Uncluttered single-image visualization of the abdominal aortic vessel tree: Method and evaluation

Joong-Ho Won^{a)}

Department of Electrical Engineering and Department of Health Research and Policy, Stanford University, Stanford, California 94305

Jarrett Rosenberg,^{b)} Geoffrey D. Rubin,^{c)} and Sandy Napel^{d)}

Department of Radiology, Stanford University, Stanford, California 94305

(Received 26 January 2009; revised 8 September 2009; accepted for publication 16 September 2009; published 15 October 2009)

Purpose: The authors develop a method to visualize the abdominal aorta and its branches, obtained by CT or MR angiography, in a single 2D stylistic image without overlap among branches.

Methods: The abdominal aortic vasculature is modeled as an articulated object whose underlying topology is a rooted tree. The inputs to the algorithm are the 3D centerlines of the abdominal aorta, its branches, and their associated diameter information. The visualization problem is formulated as an optimization problem that finds a spatial configuration of the bounding boxes of the centerlines most similar to the projection of the input into a given viewing direction (e.g., anteroposterior), while not introducing intersections among the boxes. The optimization algorithm minimizes a score function regarding the overlap of the bounding boxes and the deviation from the input. The output of the algorithm is used to produce a stylistic visualization, made of the 2D centerlines modulated by the associated diameter information, on a plane. The authors performed a preliminary evaluation by asking three radiologists to label 366 arterial branches from the 30 visualizations of five cases produced by the method. Each of the five patients was presented in six different variant images, selected from ten variants with the three lowest and three highest scores. For each label, they assigned confidence and distortion ratings (low/medium/high). They studied the association between the quantitative metrics measured from the visualization and the subjective ratings by the radiologists.

Results: All resulting visualizations were free from branch overlaps. Labeling accuracies of the three readers were 93.4%, 94.5%, and 95.4%, respectively. For the total of 1098 samples, the distortion ratings were low: 77.39%, medium: 10.48%, and high: 12.12%. The confidence ratings were low: 5.56%, medium: 16.50%, and high: 77.94%. The association study shows that the proposed quantitative metrics can predict a reader's subjective ratings and suggests that the visualization with the lowest score should be selected for readers.

Conclusions: The method for eliminating misleading false intersections in 2D projections of the abdominal aortic tree conserves the overall shape and does not diminish accurate identifiability of the branches. © 2009 American Association of Physicists in Medicine. [DOI: [10.1118/1.3243866](https://doi.org/10.1118/1.3243866)]

Key words: single-image visualization, abdominal aorta, articulated object model, simulated annealing

I. INTRODUCTION

In diagnosing and reporting vascular abnormalities from computed tomography angiography (CTA) or magnetic resonance angiography (MRA), it is often desirable to visualize all clinically relevant branches at once. A single schematic image that summarizes the key attributes of a subject's condition can provide a useful map to a more complex anatomical and physiological information described using a controlled vocabulary such as RadLex (Ref. 1) or MeSH (Ref. 2) and can be made available in a multimedia medical record that incorporates all disparate data into a single view. This concept of summary image resembles one that is often hand drawn by physicians in medical charts to convey a schematic representation of the subject's anatomy. Rather than serving for primary interpretation, the images communicate key fea-

tures efficiently through annotation of the drawing. Conventional computerized summary visualization methods, such as volume rendering (VR) and maximum intensity projection (MIP), although they provide an overview of the many slices of tomographic images in a realistic three-dimensional (3D) form, have limitations for use in this context. These methods project the reconstructed 3D data to produce two-dimensional (2D) images so that they can be displayed on computer screens and thereby often cause misleading intersections of vessels to appear. In order to address this problem, VR or MIP requires user interaction to try several projection directions, in turn, or to examine only small portions at a time. If all clinically relevant vessels can be shown without overlaps, a single image will be enough for summary visualization.

In this paper, we introduce the uncluttered single-image

visualization (USIV), a summary visualization method for the abdominal aorta and its branches (abdominal aortic vessel tree), as it might be captured by CTA and MRA, in a single 2D image. Our method presents all clinically relevant vessels in a single image without misleading overlaps, with minimal distortion, thus preventing unnecessary user interaction and enhancing the readability of the 2D visualization. In addition, our visualization method makes the visualization independent of the user interface devices. The output visualization can be displayed simply on a monitor or even printed on a sheet of paper.

Prior work closely related to ours is the research of Kanitsar *et al.*,³ developed in the context of the curved planar reformation (CPR) visualization method, which has been used to visualize the lumen of a blood vessel on a 2D plane.⁴ The wide use of CPR has been restricted to a single vessel segment because of the difficulty of defining a curved plane spanning multiply branching vessels. As an extension of CPR to multiple vessels, Kanitsar *et al.* proposed two methods for compositing individual CPRs: Multipath CPR, which preserves the spatial coherency of vessels while allowing significant overlap, and untangled CPR, which relaxes spatial coherency by hierarchically enclosing vessel centerlines in circular sectors (called vessel hulls) and rotating them in order to avoid overlaps. Although visualization of the lumen is not a primary goal of this work, the methodology of Kanitsar *et al.* provides important insight into how to handle vascular geometry: The use of the enclosures of vessel centerlines and the relaxation of spatial relations.

Our method models the anatomy of the abdominal aortic vessel tree as an articulated object whose underlying topology is a rooted tree. Our method does not involve knowledge of anatomy beyond the starting point (root) of the vessel tree and does not require branch identification.⁵ The inputs are the 3D centerlines of the vessels and their associated diameter information. The outputs are the 2D centerlines of the vessels that resemble the orthogonal projection of their 3D counterparts onto a predefined viewing direction but are free from false intersections, with their widths modulated by the local diameter of the vessel, thereby producing a stylistic visualization of the abdominal aortic vessel tree. This stylistic visualization is similar to the previous works of Oeltze and Preim,⁶ which models the vascular wall with convolution surfaces, and Hahn *et al.*,⁷ which uses truncated cones to represent schematically the vessels of the liver. Our work, however, focuses on producing a visualization with 2D geometry rather than 3D surfaces.

To test our method, we assembled a database of abdominal aortic vessel trees from multidetector CTAs. We conducted a preliminary evaluation by radiologists to see if the uncluttered single-image visualization produced by our method is recognizable by and acceptable to clinicians. We also evaluated quantitatively the distortion of the branches in the resulting images and studied the association between the quantitative distortion metrics and the subjective ratings of the readers.

The rest of this paper is organized as follows. In Sec. II, we review some relevant background concepts. In Sec. III,

FIG. 1. A schematic diagram of the canonical anatomy of the abdominal aortic tree.

we describe our visualization algorithm and how it was evaluated. In Sec. IV, we present the results. Discussion and concluding remarks follow thereafter.

II. BACKGROUND

II.A. Anatomy of the abdominal aortic vessel tree

The abdominal aorta, the lower half of the largest artery in the human body, begins at the aortic hiatus of the diaphragm and runs down the posterior wall of the abdomen until it bifurcates into the left and right common iliac arteries (Fig. 1). Not all aortic trees are identical. For example, the classical configuration of the celiac trunk occurs in 55%–65% of cases.⁸ Variations include, but are not limited to, hepatosplenic-mesenteric trunk and celiac-mesenteric trunk, where the superior mesenteric arteries (SMAs) merge into the celiac trunk. The hepatic artery also has variations in more than 40% of individuals.⁸ These include a replacement of the common hepatic, arising from the SMA, or a replacement of the right hepatic artery, which is a branch of the common hepatic in the classical configuration, also from the SMA. Similar variations are seen in the renal artery number and location of origin.

