settings. As for high-risk settings in college environments, these are well-established (e.g., off-campus parties, off-campus bars, and fraternity parties [12,13,42]), and the behavior of both hosts and patrons within these can be controlled through environmental management strategies such as increased enforcement of DUI laws and ID checks, use of decoy operations at bars and parties, limitations on promotions and advertising, and restrictions on the concentration of bars near campus [43,44]. Unfortunately, most college-based alcohol prevention programs employ educational and normative approaches that have little or no effect [45,46,47]. Like all static one-size-fit-all interventions, such programs are unlikely to influence drinking at the population level since they fail to consider the fundamental impact of dynamic

interactions between individuals and the changing “quality” of the environments in which these associations take place [17,48].

The findings of this study and their potential implications must be understood in terms of the limitations of mathematical models in general and the specific challenges encountered in applying these approaches to the study of the initiation, maintenance, and cessation of alcohol use. Social interactions are difficult to quantify and calibrate [49]. Moreover, the data available from empirical studies are not necessarily what is needed to estimate all the parameters in a dynamic process model. This is especially true of the dynamics of drinking behavior since movement into and out of drinking states such as “moderate” or “heavy” drinking is extremely fluid [50]. Furthermore, even if appropriate studies exist, the parameters described in these studies frequently do not relate directly to those that one has in mind in developing a specific model. For example, while we are interested in the amount of time that college students spend in very specific drinking environments and what the implication of this is for escalation in consumption, empirical studies describing such micro-patterns of behavior and the degree of risk associated with drinking in one setting (e.g., a fraternity party) versus another (e.g., a college dormitory) are unavailable. We hope that the results of this study will highlight the importance of finding ways of characterizing places of drinking as well as estimating the distribution of times that various types of drinking individuals spend in highly distinct drinking environments.

It is worth stressing again that the point of this study is to *highlight the influence of nonlinear dynamical interactions on drinking patterns*. We focused on the study of the impact of various social influences on drinking progression outcomes when drinking took place in two highly distinct and artificial drinking environments. We identified unexpected trends, and it is our hope that the theoretical results obtained from this simplified model of drinking environments will stimulate interest in mathematical modeling among researchers in the field of alcohol studies.

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Appendix A

A.1 Model: System of Equations

The assumptions and definitions mentioned in the main text (Section 2) give rise to the following model (see Figure 1 for flow diagram):

$$\begin{aligned}
 \frac{dS}{dt} &= -\beta_0 \frac{SM_1}{(S+M_1)} + \mu N - \mu S - \beta_2 \frac{SM_2}{(S+M_1+M_2)}, \\
 \frac{dM_1}{dt} &= \beta_0 \frac{SM_1}{(S+M_1)} - \mu M_1 - \gamma_1 M_1 + \beta_2 \frac{SM_2}{(S+M_1+M_2)} + \gamma_2 M_2, \\
 \frac{dM_2}{dt} &= \gamma_1 M_1 - \gamma_2 M_2 - \mu M_2 - \alpha M_2, \\
 \frac{dH}{dt} &= \alpha M_2 - \mu H,
 \end{aligned} \tag{A.5}$$

where $N = S + M_1 + M_2 + H$.

A.2 Analytical Result

Asymptotic analysis of the System (A.5) suggests the existence of two equilibria $D_0 \equiv (N, 0, 0, 0)$ and $D_1 \equiv (S^*, M_1^*, M_2^*, H^*)$, as stated in the following theorem (proved in the Appendix B).

Theorem

Let R_d be given by Expression (1)

- i. If $R_d \leq 1$ then the abusive drinking free equilibrium $D_0 \equiv (N, 0, 0, 0)$ is locally asymptotically stable. In fact, D_0 is globally asymptotically stable, i.e., moderate and heavy drinking will eventually die out, starting from any initial values of S , M_1 , M_2 , and H , as long as $R_d \leq 1$
- ii. If $R_d > 1$ then D_0 becomes unstable, and a unique endemic drinking equilibrium $D_1 \equiv (S^*, M_1^*, M_2^*, H^*)$ (all drinking levels above zero) exists. D_1 is locally

asymptotically stable when R_d is slightly larger than one, that is, the numbers in the classes approach the values in D_1 as long as they are already close, as time increases.

