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Compensating for Non-Stationary Blurring by Further Blurring and Deconvolution

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Abstract

In many imaging systems, the point spread function (PSF) is non-stationary. Usually a computation-intensive iterative algorithm is used to de-blur the non-stationary PSF. This paper presents a new idea of using a non-iterative method to compensate for the spatially variant PSF. This method first further blurs the image with a non-stationary kernel so that the resultant image has a stationary PSF, then deblurs the resultant image using an efficient deconvolution technique. The proposed method is illustrated and implemented by SPECT (single photon emission computed tomography) applications.

Index Terms

imaging deblurring with shift variant PSF; analytical deblurring; SPECT; PET

I. INTRODUCTION

A SPECT (Single Photon Emission Computed Tomography) collimator has a finite hole diameter and a finite hole length. This gives the collimator hole a non-zero acceptance angle, which determines the distance-dependent spatial resolution of the collimator [1]. The resolution worsens as the distance is increased from the collimator to the object of interest.

This distance-dependent collimator blurring can be compensated for during an iterative image reconstruction or in a pre-processing procedure. In an iterative reconstruction algorithm, the collimator's spatially variant point spread function (PSF) is modeled in the projector/backprojector pair. Many researchers have used this method to compensate for the spatially variant PSF [2]–[10]. This approach is considered the state-of-the-art in compensating for spatially variant PSF. However, this approach is usually time-consuming.

In addition to the iterative methods, the best known analytical method is to pre-process the projection data using the frequency-distance principle [11]–[17]. In this method, a two-dimensional (2D) Fourier transform is first applied to the projection sinogram. A sinogram is a 2D representation of the projection data, and each row of the sinogram contains the projection data at each view. The slope of a line in the Fourier transformed sinogram passing through the DC point corresponds to the distance to the detector. Therefore, distance-dependent deblurring can be achieved by using a slope-dependent filter upon the Fourier transformed sinogram along each direction passing through the origin. This frequency-distance principle is an approximation, and it works well if the collimator introduces only a small amount of blurring. The approximation is poor at the locations near the center of detector rotation and at low frequency components. Also, this principle assumes attenuation-less projections. The main drawback of this pre-process is that it usually generates very noisy images when compared with iterative methods.

To our knowledge, post-processing methods have not been explored to combat the collimator blurring. An efficient post-processing method is developed in this paper. The post-processing method is expected to be more efficient than the intrinsic iterative methods and provide more accurate and less noisy images than the pre-processing method. This post-processing method consists of three steps: (i) Reconstruct the image with attenuation compensation but without deblurring; (ii) Further blur the raw reconstruction with rotational and axial convolution, obtaining an image with spatially invariant PSF; (iii) Deblur the image with an efficient shift-invariant filter.

Image non-stationary blurring is not restricted to SPECT, and it can be found in many imaging applications, for example, in PET (positron emission tomography) [18]. The general strategy of the proposed deblurring method is first to further-blur the image to give a stationary PSF, then to deblur the image by an efficient deconvolution, for example, Fourier domain filtering. The proposed algorithm is expected to be very efficient and practical in real-time applications. Even though the proposed method has a wide range of applications, this paper will use SPECT application to illustrate the idea.

II. METHODS

A. Main Idea

As a post-processing method, the proposed deblurring technique is applied in the image domain, to a reconstructed image, which is referred to as a *raw reconstruction* in this paper. The raw image is assumed to be reconstructed by an efficient analytical [19]–[23] or iterative algorithm that corrects for attenuation. The raw reconstruction has been corrected with attenuation but does not have blurring correction. The imaging geometry is arbitrary, and can be parallel-beam, fan-beam, cone-beam, and so on. Due to collimator blurring, the raw image has a spatially variant PSF.

After the raw image is obtained, the next step is to further blur the raw image with a spatially variant kernel so that the resultant image has a spatially invariant PSF. It is in this step that we use efficient rotational convolution and axial convolution which will be introduced next. This step is the essential part of the proposed method, which converts a raw image with a spatially *variant* PSF to a further-blurred image with a spatially *invariant* PSF.