II.B. Stretched curved planar reformation

The main idea of CPR is to map the ruled surface [a (curved) surface S is ruled if through every point of S there is a straight line that lies on S] defined by the centerlines of the vessel and a chosen fixed scan direction (ruling vector) onto a display plane. CPR can be considered inverse texture mapping, working from a curved surface to a flat surface. In the stretched CPR,^{9,10} the mapping from the ruled

surface to the flat surface preserves distances. As a result, this mapping preserves the arc length of the centerline and is one-to-one between the points on the given 3D centerline and the resulting 2D centerline. Two consequences of this bijectivity are that there is no self-intersection within the flattened centerline, and that the distal point is always the farthest point from the proximal. The latter property allows us to use a convex enclosure, such as the vessel hull³ or the bounding box (to be introduced in Sec. III A 1), to simplify the geometry of multiply branching vessel centerlines.

II.C. Articulated object models

An articulated object is a piecewise rigid object in which the rigid parts of the object are connected by joints. Each joint imposes kinematic constraints on the parts, which are either translational (in a prismatic joint) or rotational (in a revolute joint). The kinematic constraints imposed on the articulated object are then modeled as a graph. Each vertex in the graph corresponds to a part. Two vertices are connected by an edge if the two corresponding parts are connected by a joint.

Reviewing how the articulated object model has been used in the human posture estimation problem^{11–15} in computer vision illustrates how the articulated object model is applied to our method. In the articulated object model of the human body, body parts (e.g., head, torso, arms, and legs) are treated as rigid objects and kinematic constraints are imposed with only rotational degrees of freedom. This topology is represented by a rooted tree. A tree, however, cannot model relations between parts that do not constitute a parent-child pair in the structure. For example, if the left hand is occluded by the right hand, the system gets confused since it is unable to find the left hand. Much effort has been made to overcome this limitation.^{13,14} Popular approaches include augmentation of the tree by adding additional edges that represent occlusion constraints among parts that can occlude each other.

In our method, we adopt the articulated model to represent the anatomy of the abdominal aortic vessel tree. The tree structure, which is a natural representation of the anatomical topology, is converted directly to the kinematic constraints of the parts. The constraints on overlaps can be interpreted as the occlusion constraints among parts. There are two distinct features of the aortic tree visualization problem when applying the articulated object model. First, individual variability of vascular anatomy entails that a single underlying tree cannot be applied to all cases. Second, the choice of joints is somewhat arbitrary. The first aspect means that the tree structure needs to be constructed from data. The latter feature can be used to maintain the size of the tree.

III. METHODS

III.A. Algorithm

Figure 2 illustrates graphically the steps of our algorithm, described in the following subsections.

III.A.1. Preprocessing

We assume that each vessel segment of the abdominal aortic vessel tree is described by a sampled centerline and that each sample point of the centerline is associated with the diameter of the vessel at the point [Fig. 2(a)]. The topology of the branches is described naturally as a rooted tree, where the most proximal aortic vessel segment becomes the root. In order to obtain a 2D representation, the mapping used in the stretched CPR of an individual branch is applied to individual centerlines. Given a viewing direction, the ruling vector for the stretched CPR is chosen as one perpendicular to the viewing direction and the principal direction of the centerline, obtained by a principal component analysis of the sample points, as suggested by Saroul *et al.*¹⁰ Then, the obtained centerlines are rendered so that the proximal point of a child vessel coincides with the distal point of the parent. In this way, the tree topology is maintained in the 2D representation. The resulting 2D tree is essentially a multipath CPR (Ref. 3) without lumen information and may exhibit intersections of the projected centerlines [Fig. 2(b)].

For each vessel segment of the 2D representation of the abdominal aortic vessel tree, the tubular shape comprised of the stretched centerline, and the diameter along its length is enclosed by a bounding box, aligned with the vector connecting the proximal and the distal points of the stretched centerline. The height of the bounding box is the length of this vector plus the radii associated with the two endpoints of the stretched centerline. The width of the bounding box is the sum of the distances of the centerline points farthest from the vector on each side, with the radii of the two farthest points added.

By fixing the width and the height, we consider the bounding box a rigid part of the articulated object model of the 2D abdominal aortic vessel tree. We reserve the term *vessel part* in order to refer to the bounding box that encloses the vessel segment. Revolute joints are placed onto the vessel part at the locations of the proximal and the distal points of the vessel segment. A natural kinematic constraint, which the proximal joint of a child vessel part must coincide with the distal joint of the parent, is assumed for each parent-child pair. With this constraint, the configuration of the vessel parts is described fully by the orientations of the individual vessel parts. The resulting topology of the vessel parts is a rooted tree identical to the topology of the vessel segments, where the vessel parts correspond to vertices and the kinematic constraints on the joints correspond to edges [Fig. 2(c)].

In order to approximate the tubular geometry of a vessel segment, the shape of bounding boxes is kept thin. If the aspect ratio (width/length) of the bounding box corresponding to a vertex of the tree is beyond a threshold, the vertex is subdivided into two vertices and an edge. New bounding boxes are computed for the two subdivided vertices. In this way, the tree structured articulated object model is constructed in a data-dependent manner. The subdivision also increases the degrees of freedom of the articulated object model and reduces the chance of overlap between large bounding boxes [Fig. 2(d)]. The configuration of the subdivi-

FIG. 2. Graphical illustration of the algorithm. (a) Surface rendering of the abdominal aortic vessel tree, obtained from the CTA of a patient, together with the 3D centerlines in the anteroposterior viewing direction. (b) 2D centerlines of this vessel tree and the associated vessel diameters. (c) Bounding box representations of the vessel tree, obtained as described in Sec. III A 1. Note that the shape of the bounding box is determined by the vessel diameters as well as by the vessel centerline. (d) Subdivided bounding boxes, obtained as described in Sec. III A 1. These subdivided bounding boxes follow the shape of the arteries more faithfully than the bounding boxes in (c). (e) Uncluttered bounding box configuration, obtained using the optimization procedure described in Sec. III A 3. The 2D vessel centerlines and diameters are transformed according to the new bounding box configuration. (f) Stylistic visualization of (e), produced using our distance surface renderer. The vessel segments were drawn with widths proportional to the associated vessel diameters, and color coded by the percentage relative to the median diameter of each segment.

vided vessel parts is again represented by the orientations of the vessel parts with respect to a common coordinate system. The set of orientations of the subdivided vessel parts is the variable of the optimization problem described in the following sections, and its initial value serves the role of the reference configuration with respect to which a new configuration is evaluated.

III.A.2. Problem formulation

Two criteria. Given the articulated object model of the abdominal aortic vessel tree, our objective is to find a con-

figuration of the vessel parts that (1) minimizes overlap among bounding boxes and (2) minimizes the deviation of the visualization from the reference configuration. Obviously, unless the reference configuration is free from overlap, these two goals conflict. This is a typical multicriteria optimization, and we formulate the metrics of each criterion and construct a single score function comprised of a linear combination of both criteria.

Overlap metrics. The overlap between two vessel segments is quantified by the sum of the pairwise overlap metrics for all pairs of vessel parts in their respective configura-

FIG. 3. An illustration of a configuration of a pair of bounding boxes. Notations are introduced in the text.

tions. For any vessel part pair (v, w) , the areas of two bounding boxes and their intersection area are denoted by A_v , A_w , and A_{vw} , respectively (Fig. 3). The orientations of the bounding boxes in the pair (v, w) are denoted by Θ_v and Θ_w . Note that A_{vw} is a function of Θ_v and Θ_w , i.e., $A_{vw} = A_{vw}(\Theta_v, \Theta_w)$. Then, the function

$$\rho(\Theta_v, \Theta_w) = \max(A_{vw}(\Theta_v, \Theta_w)/A_v, A_{vw}(\Theta_v, \Theta_w)/A_w) \quad (1)$$

is the larger of the two ratios of the intersection area to the vessel part area. This function is between 0 and 1, minimized at 0 when the two vessel parts do not overlap, and maximized at 1 when the two overlap completely.