Appendix B

In this Appendix we state and prove some of the mathematical results of our model (System (A.5)), whose interpretations are discussed in the main text. We start with the derivation (Lemma 1) of the “drinking reproduction number” and the proof of local stability (Lemma 1) of the abusive drinking free equilibrium (Section B.1). Global stability of the abusive drinking free equilibrium is proved in Lemma 2 (Section B.2). We then show the existence and uniqueness (Lemma 3 and Lemma 4) of the endemic drinking equilibrium (i.e., state of a “mixed drinking community”). This section ends with the proof of the Theorem mentioned in the Appendix A.2.

Consider the system given in (A.5) along with the initial condition for the model variables. The solutions starting in the positive orthant $\mathbb{R}_+^4$ either enter or will remain in the subset of $\mathbb{R}_+^4$, defined by

$$D = \{V = (S, M_1, M_2, H) \in \mathbb{R}_+^4 : V \geq 0 \text{ and } N \text{ constant}\}.$$

Thus, it suffices to consider solutions in the positively invariant region D . Therefore, a solution of the initial value problem starting in D and defined by our model, along with the initial conditions, exists and is unique on a maximal interval $[0, b)$. Since all solutions are bounded in the region D , the maximal interval is $[0, \infty)$. Thus, the initial value problem is *well posed* (see details in [51]). That is, if the initial conditions are non-negative, then all classes remain non-negative.

B.1 Derivation of Drinking Reproduction Number and Local Stability of the Abusive Drinking Free Equilibrium

Lemma 1 (given below) derives the threshold (R_d) expression for the System (A.5). It also proves local stability of the abusive drinking free equilibrium.

Lemma 1

The basic drinking reproduction number for the model defined by (A.5) is

$$R_d = \frac{\beta_0(1 - \gamma) + \beta_2\gamma}{\mu + \gamma\alpha}$$

where $\gamma = \frac{\gamma_1}{\mu + \gamma_1 + \gamma_2 + \alpha}$. Furthermore, if $R_d < 1$ then the abusive drinking free equilibrium is locally stable. This form of R_d is equivalent to the expression (1)

Proof

Diekmann et al. define R_0 (the basic reproduction number) for a compartmental model like Model (A.5) as the spectral radius of the “next generation matrix” [37]. It is shown that R_0 is a threshold parameter for the existence of a locally stable disease free equilibrium [37]. Since our model is analogous, we can use a similar approach to find R_d . Later in 2002, their approach was refined and restated by Van den Driessche and Watmough [38].

Following the method of Van den Driessche et al., we first reorder the compartments so that the first two correspond to compartments consisting of influential moderate drinkers (M_1 , M_2 , H , and S). Then the terms in each equation of the System (A.5) are categorized into terms describing newly generated moderate drinkers, stored in a matrix F , and transitions terms, stored in a matrix V .

We then consider an initial population near the abusive drinking free state (DFS), i.e., $(M_1, M_2, H, S) = (0, 0, 0, N)$. We construct a matrix FV^{-1} , where $F = \left[\frac{\partial F_i}{\partial x_j} \right]_{(DFS)}$ and $V = \left[\frac{\partial V_i}{\partial x_j} \right]_{(DFS)}$ with $1 \leq i, j \leq 4$ (note that F and V are part of the Jacobian or linearized matrix evaluated at DFS). The (j,k) entry of V^{-1} is the average length of time an infectious (i.e., a “typical” moderate drinker) individual spends in the compartment ‘j’ during its lifetime, assuming the population remains near DFS. The (i,j) entry of F is the rate at which a moderate drinker in compartment j produce new drinkers in compartment i. Hence, the (i,k) entry of the product FV^{-1} is the expected number of new drinkers in compartment i produced by the “typical moderate drinker” individual originally introduced into compartment k. We call FV^{-1} the next generation matrix for the model and set

$$R_d = \rho(FV^{-1}),$$

where $\rho(A)$ denotes the spectral radius of matrix A .