Since the further-blurred image has a shift-invariant PSF, an efficient deblurring technique can be applied to compensate for the blurring. Usually a frequency domain filtering method is used. In fact, an efficient iterative or non-linear filter can also be adopted in this final step.

We believe that to further blur an image is easier and more efficient than to deblur an image when the PSF is spatially variant. We also believe that to deblur an image that has a spatially invariant PSF is easier and more efficient than to deblur an image that has a spatially variant PSF.

B. The Characteristics of the Image Domain PSF

We now study the effect of collimator distance-dependent blurring in the reconstruction domain, so that we can correct for this effect using a post-processing technique. Fig. 1(a) shows a computer generated phantom that contains small circular discs (or dots). Simulated projections are generated analytically with the distance-dependent collimator blurring effect. The projections are attenuated with a uniform attenuator. The attenuator is a large uniform disc with an attenuation coefficient of water at 140 keV. Finally the image is reconstructed as Fig. 1(c) with Novikov's FBP (filtered backprojection) algorithm [19] that corrects for the attenuation effect, without performing collimator blurring correction. It is observed that

collimator blurring causes an elongated point response. The elongation gets worse as the distance to the axis of rotation increases.

The image intensity is also a function of the distance from the center of rotation. The image intensity is higher if the location is farther away from the center of detector rotation. We must point out that this non-uniform intensity is not caused by attenuation, because the attenuation effect has already been compensated for in the FBP reconstruction algorithm. To verify this, we generated attenuation-less projections and used a standard FBP algorithm to reconstruct the image, the resultant image (Fig. 1(b)) was almost the same as that in Fig. 1(c). A collimator hole in a typical general-all-purpose (GAP) collimator has an acceptance angle of 6° . In this study, we use an acceptance angle of 16° to simulate a high-sensitivity parallel-hole collimator. It is concluded that the collimator's distance-dependent blurring can be reflected in the reconstructed image. The PSF in the image domain is systematic: The elongation of the point response function in the radial direction depends on the distance to the axis of rotation.

We also performed an iterative ML-EM reconstruction, and obtained almost the same PSF. Once the PSF is characterized, a post-processing technique can be used to correct for this PSF.

Due to the symmetry of the PSF, we only need to characterize the PSF for locations on the positive x -axis, as shown in Fig. 2. In this case, the width of the PSF h in the radial direction (i.e., the x -axis in Fig. 2) is stationary, and the width is determined by the distance of the detector (at 90° position) to the origin, which is the center of rotation. The width of the PSF h in the tangential direction (i.e., the y -axis in Fig. 2) decreases as the distance from the origin (i.e., the value $|x|$ in Fig. 2) gets larger. The width of h is determined by the distance of the detector (at 0° position) to the location of interest. In fact, the tangent blurring is the sum of a near field Gaussian blurring (say, at 0°) and a far field Gaussian blurring (say, at 180°). The near field Gaussian dominates the PSF at the FWHM level.

After attenuation correction, the PSF is almost independent of the attenuation medium. This observation is quite different from the detector-domain PSF, which has a much smaller magnitude for an attenuating medium. It is observed that the image-domain PSF is systematic, and is rotationally invariant. Therefore, the image-domain PSF estimation can be achieved by considering a point source on the positive x -axis with various radial locations $(x_0, 0)$.

The PSF h in the axial direction is the same as in the transaxial direction, as illustrated in Fig. 2, except that the y -axis in Fig. 2 is relabeled as the z -axis (i.e., the axis of rotation).

The width of h can be calculated by the collimator's acceptance angle and the distance. The magnitude of h is determined by the fact that the total integral of the function h is unity for parallel-hole collimators.