There is a question of whether to count an overlap of a joint-sharing pair of vessel parts, i.e., the parts that constitute either a parent-child pair or a sibling, because some overlap of the parts at the joint is unavoidable. If the overlap is counted, the relative orientation between the vessel parts would tend toward $\pm 90^\circ$ or 180° since at these orientations the overlap area is minimized. On the other hand, if the overlap is not counted, the two parts of a pair that hide the majority of each other could tend to retain their relative orientation. Thus, it is desirable to keep the overlap metrics for joint-sharing vessel part pairs small and regularize them by deviation metrics.

These observations suggest a pairwise overlap metric

$$\omega(\Theta_v, \Theta_w) = \begin{cases} \alpha \cdot \tilde{\rho}(\Theta_v, \Theta_w) & \text{if } v \text{ and } w \text{ share a joint} \\ \rho(\Theta_v, \Theta_w) & \text{otherwise,} \end{cases} \quad (2)$$

where $0 < \alpha < 1$ and

$$\tilde{\rho}(\Theta_v, \Theta_w) = \begin{cases} 0, & 0 \leq \rho(\Theta_v, \Theta_w) < a \\ \frac{1}{b-a}(\rho(\Theta_v, \Theta_w) - a), & a \leq \rho(\Theta_v, \Theta_w) < b \\ 1, & b \leq \rho(\Theta_v, \Theta_w) \leq 1 \end{cases} \quad (3)$$

with $0 \leq a < b \leq 1$. The $\tilde{\rho}(\Theta_v, \Theta_w)$ is designed so that unavoidable overlap is penalized less severely than avoidable overlap among non-joint-sharing pairs.

The total overlap of the given configuration is defined by the sum of the pairwise metrics over all pairs of vessel parts; thus,

FIG. 4. An illustration of a configuration of a pair of joint-sharing bounding boxes. Notations are introduced in the text. (a) The reference configuration. (b) A new configuration for which the deviation from the reference configuration is evaluated.

$$\Omega(\Theta_1, \dots, \Theta_{|V|}) = \sum_{(v,w) \in V \times V} \omega(\Theta_v, \Theta_w) = \sum_{(v,w) \in J} \rho(\Theta_v, \Theta_w) + \alpha \cdot \sum_{(v,w) \in (V \times V) \setminus J} \tilde{\rho}(\Theta_v, \Theta_w). \quad (4)$$

Here, V denotes the index set (vertex set) of the vessel parts. $J = E_k \cup E_s$ is a set of joint-sharing vessel part pairs. The set

$$E_k = \{(v, w) | v, w \in V, \text{ vessel part } v \text{ is the parent of vessel part } w \text{ or vice versa}\}$$

is the edge set of the underlying tree $T = (V, E_k)$ of the articulated object model constructed in Sec. III A 1, and the set

$$E_s = \{(v, w) | \exists u \in V \text{ s.t. } (v, u), (u, w) \in E_k \text{ and } u \neq v, w\}$$

is the additional edge set for the sibling pairs. The scale factor α introduced in Eq. (2) has an interpretation here as the contribution of the joint-sharing pairs of vessel parts to the total overlap.

Using the rectangular geometric primitive greatly simplifies computation of the overlap. In Eq. (1), the areas A_v and A_w of the bounding boxes v and w are trivial to compute. The area of the intersection $A_{vw}(\Theta_v, \Theta_w)$ can be computed efficiently using the intersection area algorithm for two convex planar polygons.¹⁶

Deviation metrics. We measure the deviation of a configuration from the reference configuration as the sum of the pairwise deviation metrics for all pairs of joint-sharing vessel parts. The orientations of the bounding boxes in the pair (v, w) for the reference configuration are denoted, respectively, by $\Theta_v^{(\text{ref})}$ and $\Theta_w^{(\text{ref})}$ [Fig. 4(a)], and their orientations in a deviated configuration are denoted by Θ_v and Θ_w [Fig. 4(b)]. Then, the pairwise deviation metric is defined as the squared Euclidean norm of the difference in the unit vector

corresponding to the relative representations of the orientation between pairs

$$\delta(\Theta_v, \Theta_w) = \left\| \begin{pmatrix} \cos(\Theta_v - \Theta_w) \\ \sin(\Theta_v - \Theta_w) \end{pmatrix} - \begin{pmatrix} \cos(\Theta_v^{(\text{ref})} - \Theta_w^{(\text{ref})}) \\ \sin(\Theta_v^{(\text{ref})} - \Theta_w^{(\text{ref})}) \end{pmatrix} \right\|^2. \quad (5)$$

The pairwise deviation metric is designed so that the sum of the metrics over all joint-sharing vessel part pairs is minimized (zero) when there is no change in their relative orientations, i.e., for the reference configuration. The use of trigonometric functions automatically handles the periodicity of the angle.

The total deviation of the given configuration is defined by the sum of the pairwise metrics over all pairs of joint-sharing vessel parts,

$$\Delta(\Theta_1, \dots, \Theta_{|V|}) = \sum_{(v,w) \in J} \delta(\Theta_v, \Theta_w). \quad (6)$$

Score function. The score function evaluating the objective of deviation and overlap minimization for the USIV problem is now defined as follows:

$$\mathcal{E}(\Theta_1, \dots, \Theta_{|V|}) = \Omega(\Theta_1, \dots, \Theta_{|V|}) + \lambda \cdot \Delta(\Theta_1, \dots, \Theta_{|V|}), \quad (7)$$

where λ is the Lagrangian multiplier (i.e., the weight) given to the total deviation. The USIV problem is then cast as an optimization problem that minimizes the score function $\mathcal{E}$ over the configuration space of $(\Theta_1, \dots, \Theta_{|V|})$.

Although the underlying graph of the articulated object model is a tree, Eq. (7) is defined over the complete supergraph $K_{|V|} = (V, V \times V)$ of T . Note that the complete supergraph $K_{|V|}$ is the interaction graph of Eq. (7). There are $O(|V|^2)$ interaction terms.

III.A.3. Optimization

Simulated annealing. Because of the large number of interaction terms (e.g., 780 interaction terms for a model with 40 vessel parts) and the nonlinearity of each term, minimization of Eq. (7) seems difficult. We use simulated annealing (SA),¹⁷ a stochastic optimization algorithm, to minimize the score function of Eq. (7).

The key idea of SA is to interpret the optimization variables as state variables of a probabilistic system and change the internal state according to the transition probabilities of the state variables. SA has many variants according to the choice of the state transition strategy. We chose adaptive simulated annealing (ASA), formerly known as very fast simulated reannealing,¹⁸ because of its popularity and the availability of software.

Constraining the search space. We use domain knowledge to reduce the search space. As the largest artery of the body, the aorta can be considered a frame of reference in relation to which other branches are recognized. The common iliac bifurcation serves a similar role, as it is the most noticeable bifurcation in the abdominal aortic vessel tree. Anchoring the vessel parts that correspond to the aorta and

common iliac arteries creates a configuration that does not differ much from what is expected from the model anatomy (Sec. II A), while reducing the dimension of the problem variables.