The DFS is locally asymptotically stable if all the eigenvalues of the Jacobian matrix of the linearised system have negative real parts and unstable if any eigenvalue of the Jacobian matrix has a positive real part. The maximum real part of all the eigenvalues of the matrix $F - V$ (i.e., $s(F - V)$, where $F - V$ is linearised matrix) is negative if and only if $\rho(FV^{-1}) < 1$; is zero if and only if $\rho(FV^{-1}) = 1$; and is positive if and only if $\rho(FV^{-1}) > 1$ (see details in [38]). For our model we get

$$FV^{-1} = \begin{bmatrix} \frac{\beta_0(\mu+\gamma_2+\alpha)}{\mu(\mu+\gamma_2+\gamma_1)} + \frac{\beta_2\gamma_1}{\mu(\mu+\gamma_2+\gamma_1)} & \frac{\beta_0\gamma_2}{\mu(\mu+\gamma_2+\gamma_1)} + \frac{\beta_2(\mu+\gamma_1)}{\mu(\mu+\gamma_2+\gamma_1)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Using the definition of spectral radius and the notation $\gamma = \frac{\gamma_1}{\mu+\gamma_1+\gamma_2+\alpha}$ we obtain the stated expression for R_d . Also, if $R_d < 1$ then all the eigenvalues of the Jacobian matrix of the linearised system will have negative real parts, and hence abusive drinking free equilibrium will be locally stable.

B.2 Global Stability Analysis of the Abusive Drinking Free Equilibrium

Lemma 2

If $R_d < 1$ then the abusive drinking free equilibrium is globally stable.

Proof

Consider the expression for R_d given in (1). Substituting $H = 0$ in the equilibrium equations we get $M_2 = 0$, $M_1 = 0$, and $S = N$. Hence, the drinking free steady state, $D_0 = (N, 0, 0, 0)$, always exists.

If $R_d < 1$, global stability of the drinking free state is proved as follows, using the fluctuation method [51]. Since the total population is constant, $S(t)$, $M_1(t)$, $M_2(t)$, and $H(t)$ are bounded

functions for $t \in \mathbb{R}$. Also, we have assumed that S, M_1, M_2 , and H are differentiable functions. So there is a sequence $\{e_n\}$ such that $\dot{M}_2(e_n) \rightarrow 0, M_2(e_n) \rightarrow M_2^\infty$, as $n \rightarrow \infty$ and $\limsup_n M_1(e_n) \leq M_1^\infty$.

By using $\dot{M}_2$ equation of System (A.5), we get

$$\begin{aligned} \lim_{n \rightarrow \infty} \dot{M}_2(e_n) &= \gamma_1 \lim_{n \rightarrow \infty} M_1(e_n) - (\mu + \gamma_2 + \alpha) \lim_{n \rightarrow \infty} M_2(e_n), \\ 0 &\leq \gamma_1 M_1^\infty - (\mu + \gamma_2 + \alpha) M_2^\infty, \\ M_2^\infty &\leq \frac{\gamma_1}{(\mu + \gamma_2 + \alpha)} M_1^\infty. \end{aligned} \quad (\text{B.6})$$

Since $\frac{s}{s+M_1} \leq 1$ and $\frac{s}{s+M_1+M_2} \leq 1$, therefore the $\dot{M}_1$ equation of System (A.5) yields

$$\begin{aligned} \dot{M}_1 &\leq \beta_0 M_1 \\ &\quad - (\mu + \gamma_1) M_1 + (\beta_2 + \gamma_2) M_2. \end{aligned} \quad (\text{B.7})$$

Similarly, there is a sequence $\{t_n\}$ such that $\dot{M}_1(t_n) \rightarrow 0, M_1(t_n) \rightarrow M_1^\infty$, as $n \rightarrow \infty$ and $\limsup_n M_2(t_n) \leq M_2^\infty$, and so by using inequality (B.6) and (B.7) we get

$$\begin{aligned} 0 &\leq \beta_0 M_1^\infty - (\mu + \gamma_1) M_1^\infty + (\beta_2 + \gamma_2) M_2^\infty \\ 0 &\leq \left(\beta_0 - (\mu + \gamma_1) + \frac{(\beta_2 + \gamma_2) \gamma_1}{(\mu + \gamma_2 + \alpha)} \right) M_1^\infty. \end{aligned} \quad (\text{B.8})$$