C. Estimation of the SPECT point spread function (PSF) in the reconstructed image

We assume that the half-maximum contour of the PSF is an ellipse with the major axis in the x -direction and the minor axis in the y -direction. The image-domain PSF h_{det} is the convolution of the collimator PSF function h_{coll} and detector's intrinsic PSF $h_{intrinsic}$:

$$h_{det}(x - x_0, y; x_0, 0) = h_{coll}(x - x_0, y; x_0, 0) * h_{intrinsic}(x - x_0, y). \quad (1)$$

If we approximately model both collimator and intrinsic PSFs as Gaussian, then the overall PSF h_{det} is also Gaussian. The intrinsic PSF $h_{intrinsic}$ is stationary, while the collimator PSF h_{coll} is not and can be modeled as:

$$h_{coll}(x - x_0, y; x_0, 0) = \frac{1}{2\pi\sigma_{R-x_0}\sigma_R} \exp\left[\frac{-y^2}{2\sigma_{R-x_0}^2}\right] \exp\left[\frac{-(x - x_0)^2}{2\sigma_R^2}\right] \quad (2)$$

where $\sigma_{R-x_0}^2$ is the “variance” in the y -direction, σ_R^2 is the “variance” in the x -direction at location $(x_0, 0)$, and R is the detector rotation radius. We can further assume that the “variance” for the intrinsic PSF $h_{intrinsic}$ is σ_0^2 , then (by letting $x_0 = R = 0$):

$$h_{intrinsic}(x, y; 0, 0) = \frac{1}{2\pi\sigma_0^2} \exp\left(\frac{-y^2}{2\sigma_0^2}\right) \exp\left(\frac{-x^2}{2\sigma_0^2}\right). \quad (3)$$

By using the Fourier convolution theorem, substituting (2) and (3) into (1) yields

$$h_{det}(x - x_0, y; x_0, 0) = \frac{1}{2\pi\sqrt{(\sigma_{R-x_0}^2 + \sigma_0^2)(\sigma_R^2 + \sigma_0^2)}} \exp\left[\frac{-y^2}{2(\sigma_{R-x_0}^2 + \sigma_0^2)}\right] \exp\left[\frac{-(x - x_0)^2}{2(\sigma_R^2 + \sigma_0^2)}\right]. \quad (4)$$

If the point source is not on the x -axis, we can always rotate the point source to the x -axis, evaluate the PSF, and rotate the PSF to the point source location as illustrated in Fig. 3.

We have made some observations of the image-domain PSF. First, the parameter $\sigma_{R-x_0}^2 + \sigma_0^2$ (or equivalently, the FWHM in the tangential-direction) decreases as x_0 moves away from the center of rotation. Second, the parameter $\sigma_R^2 + \sigma_0^2$ (or equivalently, the FWHM in the radial-direction) is almost stationary, and does not vary with x_0 . Third, the maximum value of the image-domain PSF h_{det} is approximately $1/[2\pi\sqrt{(\sigma_{R-x_0}^2 + \sigma_0^2)(\sigma_R^2 + \sigma_0^2)}]$, which increases as x_0 moves away from the center of rotation. These observations agree with the computer simulation shown in Fig. 1.

The image-domain PSF at origin is the goal PSF, and the further blurring is only carried out in the tangential (y) direction. The further blurring kernel $h_{further}$ can be expressed as

$$h_{further}(x - x_0, y; x_0, 0) = \sqrt{\frac{\sigma_{R-x_0}^2 + \sigma_0^2}{\sigma_R^2 + \sigma_0^2}} \exp\left[\frac{-y^2}{2(\sigma_R^2 - \sigma_{R-x_0}^2)}\right]. \quad (5)$$

For a parallel-hole collimator in SPECT, the system resolution is approximately a linear function of the distance to the detector, $R - x_0$, that is,

$$\sigma_{R-x_0}^2 = k(R - x_0) \quad (6)$$

for a certain constant k , which is determined by the collimator hole size and hole length. Substituting (6) into (5) yields

$$h_{further}(x - x_0, y; x_0, 0) = \sqrt{\frac{k(R - x_0) + \sigma_0^2}{kR + \sigma_0^2}} \exp\left[\frac{-y^2}{2kx_0}\right]. \quad (7)$$

In Part *D* and Part *E* below, we will introduce an efficient further-blurring method based on (7). We must point out that the PSF is different for different imaging system, and the PSF needs to be developed in a case-by-case manner.