In order to identify the vessel parts corresponding to the aorta and the common iliac bifurcation, we use clues given by the branching pattern to identify the dominant branch. Given a vessel part, the child with the largest diameter is considered dominant and is recognized as the continuation of the parent vessel part. The other children are considered branches. If the widths of the children are nearly equal, the child that diverges least from the direction of the parent is declared the dominant branch.¹⁹ A recursive run of this algorithm yields a path of vessel parts starting from the aorta and ending with a branch of one of the two common iliac arteries. A path starting from the other common iliac artery is identified easily and is attached to the first path at the common iliac bifurcation. This combination of the two aorto-iliac paths is fixed and is called a “backbone” of the abdominal aortic vessel tree.

Two-pass optimization. It is possible that SA will not converge to a configuration that eliminates all vessel intersections. In this case, we run a second-pass SA optimizer for only the intersecting branches. The branch intersections are measured indirectly by the sum of the pairwise overlap metrics (2) for the subtrees of the backbone. The second pass has much smaller dimensional problem variables than the first and typically performs faster because we can regard the other nonintersecting branches as a new backbone. More than two passes of SA are undoubtedly possible, but experience shows that two are enough.

III.A.4. The USIV output

From the optimized configuration of the vessel parts, the centerlines enclosed by the bounding boxes are reconstructed [Fig. 2(e)]. Then, a stylistic visualization is generated by modulating the widths of the centerlines by the diameters, thereby constituting an uncluttered single-image visualization [Fig. 2(f)].

III.B. Evaluation

We conducted a preliminary evaluation of our method using a set of real abdominal aortic vessel trees. As our goal is to produce a drawing that could be used as a map for display of anatomical and physiological information, it is important to know if the images that our method produces are recognizable as a map of the underlying structure. We designed our evaluation around investigating this, i.e., whether branches are recognizable, and whether distortion is tolerable, at least subjectively.

III.B.1. Patient data and reference

Five patient scans of abdominal CT angiography, chosen to contain normal and typical anatomic variation, were retrospectively collected from our CTA archive. One had a replaced hepatic artery off of the SMA and two had an acces-

TABLE I. Patient scan parameters.

Parameter	Value
Section resolution	512 × 512
Field of view	299–447 mm
Section thickness	0.625–1.25 mm
No. of slices per patient	508–693

sory renal artery (one on the left side of the abdominal aorta and the other on the right). Among the two cases with typical configuration of anatomy, one case had a very short SMA and common iliac arteries. Table I provides the CT scan parameters. Under the approval of our Institutional Review Board, all the CT images were anonymized as required by HIPAA.

For each of the patient cases, the CTA images of the abdominal aortic vessel tree were segmented using the vascular modeling toolkit (VMTK).²⁰ A radiologist who specialized in vascular imaging labeled the major aortic branches of the resulting segmentation. We used this labeling as our reference for the subjective evaluation (Sec. III B 4). Table II summarizes the labeled branches and describes the anatomical variation among the subjects.

III.B.2. Generating outputs for the patient data

Using the algorithm described in Sec. III A, we obtained uncluttered single-image visualizations in the antero posterior viewing direction for each of the five cases. To address the stochastic nature of the algorithm, ten different variants of USIV outputs, the outcomes of our method with randomly chosen seeds for the SA optimizer, were generated for each case. Centerlines of the arterial branches and the associated vessel diameter information were obtained from the segmented abdominal aortic vessel tree, again using VMTK. The tree structured articulated object model and the reference configuration of vessel parts were built upon the information that VMTK provided as described in Sec. III A 1.

We implemented our algorithm in MATLAB and C on a

TABLE II. Branch composition of the cases. Abbreviations: C=celiac trunk, SA=splenic artery, HA=hepatic artery, SMA=superior mesenteric artery, LRA=left renal artery, RRA=right renal artery, LIIA=left internal iliac artery, RIIA=right internal iliac artery, LEIA=left external iliac artery, REIA=right external iliac artery, RRA1=main RRA, RRA2=secondary RRA, LRA1=main LRA, and LRA2=secondary LRA.

Case	Branch composition
A	C, SA, HA, SMA, LRA, RRA1, RRA2, LCIA, LEIA, LIIA, RCIA, REIA, RIIA
B	C, HA, SA, SMA, LRA, RRA, LCIA, LEIA, LIIA, RCIA, REIA, RIIA
C	C, HA, SA, SMA, LRA1, LRA2, RRA, LCIA, LEIA, LIIA, RCIA, REIA, RIIA
D	C, HA, SA, SMA, LRA, RRA, LCIA, LEIA, LIIA, RCIA, REIA, RIIA
E	SA, HA (replaced), SMA, LRA, RRA, LCIA, LEIA, LIIA, RCIA, REIA, RIIA

TABLE III. Algorithm parameters.

Category	Parameter	Value
Subdivision	Aspect ratio	10
Total deviation weight	λ	4×10^{-3}
Joint-sharing overlap weight	α	2×10^{-4}
Joint-sharing pairwise overlap linear region	a	0.2
	b	0.5

GNU/Linux system with a 3 GHz AMD processor with 8 Gbyte RAM. The publicly available ASA library²¹ and its MATLAB gateway²² were used in the software. Table III gives the values of the parameters used, which were selected by trial and error for case A and applied to the other four cases. Note that while we did not tune these parameters for more than one dataset, we did perform an analysis to determine the sensitivity of the results to these settings (Sec. V G). We stopped the first pass of the algorithm after 20 000 iterations. The second pass, if required, was stopped after 5000 iterations. The execution time of the implemented method varied over the five cases; mean $\pm$ standard deviation and the number of vessel parts for each case were (A) 176 ± 0.175 s and 53, (B) 258 ± 51.6 s and 59, (C) 223 ± 0.392 s and 61, (D) 314 ± 26.8 s and 65 (E) 732 ± 53.8 s and 72. The mean and standard deviation for all 30 variants were 341 and 207 s, respectively. Three variants of them required the second-pass optimization, whose execution time was included in the aforementioned measurement.

For each variant, from the vessel part configuration that our method produced, and with the centerlines and the diameter information associated with the parts, a stylistic visualization (USIV) was generated using a distance surface renderer.²³ These visualizations were evaluated both quantitatively and subjectively, as follows.

III.B.3. Using the score to determine candidate visualizations for evaluation

From the ten variants for each of the five cases, we chose six different variants of USIV outputs. We selected the three variants with the lowest scores [SA-optimized values of Eq. (7)] and the three with the highest.

For each case, the score rankings of the six variants were kept for cross-case comparison of the visualizations. Note that the SA optimization scores were only comparable among the variants of a given case, not across cases. The rankings of the variants, however, can be compared across the cases.

III.B.4. Subjective rating

The purpose of this subjective rating study was to see whether the distortion of the visualizations introduced by our method, with regard to the expected appearance of branches, was acceptable to clinicians. Three readers independently labeled the branches on the 30 stylistic visualizations produced by our method. All the readers were trained radiologists and are familiar with the 3D anatomy of the abdominal aorta. For

each branch label of a visualization, the radiologists labeled the branches with their anatomical names, reported their confidence in this labeling, and rated distortion with regard to the expected appearance of the branch on a three-point scale (0=low, 1=medium, and 2=high). The labels were compared to the ground truth set by the independent radiologist in Sec. III B 1.

III.B.5. Quantitative metrics

The overlap among the branches in the reference configuration and the configuration produced were evaluated according to the following metrics.

Centerline intersection. This metric counts the intersection of the centerlines in each branch b_i of the backbone with the centerlines of all other branches. An overlap due to the diameter of the branches is not included in this metric.