Simplification gives us

$$0 \leq \left(\frac{(\mu + \gamma_1)(\mu + \gamma_2 + \alpha) - \gamma_1 \gamma_2}{(\mu + \gamma_2 + \alpha)} \right) (R_d - 1) M_1^\infty. \quad (\text{B.9})$$

If $R_d < 1$ then inequality (B.9) can hold only if $M_1^\infty = 0$. But we know $0 \leq M_1 \leq M_1^\infty$, so $M_1 = 0$. Hence, we also get $M_2^\infty = 0$ (or $M_2 = 0$) by (B.6).

We can use the same approach in the $\dot{S}$ equation of System (A.5). That is, there is a sequence $\{s_n\}$ such that $\dot{S}(s_n) \rightarrow 0, S(s_n) \rightarrow S_\infty$, as $n \rightarrow \infty$, $\limsup_n M_1(s_n) \leq M_1^\infty$ and $\limsup_n M_2(s_n) \leq M_2^\infty$. Since $-1 \leq -\frac{s}{s+M_1}$ and $-1 \leq -\frac{s}{s+M_1+M_2}$, therefore the $\dot{S}$ equation becomes

$$\dot{S} \geq -\beta_0 M_1 + \mu N - \mu S - \beta_2 M_2. \quad (\text{B.10})$$

Hence, by the fluctuation method

$$0 \geq -\beta_0 M_1^\infty + \mu N - \mu S_\infty - \beta_2 M_2^\infty. \quad (\text{B.11})$$

But we have proved above that if $R_d < 1$ then $M_1^\infty = 0$ and $M_2^\infty = 0$. Therefore, we get $S_\infty \geq N$. Since the total population N in our system is bounded, hence $S_\infty = N$. So $S = N$. This proves the global stability of the drinking free steady state, if $R_d < 1$.

B.3 Existence of Endemic Drinking Equilibrium (i.e., State of Mixed Drinking Community)

Lemmas 3 and 4 prove the existence of a unique endemic drinking equilibrium. In order to find endemic drinking equilibrium solutions we set $\frac{dS}{dt} = 0$, $\frac{dM_1}{dt} = 0$, $\frac{dM_2}{dt} = 0$, and $\frac{dH}{dt} = 0$ in the System (A.5) and assume $H \neq 0$. We then solve for the endemic values S^* , M_1^* , M_2^* , and H^* using these equations. The endemic values are given as follows

$$\begin{aligned} S^* &= N - \eta H^*, \quad M_1^* = \xi \theta H^* \\ &, \quad M_2^* = \theta H^*, \end{aligned} \quad (\text{B.12})$$

where η , ξ , θ are given in Lemma 3, $H^* = \tilde{H}N$, and $\tilde{H}$ satisfies the quadratic equation:

$$a\tilde{H}^2 - b\tilde{H} + c = 0. \quad (\text{B.13})$$

The coefficients a , b , and c of the quadratic equation (B.13) are given in Lemma 3.

Lemma 3

Let $H \neq 0$, then the coefficients 'a', 'b', and 'c' of the quadratic equation (B.13) are given by

$$\begin{aligned} a &= \theta \left(\xi + \frac{\beta_2}{\alpha} \right) + \eta(R_d - 1), \\ b &= \theta \left(\xi + \frac{\beta_2}{\alpha\eta} \right) + (1 + \eta)(R_d - 1), \\ c &= (R_d - 1), \end{aligned} \quad (\text{B.14})$$

where $\theta = \frac{\mu}{\alpha}$, $\eta = \frac{\mu + \gamma\alpha}{\gamma\alpha}$, $\xi = \frac{1 - \gamma}{\gamma}$, $\gamma = \frac{\gamma_1}{\mu + \gamma_1 + \gamma_2 + \alpha}$, and $\tilde{H} = \frac{\mu}{N}$. If $R_d > 1$, then the coefficients a , b and c are positive.