Overlap proportion. This metric measures the overlap due to the diameter of the branches. For every vessel part v in a given branch b_i of the backbone, and for every vessel part w of the other branch b_j of the backbone, we compute the intersection area ratio A_{vw}/A_v , where A_{vw} and A_v are as defined in Sec. III A 2. The average of the ratios of intersection area, denoted as r_{ij} , is roughly a measure of the proportion of b_i overlapped by b_j because of their widths. Finally, the average of r_{ij} over all b_j 's, weighed by the number of vessel parts in b_j , and the maximum of r_{ij} over all b_j 's are taken and used as the overlap proportion metrics of b_i in the presence of all the other branches. Since average ratio r_{ij} overestimates the proportion of the area of b_i overlapped by b_j , the overlap proportion metrics may exceed unity.

III.B.6. Association study

To verify the formulation of the score function, we evaluated the association between the score rankings introduced in Sec. III B 3 and the subjective ratings by the radiologists (Sec. III B 4) using statistical methods. We studied whether the score ranking was a reasonable predictor of the reader's subjective distortion ratings.

IV. RESULTS

IV.A. Subjective Ratings

The labeling accuracies of the three readers were 93.4%, 94.5%, and 95.4%, respectively, for the 366 branches of the 30 visualizations. The corresponding 95% confidence intervals were, respectively, 90.4%–95.7%, 92.7%–97.3%, and 91.7%–96.6%. For the 1098 total samples (366 branches $\times$ 3 readers), the combined labeling accuracy was 94.4% (with 95% confidence interval 92.9%–95.7%), the subjective distortion ratings were 77.4%, low; 10.5%, medium; and 12.1%, high. The confidence ratings were 5.56%, low; 16.5%, medium; and 77.9%, high.

The three subjective metrics were correlated. The distortion ratings were negatively correlated with the confidence ratings (Spearman $r = -0.36$, $p < 0.0001$). Higher accuracy was associated with higher confidence ratings (low: 77% accuracy; medium: 88%; high: 97%; $p < 0.0001$, Fisher's exact

test). Incorrectly labeled branches had higher distortion ratings than correctly labeled ones (mean 1.95 vs 1.31; $p < 0.0001$, Mann–Whitney test).

IV.B. Quantitative metrics

Table IV reports the overlap metrics for the 30 visualizations. For each case, the table lists the number of centerline intersections and the percentage reduction in the number of centerline intersections, the average overlap proportion metric, and the maximum overlap proportion metric of the configurations produced. For the three metrics, the mean and the standard deviation of the six variants are reported for each branch.

The overlap metrics agreed reader's visual inspection; no branch crossing were reported. Table IV and Fig. 5 show that the intersections among the centerlines were completely removed for all the variants in each of the five cases. The average and the maximum overlap proportion metrics were also significantly reduced. This reduction was most noticeable for the splenic, the hepatic, and the superior mesenteric arteries, as well as for the backbone.

IV.C. Association study

There was no significant monotonic trend in the association between the variant rankings and the total subjective distortion rating, defined as the sum of each subjective distortion rating given by a reader for each branch of each variant ($p = 0.1132$, Jonckheere–Terpstra test). However, the lowest ranking variants had significantly lower distortion ratings than the highest ranking variants ($p < 0.02$, Mann–Whitney test).

IV.D. Visual display of outputs

Figure 5 provides a visual display of the outputs of our method. All of the 30 visualizations are presented by case and by variant, sorted from the lowest score (best) to the highest score (worst).

V. DISCUSSION

V.A. Subjective ratings

The reader's labeling accuracy was high overall (94.4%). Errors tended to occur in pairs. Of the mislabelings that take only 5% of the branches, one-third involved the external and internal iliac arteries (whether right or left). This mislabeling is plausible since given a prior knowledge of anatomy, if branch b_A is mislabeled b_B , then highly correlated branch b_B is likely to be also mislabeled b_A . Section V D discusses this subject further.

V.B. Quantitative metrics

The noticeable reduction in overlap on the splenic, the hepatic, and the superior mesenteric arteries indicates that the false intersection due to the anteroposterior projection generally involves these three branches. The right renal artery (RRA) of case D exhibited a small increase in overlap

TABLE IV. Branch-level overlap metrics. For each of the metrics, the mean of the corresponding metric is taken over the size variants, and its percentage *reduction* from the corresponding metric of the reference configuration is presented on the first line, by case and branch. A negative entry means an increase in the overlap proportion. The metrics of the reference configuration are given in parentheses on the second line. Abbreviations follow the convention of Table II.

Case	Category	Value							
A	Branch label	Backbone	C+SA+HA	SMA	LRA	RRA1	RRA2	LIIA	RIIA
	Centerline intersections	100% (5)	100% (2)	100% (5)	100% (0)	100% (3)	100% (2)	100% (0)	100% (1)
	Avg. overlap proportion	65.80% (0.082)	67.19% (0.060)	92.77% (0.210)	23.86% (0.046)	43.07% (0.105)	46.11% (0.148)	19.45% (0.128)	72.33% (0.195)
	Max. overlap proportion	74.64% (0.187)	72.20% (0.215)	94.56% (0.971)	10.26% (0.189)	-1.54% (0.266)	27.45% (0.499)	19.45% (0.733)	72.33% (1.098)
B	Branch label	Backbone	C+SA+HA	SMA	LRA	RRA	LIIA	REIA	
	Centerline intersections	100% (3)	100% (1)	100% (3)	100% (1)	100% (0)	100% (0)	100% (0)	
	Avg. overlap proportion	67.49% (0.087)	78.08% (0.086)	89.91% (0.228)	38.82% (0.072)	3.38% (0.066)	10.14% (0.168)	10.38% (0.061)	
	Max. overlap proportion	71.68% (0.167)	74.07% (0.156)	89.12% (0.494)	-3.14% (0.111)	0.02% (0.161)	10.14% (0.472)	9.02% (0.165)	
C	Branch label	Backbone	C+SA+HA	SMA	LRA1	LRA2	RRA	LEIA	REIA
	Centerline intersections	100% (4)	100% (4)	100% (5)	100% (3)	100% (0)	100% (2)	100% (0)	100% (0)
	Avg. overlap proportion	54.24% (0.082)	75.73% (0.073)	85.91% (0.399)	72.12% (0.288)	6.87% (0.076)	70.44% (0.148)	0.72% (0.155)	-0.56% (0.199)
	Max. overlap proportion	59.31% (0.146)	74.76% (0.163)	88.87% (1.036)	58.86% (0.603)	-11.61% (0.218)	54.21% (0.292)	0.72% (0.561)	-0.56% (0.721)
D	Branch label	Backbone	C+SA+HA	SMA	LRA	RRA	LIIA	RIIA	
	Centerline intersections	100% (0)	100% (0)	100% (0)	100% (0)	100% (1)	100% (0)	100% (0)	
	Avg. overlap proportion	37.65% (0.044)	69.41% (0.099)	74.29% (0.267)	-0.15% (0.051)	-7.75% (0.055)	5.24% (0.178)	0.70% (0.311)	
	Max. overlap proportion	33.98% (0.073)	61.32% (0.213)	67.10% (0.625)	0.97% (0.135)	-7.39% (0.148)	5.24% (0.566)	0.70% (0.992)	
E	Branch label	Backbone	SA	SMA+HA	LRA	RRA	LIIA	RIIA	
	Centerline intersections	100% (1)	100% (1)	100% (6)	100% (3)	100% (2)	100% (0)	100% (0)	
	Avg. overlap proportion	80.48% (0.141)	43.37% (0.060)	90.06% (0.287)	18.30% (0.074)	49.22% (0.123)	43.85% (0.219)	-0.44% (0.103)	
	Max. overlap proportion	83.00% (0.317)	58.56% (0.134)	91.91% (0.756)	7.79% (0.142)	42.55% (0.269)	43.85% (0.808)	-0.44% (0.378)	

proportion. This is because the RRA in the reference configuration takes off the aorta at almost 90° , and our method produced configurations with the take-off angle at other values. Since the overlap between two bounding boxes tends to be minimized when the two meet at a right angle, deviation from a 90° angle is somewhat over-represented in the overlap proportion metrics. The secondary left renal artery (LRA2) of case C exhibited a similar phenomenon, although less severe than case D, with an increase only in the maximum overlap proportion.