Proof

Setting derivatives equal to zero in the System (A.5) leads to four different equations in variables S , M_1 , M_2 and H . Let any solution of these equations be denoted by S^* , M_1^* , M_2^* and H^* . Then the equation $\frac{dH}{dt} = 0$ gives

$$M_2^* = \frac{\mu}{\alpha} H^* (= \theta H^*), \quad (\text{B.15})$$

and the equation $\frac{dM_2}{dt} = 0$ gives

$$M_1^* = \left(\frac{1-\gamma}{\gamma} \right) M_2^* (= \xi M_2^*). \quad (\text{B.16})$$

By substituting (B.15) in (B.16) we get

$$M_1^* = \xi \theta H^*. \quad (\text{B.17})$$

Adding the equations $\frac{dS}{dt} = 0$ and $\frac{dM_1}{dt} = 0$ and using values of M_1^* and M_2^* from (B.17) and (B.15), respectively, gives

$$S^* = N - \eta H^*. \quad (\text{B.18})$$

Using values from (B.18), (B.17), and (B.15) in the equation $\frac{dS}{dt} = 0$, we get the quadratic equation in H^* ,

$$\begin{aligned} & \left[1 + \theta - \eta - \frac{\beta_2 \theta}{\alpha} + (1 - R_d) \eta \right] H^{*2} \\ & - \left[1 + \theta - \eta - \frac{\beta_2 \theta}{\alpha \eta} + (1 - R_d)(\eta + 1) \right] N H^* \\ & + (1 - R_d) N^2 = 0. \end{aligned} \quad (\text{B.19})$$

After further simplification, and using $\tilde{H} = \frac{H}{N}$, we get the quadratic equation (B.13) with coefficients given in (B.14).

Since all the parameters in the System (A.5) are positive and γ lies between 0 and 1, therefore θ , η and ξ are positive. This implies the sign of the coefficients a, b, and c will depend on the factor $(R_d - 1)$. Hence, if $R_d > 1$ then all coefficients will be positive.

Clearly by (B.12), $M_1^* > 0$ and $M_2^* > 0$ provided $H^* > 0$. So in order to have a positive endemic equilibrium we must have $S^* > 0$ and $H^* > 0$. Let $H^* > 0$ (i.e., $\tilde{H} > 0$). If $H^* < \frac{N}{\eta}$ (or, $\tilde{H} < \frac{1}{\eta}$), then from (B.12), $S^* > 0$.

Lemma 4

If $R_d > 1$ then there exists exactly one positive real root $\tilde{H}^*$ of the quadratic equation (B.13) such that $\tilde{H}^* \in \left(0, \frac{1}{\eta} \right)$.

Proof

By lemma 3, if $R_d > 1$ then $a > 0$, and so the parabola defined by (B.13) will be concave up. Let $f(x) = ax^2 - bx + c$ be a function with a, b, and c given by expressions in (B.14). Then $f(0) = c$ and $f\left(\frac{1}{\eta}\right) = \frac{a}{\eta^2}(1 - \eta)$. Using the expression for η , we find $f\left(\frac{1}{\eta}\right) < 0$. Since $R_d > 1$, therefore by (B.14), $f(0) > 0$. Hence, by the intermediate value theorem there exists at least one root between 0 and $\frac{1}{\eta}$. But since $f(x)$ is a parabola facing up, there exists exactly one positive real

value ‘z’ between 0 and $\frac{1}{\eta}$ such that $f(z) = 0$. This is equivalent to saying that there exists exactly one positive real root $\tilde{H}^*$ of the quadratic equation (B.13) such that $\tilde{H}^* \in (0, \frac{1}{\eta})$.

B.4 Proof of the Theorem in Appendix A.2

Lemma 1 shows that the abusive drinking free steady state is locally asymptotically stable if $R_d < 1$ and unstable if $R_d > 1$.