V.C. Association study

We can claim that the lowest ranking variants (in fact, the best among the ten in terms of score) have lower distortion ratings than the highest ranking variants (the worst of the

ten) in spite of the lack of monotonic association between the variant rankings and the total subjective distortion rating.

The significant difference between the best and the worst variants illustrates that our heuristic of running the algorithm several times with different random seeds and selecting one with the best score is reasonable. Although we cannot guarantee that the sorting of the outputs by score will yield subjective ratings of distortion in that order, the best variants (with the lowest score) will have the lowest overall subjective distortion rating.

V.D. Effect of anatomical variants

We examined if the anatomical variants among the cases affected a reader's subjective ratings. There were differences in subjective distortion ratings among the five cases (p

FIG. 5. Visual display of the outputs of the USIV method. Each row of the array of visualizations begins with the anteroposterior projection of the abdominal aortic vessel tree reconstructed from the segmentation and the surface extraction. The USIV outputs of the corresponding vessel tree are presented, arranged with the score ranking, from the best to the worst.

<0.0001 , Kruskal–Wallis test). Indeed, it was pointed out by the radiologist who set the reference standard that case D, characterized by the short SMA and common iliac arteries, and case E, characterized by the replaced hepatic artery, exhibited particularly abnormal anatomies. A more elaborated regression study (not shown) found that peculiar anatomy had a positive (i.e., increasing distortion) effect in subjective distortion rating.

V.E. Choice of viewing direction

We examined the effect that a different choice of viewing direction might have on the number of overlapping branches

by looking at alternate reference configurations generated over a range of viewing angles about the cranial-caudal axis. For each angle, we tabulated the number of centerline intersections, by case and by branch, in the same manner as described in Sec. III B 5. Table V tabulates these results, which show that in no case are the numbers of intersections reduced to zero, and that cannot it be guaranteed that the number of intersections will be reduced. We acknowledge that using one of these alternate configurations as the starting point for our optimization could produce results with less distortion, but given the variance in the resulting distortion [Fig. 6(b)] due to the randomization provided by the algo-

TABLE V. Change in viewing directions. The centerline intersection metric (Sec. III B 5) is presented for each viewing direction, by case and branch. The viewing direction is represented by the azimuthal angle of the vessel tree, which varies from -30° to $+30^\circ$ about the cranial-caudal axis, with the zero azimuthal angle corresponding to the reference configuration given in Table IV. Abbreviations follow the convention of Table II.

Case	Angle (deg)	Branch label								(Total)
		Backbone	C+SA+HA	SMA	LRA	RRA1	RRA2	LIIA	RIIA	
A	-30	3	6	3	1	2	2	0	0	17
	-20	3	6	3	2	2	2	0	0	18
	-10	5	3	4	1	2	2	0	1	18
	0	5	2	5	0	3	2	0	1	18
	10	6	1	5	0	2	2	0	0	16
	20	2	1	3	0	1	1	0	0	8
	30	2	1	3	0	1	1	0	0	8
B		Backbone	C+SA+HA	SMA	LRA	RRA	LIIA	REIA		(Total)
	-30	3	1	2	1	0	0	1		8
	-20	3	1	2	1	0	0	1		8
	-10	3	1	3	1	0	0	0		8
	0	3	1	3	1	0	0	0		8
	10	5	2	7	2	0	0	1		17
	20	8	2	9	1	0	0	1		21
	30	7	1	8	2	0	1	1		20
C		Backbone	C+SA+HA	SMA	LRA1	LRA2	RRA	LEIA	REIA	(Total)
	-30	3	4	5	1	2	2	0	0	17
	-20	3	2	6	1	2	1	0	0	16
	-10	2	4	5	3	1	2	0	0	17
	0	4	4	5	3	0	2	0	0	18
	10	2	4	3	1	0	2	0	0	12
	20	3	5	5	1	0	3	1	0	18
	30	2	7	5	1	0	3	1	0	19
D		Backbone	C+SA+HA	SMA	LRA	RRA	LIIA	RIIA		(Total)
	-30	1	2	2	2	0	0	0		7
	-20	2	3	3	0	1	0	0		9
	-10	0	2	2	0	1	0	0		5
	0	0	0	0	0	1	0	0		1
	10	1	1	0	0	1	0	0		3
	20	1	1	0	0	1	0	0		3
	30	1	1	0	0	0	0	0		2
E		Backbone	SA	SMA+HA	LRA	RRA	LIIA	RIIA		(Total)
	-30	3	1	5	2	2	0	1		14
	-20	2	1	5	2	1	0	0		13
	-10	4	2	6	1	2	0	0		15
	0	1	1	6	3	2	0	0		13
	10	5	2	6	0	2	1	0		16
	20	2	2	4	0	2	1	0		11
	30	0	3	3	0	3	0	0		9

rithm, it is not clear adding another degree of freedom (i.e., the choice of reference configuration) would be fruitful.

V.F. Score function

V.F.1. SA as local optimization

The use of SA in this paper can be considered an enhanced local optimization method that may escape a few but not all local minima of the score function that is difficult to minimize. SA is an instance of a stochastic global optimization algorithm that finds the globally optimal solution only

probabilistically, after an indefinite number of iterations.²⁴ In practical problems in science and engineering, however, since there is no deterministic guarantee of reaching the global minimum, SA has been used in applications where a few “good enough” solutions that reduce the score function by a sufficient amount are tolerated, stopping after a finite number of iterations.²⁵ Our application falls into this category of problems. As demonstrated by experiments, there may be several solutions that are equally useful; i.e., we do not necessarily need the global minimum to have a useful result.

FIG. 6. Sensitivity to the weight of the total deviation. The mean and standard error of (a) average branch rotation, (b) average branch distortion, and (c) average overlap reduction proportion, computed as described in Sec. III B 5, taken from the six variants of each case. (d) Overlap count: Number of variants with branch overlaps, visually counted for all six variants of each case. Relative weight 1 corresponds to the value of λ reported in Table III. The scale of the relative weights is linear for the values from 0.8 to 1.2 and is nonlinear from 0.1 to 0.8 and from 1.2 to 10.

V.F.2. The weight factors

Since the weight factors α and λ reported in Table III are very small, there might be a concern regarding the usefulness of the joint optimization. The key is in the scales of the terms combined by these factors. In Eq. (4), α is multiplied to the sum of $|V|^2 - |J|$ pairwise overlap metrics (non-joint-sharing). This sum is added to the sum of $|J|$ pairwise overlap metric (joint-sharing). Since $|V|^2 \gg |J|$, α scales down the effect of the non-joint-sharing terms. Taking this into account, the contribution of the $|V|^2 - |J|$ non-joint-sharing metrics is about 15–100 times larger than that of the $|J|$ joint-sharing

metrics. Similarly, in Eq. (7), the contribution of the total deviation term, after multiplied by λ , is about 0.1–2 times that of the total overlap term.

V.G. Sensitivity of parameters

This section describes experiments we performed with the USIV method to determine the sensitivity of the results to the relative change in coefficients in the score function, i.e., the weight λ given to the total deviation $\Delta(\cdot)$ in Eq. (7) and the contribution α of joint-sharing pairs to the total overlap $\Omega(\cdot)$ in Eq. (4). We also examined the sensitivity of the parameters a and b of the pairwise overlap metric (3) for joint-

sharing pairs. We perturbed the values reported in Table III, as described below, in order to study the sensitivity of the results to these parameters.