If $R_d > 1$, then by Lemmas 3 and 4 we have a unique H^* such that $H^* > 0$ and $S^* > 0$. Therefore, there exists a unique endemic drinking equilibrium for the model. If $R_d = 1$, then the quadratic equation (B.13) with coefficients given in (B.14) becomes a linear equation if $H \neq 0$. Solving

the linear equation gives $\tilde{H}^* \geq \frac{1}{\eta}$ (i.e., $S^* \leq 0$). Hence, in this case no endemic drinking equilibrium in the positive quadrant exists. If $R_d < 1$, then global stability of the drinking free steady state rules out the existence of an endemic drinking equilibrium for the model.

We now prove the claimed stability at and near $R_d = 1$ (including the case when R_d is slightly larger than one). For simplicity we choose β_0 as the bifurcation parameter instead of R_d . If $R_d = R_d^*$ (i.e., $R_d = 1$) or $\beta_0 = \beta_0^*$ (i.e., $\beta_0 = \sigma - \frac{\gamma_1 \delta}{\theta}$), where $\theta = \mu + \gamma_2 + \alpha$, $\sigma = \mu + \gamma_1$ and $\delta = \beta_2 + \gamma_2$, then the drinking “free” equilibrium (DFE) is non-hyperbolic, and Center Manifold Theorem (CM Theorem) & Reduction Principle can be used to obtain the stability of the DFE. According to the CM Theorem and Reduction Principle, under these conditions the original system will have the same dynamics as the system reduced on the center manifold. Calculations lead us to the following normal form of the reduced system on the center manifold:

$$\frac{dX}{dt} = m \left(pX + \frac{n}{m} X^2 \right) + h.o.t.$$

where $X = \frac{\gamma_1 [\theta_1 M_1 + (\beta_2 + \gamma_2) M_2]}{[\theta_1^2 + \gamma_1 (\beta_2 + \gamma_2)]}$, $\theta_1 = \mu + \gamma_2 + \alpha$, $p = \beta_0^* - \beta_0$, $n = -\frac{\theta(\sigma\theta^2 + \beta_2\gamma_1\theta\gamma_1\delta\theta + \beta_2\gamma_1^2)}{\gamma_1(\theta^2 + \gamma_1\delta)N}$, and $m = \frac{\theta^2}{(\theta^2 + \gamma_1\delta)}$.

This normal form shows that at the DFE with $p = 0$ the bifurcation is transcritical (i.e., there is a forward bifurcation). Also there are small numbers $\varepsilon > 0$ and $\delta > 0$ such that for $0 \leq p < \varepsilon$ (i.e., $\varepsilon < R_d \leq 1$) the DFE is locally asymptotically stable and for $-\delta < p < 0$ (i.e., $1 < R_d < \delta$) the DFE is unstable and a locally asymptotically stable endemic equilibrium exists [52, 34, 38].

Biographies

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Priscilla E. Greenwood is Research Professor of Mathematics and Statistics at Arizona State University, Tempe. She received her B.S. in mathematics from Duke University, Durham, NC and both her M.Sc. and Ph.D. in mathematics from the University of Wisconsin, Madison. Professor Greenwood's research has moved from sample path properties of stochastic processes in the 1960s and 1970s, to asymptotic statistics for stochastic processes in the 1980s and 1990s, and, more recently, to stochastic modeling and analysis of population, genetic, epidemic, and neuron dynamics. She has served as Associate Editor for probability and statistics journals, and refereed for these, and others. For more than three decades she was a member of the Department of Mathematics, University of British Columbia, Vancouver, Canada. Professor Greenwood is a Fellow of the Institute of Mathematical Statistics. Her recent work has been published in *Theoretical Population Biology*, *Physical Review Letters*, *Proceedings of the National Academy of Sciences U.S.A.*, *Proceedings of the Royal Society B*, and *Journal of Theoretical Biology*.

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Figure 1.

Drinking population model with three Types of Drinkers (light or occasional (S), moderate ($M = M_1 + M_2$) and heavy (H)) and two Distinct Drinking Environments (low-risk (E_1) and high-risk (E_2)). The $S - M_2$ interactions in the flow diagram are drawn within the environment type E_1 , but their nature is differentiated by the use of a dotted line in order to show that it captures the interactions happened outside E_1 and E_2 drinking environments.

(a) Proportion of S-individuals as a function of time for 5 different initial conditions. (b) Proportion of M_1 -individuals as a function of time for 5 different initial conditions.

(c) Proportion of M_2 -individuals as a function of time for 5 different initial conditions. (d) Proportion of H-individuals as a function of time for 5 different initial conditions.

Figure 2.

Using five different initial conditions (given in figure's legend), graphs support the hypothesis that the endemic drinking state, D_1 , is asymptotically stable. Here, $R_d = 2.67$ which is greater than one (condition for the existence of D_1). The parameters used for these graphs are $\mu = 0.28$, $\beta_0 = 0.71$, $\beta_2 = 2.29$, $\gamma_1 = 0.49$, $\gamma_2 = 0.49$, and $\alpha = 0.54$. For this set of parameters $\gamma = 0.27$.

Figure 3.

Plot of the surface $\gamma_2 = f(\gamma_1, \alpha)$, function 'f' obtained from the solution of $R_d = 1$, under fixed parameters $\beta_0 = 2$, $\beta_2 = 2$, and $\mu = 0.25$. The parameter definitions are in Table 1.

Figure 4.

Equilibrium class sizes for various values of α (per capita progression rate of moderate drinkers in high risk environments to heavy drinking class) as a function of proportion of residence time in high-risk environments (γ). Fixed parameters are $\beta_0 = 2$, $\beta_2 = 2$, $\mu = 0.25$, and $N = 1000$. On the vertical axis we have the number of individuals in S^* , M^* , and H^* classes; S^* represented by Solid, Green curve, M^* by Dotted, Red curve, and H^* by Dashed, Blue curve. Parameters are defined in Table 1.

Figure 5.

Equilibrium class sizes for various values of β_0 (social influence parameter in lowrisk environments) as a function of proportion of residence time in high-risk environments (γ). Fixed parameters are $\beta_2 = 2$, $\alpha = 1.5$, $\mu = 0.25$, and $N = 1000$. On the vertical axis we have number of individuals in S^* , M^* , and H^* classes; S^* represented by Solid, Green curve, M^* by Dotted, Red curve, and H^* by Dashed, Blue curve. Parameters are defined in Table 1.

Figure 6.

Equilibrium class sizes for various values of μ (per capita arrival or departure rate of drinkers population) as a function of proportion of residence time in high-risk environments (γ). Fixed parameters are $\beta_0 = 2$, $\beta_2 = 2$, $\alpha = 1.5$, and $N = 1000$. On the vertical axis we have number of individuals in S^* , M^* , and H^* classes; S^* represented by Solid, Green curve, M^* by Dotted, Red curve, and H^* by Dashed, Blue curve. Parameters are defined in Table 1.

Table 1

The definition of the model variables and parameters

Variables	Definitions	Units
t	Time variable	Years
S	Number of light or occasional drinkers (or susceptibles)	No. of individuals
M_1	Moderate drinkers in low-risk drinking environments (E_1)	No. of individuals
M_2	Moderate drinkers in high-risk drinking environments (E_2)	No. of individuals
H	Number of heavy drinkers	No. of individuals
Parameters		
β_0	Average number of social influential contacts (environment dependent) of one susceptible per unit time with M_1 individuals	No. of contacts $\times$ per unit time $\times$ per individual
β_2	Average number of social influential contacts (environment dependent) of one susceptible per unit time with M_2 individuals mediated by M_1 individuals	No. of contacts $\times$ per unit time $\times$ per individual
μ	Per capita arrival and departure rate for the system (can be referred as turnover rate in college context)	Per unit time $\times$ per individual
α	Per capita progression rate of M_2 -moderate drinkers to H class	Per unit time $\times$ per individual
γ_1	Per capita leaving rate of M_1 -moderate drinkers from E_1 -environments	Per unit time $\times$ per individual
γ_2	Per capita leaving rate of M_2 -moderate drinkers from E_2 -environments	Per unit time $\times$ per individual
γ	Proportion of time individual resides in the high-risk environments (residence time parameter)	Dimensionless