V.G.1. The weight of the total deviation

We examined the weight given to the total deviation function $\Delta(\cdot)$, which determines the relative importance of deviation with respect to overlap. We scaled the weight λ by 0.1, 0.8, 0.9, 1.1, 1.2, and 10 from the reported value. The other parameters of the score function were held constant. The scales 0.8–1.2 were intended to study the sensitivity to moderate changes. The scales 0.1 and 10 were chosen to see the effect of large changes in the weight. Figure 6 plots the means and the standard errors of the average branch rotation, the average branch distortion, the average overlap reduction proportion, and the overlap count, or number of variants with branch overlaps (visually counted for all six variants) for the results of each case, respectively. As seen from the figure, for a moderate change (0.8–1.2) in the weight, the deviation represented by branch rotation and branch distortion did not vary much within the standard error. For a large change, there is a rough trend of decrease in the deviation as the weight increases from 0.1 to 10. This is expected since a large weight was given to the (total) deviation relative to the (total) overlap. As a trade-off, however, there were a few variants whose overlaps were not completely resolved [Fig. 6, panel (d), at the relative weight $\lambda=10$].

V.G.2. The contribution of joint-sharing pairs to the total overlap

We examined the contribution of joint-sharing vessel part pairs in the total overlap function $\Omega(\cdot)$, which determines the relative importance of the overlap of joint-sharing pairs with respect to the overlap of non-joint-sharing pairs. We scaled the weight α by 0.1, 0.8, 0.9, 1.1, 1.2, and 10 from the reported value, with the other score function parameters held constant, as above. Figure 7 plots the means and the standard errors of the average branch rotation, the average branch distortion, the average overlap reduction proportion, and the overlap count, or number of variants with branch overlaps (visually counted for all six variants) for the results of each case, respectively. As seen from the figure, the deviation represented by branch rotation and branch distortion remains about the same for a large range of the change of the weight, as well as for a moderate change (0.8–1.2). This is expected since the contribution of joint-sharing pairs of vessel parts to the total overlap was small relative to non-joint-sharing pairs. However, one variant had overlaps not completely resolved, at the relative weight $\alpha=10$.

V.G.3. Parameters of the pairwise overlap metric

The parameters a and b determine the linear region of the pairwise overlap metric (3) designed for joint-sharing pairs. We reparameterized them with the width $w=b-a$ and the center $c=(a+b)/2$ and perturbed each parameter from the reported value, which was empirically chosen, while holding

the other fixed at the reported value. We examined the widths 0.1, 0.2, 0.3, 0.4, and 0.5, where 0.3 is the reported value. For the centers we tested the values of 0.15, 0.25, 0.35 (reported), 0.45, and 0.55. The result indicated a trend similar to Figs. 6 and 7, evidencing the stability of these parameters. In particular, the overlap counts were all zero, except for case C when the width was set to 0.1 and 0.2, and the center 0.15 (the overlap counts were 1).

V.H. Execution time

The execution time of our visualization method for the cases evaluated ranged from 3 to 12 min. The distribution of the execution time depended on case, suggesting an effect of the complexity of the articulated object model to the computation of the score function for each iteration of the SA optimization step. In fact, a monotonic trend between the number of vessel parts in the model and the execution time was asserted ($p < 0.0001$, Jonckheere–Terpstra test). Case D with the shortest mean execution time was comprised of 53 vessel parts, while case E with the longest mean execution time was made of 72 vessel parts. A subdivision scheme that restricts the complexity of an articulated object model for the data may improve the running time of the method.

V.I. Limitations

There are four potential limitations to our visualization method. First, our method requires a reliable extraction of the vessel centerlines and diameters. There are many ways of defining the centerline and the diameter of a blood vessel,^{26–30} and the results of these methods do not necessarily agree with each other. Of the available extraction methods, we used VMTK, which extracts a centerline as the trace of the maximum inscribed spheres of the vessel surface.³¹ We chose to use VMTK because it provides a robust decomposition of bifurcating vessels,³² important in our application which handles tens of centerlines. However, it should be noted that our method is compatible with any other centerline extraction method. Nevertheless, we rely on the centerline algorithm to give us the correct locations of branch points; incorrect determination of the branch points could result in larger deformations.

Second, there is no theoretical guarantee of complete removal of overlap among the branches. Although reducing the value of the total overlap (4) generally results in a configuration with reduced branch overlaps, this is not a guarantee of zero branch intersections. Our method takes an indirect approach to quantify the overlaps, i.e., the use of bounding boxes. A more direct approach of counting overlaps, however, may be impractical as it requires more computation time owing to the combinatorial aspect of overlap (or collision) detection. If only centerlines were involved, detection of their crossings might not be too complicated; but the associated vessel diameters along their lengths make the geometry complex and, therefore, detection of overlap computationally difficult.³³ More importantly, bifurcations or trifurcations in the abdominal aortic vessel tree make the perfect removal or detection of branch overlaps very chal-

FIG. 7. Sensitivity of the contribution of joint-sharing pairs to the total overlap. The mean and standard error of (a) average branch rotation, (b) average branch distortion, and (c) average overlap reduction proportion, computed as described in Sec. III B 5, taken from the six variants of each case. (d) Overlap count: Number of variants with branch overlaps, visually counted for all six variants of each case. Relative weight 1 corresponds to the value of α reported in Table III. The scale of the relative weights is linear for the values from 0.8 to 1.2 and is nonlinear from 0.1 to 0.8 and from 1.2 to 10.

linging. Our use of the bounding box primitives and their overlap areas as a surrogate to the overlap count is a compromise between these difficulties and the need for quantifying overlaps, as well as for reducing computational demand.

Third, this preliminary study involved only five cases with six variants for each because of the time required of the radiologists in the subjective portion of the evaluation (about 2 h per radiologists to review all 30 images). Although the results are promising, this small sample size limits conclusions that can be drawn from the study of the association between subjective distortion rating and score ranking. Also, small sample size precluded setting parameters using some studies and testing on others. We instead picked the required parameters based on a single case and then applied them to

others. We then performed a sensitivity analysis to show that our choice was not critical over reasonable ranges. Fourth, while our method presents a topology-preserving schematic of the vessel tree, other features of the branches, e.g., tortuosity, may not be preserved.

Finally, it was observed in the association study that the symmetry in the anatomy may cause confusion in identification of the branches. Much of the labeling error was due to the confusion of the internal and external iliac arteries, which come in pairs. Incorporating a visualization convention, such as fixing the orientation so that the right external iliac artery is always placed to the right of the right internal iliac artery, may help reduce confusion. This requires an identification of the internal and external arteries by the algorithm. A branch

identification system⁵ may be combined with our method to produce a overlap-reduced single-image visualization with correct labeling of the branches.

VI. CONCLUSION

We have presented a method for uncluttered single-image visualization that can be used as a map for displaying patient-specific anatomy of the abdominal aorta and its major branches. Our method assumes a geometric representation of the vessel tree, such as the centerlines and the vessel diameter information that may be obtained from other available tools, but does not assume further knowledge of anatomy, such as the names of individual vessels. Our method is capable of eliminating misleading false intersections in the conventional projection of a treelike vascular structure by applying a simulated annealing based stochastic optimization method. Our method conserves the overall topology of the vasculature, represents it with minimal distortion, and can be combined with visualization of clinical information such as vessel diameter or lumen patency in a single image for a large portion of the human vasculature.

We tested our method using 30 visualizations produced from five patient scans with representative anatomical variants in the abdominal aortic tree, evaluated the results using quantitative performance metrics and subjective responses from radiologists, and also analyzed the association between the quantitative metrics while subjective responses. The experimental results show that the method produced visualizations that are acceptable to radiologists in the sense that the branches were recognized with high accuracy and the subjective distortion was low. Having established that the visualizations are recognizable as a map, the future directions of research may be to use the map for display of multimodal information (e.g., stenosis from CT, and flow from MR) and to assess this in an increased number of cases.

ACKNOWLEDGMENTS

This work was supported by the National Institutes of Health under Grant Proposal No. 1R01HL67194 and by NIH MERIT Award No. R37EB02784. The authors would like to sincerely thank Dr. Dominik Fleischmann, Dr. Justus Roos, and Dr. Humberto Wong who participated in the reader study.

^{a)} Author to whom correspondence should be addressed. Electronic mail: jhwon@stanford.edu

^{b)} Electronic mail: jarrett.rosenberg@stanford.edu

^{c)} Electronic mail: grubin@stanford.edu

^{d)} Electronic mail: snapel@stanford.edu

¹ C. P. Langlotz, "RadLex: A new method for indexing online educational materials 1," *Radiographics* **26**(6), 1595–1597 (2006).

² National Library of Medicine, "MeSH—Medical subject headings," (<http://www.nlm.nih.gov/mesh/>).

³ A. Kanitsar, R. Wegenkittl, D. Fleischmann, and M. E. Gröller, "Advanced curved planar reformation: Flattening of vascular structures," Proceedings of the 14th IEEE Conference on Visualization, pp. 43–50, 2003.

⁴ R. Raman, S. Napel, C. F. Beaulieu, E. S. Bain, R. B. Jeffrey, Jr., and G. D. Rubin, "Automated generation of curved planar reformations from volume data: Method and evaluation," *Radiology* **223**, 275–280 (2002).

⁵ K. Mori, J. Hasegawa, Y. Suenaga, and J. Toriwaki, "Automated anatomical labeling of the bronchial branch and its application to the virtual

bronchoscopy system," *IEEE Trans. Med. Imaging* **19**(2), 103–114 (2000).

⁶ S. Oeltze and B. Preim, "Visualization of vasculature with convolution surfaces: Method, validation and evaluation," *IEEE Trans. Med. Imaging* **24**(4), 540–548 (2005).

⁷ H. Hahn, B. Preim, D. Selle, and H. Peitgen, "Visualization and interaction techniques for the exploration of vascular structures," Proceedings of the 11th IEEE Conference on Visualization, pp. 395–402, 2001.

⁸ R. Uflacker, *Atlas of Vascular Anatomy: An Angiographic Approach* (Lippincott Williams & Wilkins, Baltimore, 2006).

⁹ A. Kanitsar, D. Fleischmann, R. Wegenkittl, P. Felkel, and M. E. Gröller, "CPR—Curved planar reformation," Proceedings of the 13th IEEE Conference on Visualization, pp. 37–44, 2002.

¹⁰ L. Saroul, S. Gerlach, and R. D. Hersch, "Exploring curved anatomic structures with surface sections," Proceedings of the 14th IEEE Conference on Visualization, pp. 27–34, 2003.

¹¹ S. X. Ju, M. J. Black, and Y. Yacoob, "Cardboard people: A parameterized model of articulated image motion," Proceedings of the second International Conference on Automatic Face and Gesture Recognition, pp. 38–44, 1996.

¹² P. F. Felzenszwalb and D. P. Huttenlocher, "Pictorial structures for object recognition," *Int. J. Comput. Vis.* **61**, 55–79 (2005).

¹³ X. Lan and D. P. Huttenlocher, "Beyond trees: Common-factor models for 2D human pose recovery," ICCV 2005: Proceedings of the Tenth IEEE International Conference on Computer Vision, Vol. 1, pp. 470–477, 2005.

¹⁴ X. Ren, A. Berg, and J. Malik, "Recovering human body configurations using pairwise constraints between parts," ICCV 2005: Proceedings of the tenth IEEE International Conference on Computer Vision, Vol. 1, pp. 824–831, 2005.

¹⁵ A. Gupta, A. Mittal, and L. Davis, "Constraint integration for efficient multiview pose estimation with self-occlusions," *IEEE Trans. Pattern Anal. Mach. Intell.* **30**(3), 493–506 (2008).

¹⁶ J. O'Rourke, *Computational Geometry in C* (Cambridge University Press, New York, NY, 1994).

¹⁷ S. Kirkpatrick, C. Gelati, Jr., and M. Vecchi, "Optimization by simulated annealing," *Science* **220**(4598), 671–680 (1983).

¹⁸ L. Ingber and B. Rosen, "Very fast simulated re-annealing," *Math. Comput. Modell.* **12**(8), 967–973 (1989).

¹⁹ J. Bloomenthal, "Modeling the mighty maple," SIGGRAPH '85: Proceedings of the 12th Annual Conference on Computer Graphics and Interactive Techniques, pp. 305–311, 1985.

²⁰ L. Antiga, "VMTK—Vascular modeling toolkit," 2006 (<http://www.vmtk.org>).

²¹ L. Ingber, "Adaptive simulated annealing (ASA)," 1993 (<http://www.ingber.com/>).

²² S. Sakata, "ASAMIN: A MATLAB gateway routine to the adaptive simulated annealing software," 2006 (http://www.econ.ubc.ca/ssakata/public_html/software/).

²³ J. Bloomenthal and K. Shoemake, "Convolution surfaces," SIGGRAPH'91: Proceedings of the 18th Annual Conference on Computer Graphics and Interactive Techniques, pp. 251–256, 1991.

²⁴ A. Dekkers and E. Aarts, "Global optimization and simulated annealing," *Math. Program.* **50**, 367–393 (1991).

²⁵ S. Sun, G. D. Rubin, D. Paik, R. M. Steiner, F. Zhuge, and S. Napel, "Registration of lung nodules using a semi-rigid model: method and preliminary results," *Med. Phys.* **34**(2), 613–626 (2007).

²⁶ D. S. Paik, C. F. Beaulieu, R. B. Jeffrey, Jr., G. D. Rubin, and S. Napel, "Automated right path planning for virtual endoscopy," *Med. Phys.* **25**(5), 629–637 (1998).

²⁷ P. Golland, W. Eric, and L. Grimson, "Fixed topology skeletons," CVPR 2000: Proceedings of the 2000 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, Vol. 1, pp. 10–17, 2000 (unpublished).

²⁸ C. Kirbas and F. Quek, "A review of vessel extraction techniques and algorithms," *ACM Comput. Surv.* **36**(2), 81–121 (2004).

²⁹ Y. Fridman, S. M. Pizer, S. Aylward, and E. Bullitt, "Extracting branching tubular object geometry via cores," *Med. Image Anal.* **8**(3), 169–176 (2004).

³⁰ S. Bouix, K. Siddiqi, and A. Tannenbaum, "Flux driven automatic centerline extraction," *Med. Image Anal.* **9**(3), 209–221 (2005).

³¹ L. Antiga, B. Ene-Iordache, and A. Remuzzi, "Computational geometry for patient-specific reconstruction and meshing of blood vessels from MR

- and CT angiography,” *IEEE Trans. Med. Imaging* **22**(5), 674–684 (2003).
- ³²L. Antiga and D. A. Steinman, “Robust and objective decomposition and mapping of bifurcating vessels,” *IEEE Trans. Med. Imaging* **23**(6), 704–713 (2004).
- ³³S. Redon, M. Lin, D. Manocha, and Y. Kim, “Fast continuous collision detection for articulated models,” *J. Comput. Inf. Sci. Eng.* **5**, 126–137 (2005).