D. Rotational Convolution

In SPECT, the FWHM of the image-domain PSF in the raw image is almost stationary in the radial direction, but the FWHM gets wider in the tangential direction as the point source moves towards the center of rotation. Therefore, the image further blurring can be achieved by performing blurring only in the tangential direction as follows. Let a transaxial slice of the raw reconstruction be image g . We rotate the image g counter-clockwise by a small angle Δ (e.g., $\Delta=1^\circ$) about the axis of detector rotation (which is the origin in Fig. 4) obtaining g_Δ , and rotate g clockwise by Δ obtaining $g_{-\Delta}$. If necessary, we rotate the image g counter-clockwise by $n\Delta$ obtaining $g_{n\Delta}$, and rotate g clockwise by $n\Delta$ obtaining $g_{-n\Delta}$ for $n = 2, 3, \dots$

A weighted sum of these rotated versions of g gives a further-blurred image:

$$\widehat{q} = A(r) \sum_n a_n g_{n\Delta} \quad (8)$$

where the weighting factors a_n are normalized (i.e., added up to 1) and $A(r)$ is a function of the radial distance r from the origin. Using the results from Part *C*, $A(r)$ is calculated as

$$A(r) = \sqrt{\frac{k(R - r) + \sigma_0^2}{kR + \sigma_0^2}}. \quad (9)$$

The coefficients $\{a_n\}$ are the discrete fit of the exponential function in (7)

$$a_n \sim \exp\left[\frac{-t^2}{2kr}\right]. \quad (10)$$

where the sign “ $\sim$ ” is used, instead of the equal sign, because $\{a_n\}$ obtained via (10) will be normalized before it is utilized in rotational convolution.

In (10), the variable t is in the tangential direction. At a fixed radius r , the tangential distance t can be approximated as the arc-length $r(n\Delta)$ with a *small* rotational angle $n\Delta$. Thus (10) can be expressed as

$$a_n \sim \exp\left[\frac{-(rn\Delta)^2}{2kr}\right] = \exp\left(\frac{-n^2\Delta^2}{2k}\right). \quad (11)$$

It is important to observe that $\{a_n\}$ defined in (11) is radial distance r independent. This makes the rotational convolution realization (8) of the efficient further blurring possible.

E. Axial Convolution

The further-blurred image $\hat{q}(x, y, z)$ obtained by (8) needs to be blurred even further in the axial (z) direction. A 1D convolution kernel $ker_r(z)$ is used for the axial convolution, where $r = \sqrt{x^2 + y^2}$. Let $q(x, y, z)$ be the result of the axial convolution $\hat{q}(x, y, z)$ with $ker_r(z)$. The kernel $ker_r(z)$ can be formed using the exponential function in the right-hand-side of (7).

$$ker_r(z) \sim \exp \left[\frac{-z^2}{2kr} \right] \quad (12)$$

where the sign “ $\sim$ ” is used, instead of the equal sign, because $ker_r(z)$ obtained via (11) will be normalized before it is utilized in axial convolution. This rotational and axial convolution equivalently achieves the spatially variant further-blurring, to obtain an image with a shift-invariant PSF.

III. PHANTOM EXPERIMENTS

A. Jaszczak phantom

A flanged Jaszczak hot-rod/cold-sphere phantom (See Fig. 5 (left)) was used in an experiment, using a Philips' IRIX SPECT system. The rod diameters were 4.8, 6.4, 7.9, 9.5, 11.1 and 12.7 mm, respectively, in the hot rod section. The phantom cylinder inside diameter was 21.6 cm. Three low-energy high-resolution parallel-hole collimators were used during data acquisition. The collimators had a full-width at half-maximum (FWHM) of 0.7 cm at a distance of 10 cm, and 0.39 cm at the 0 cm. The FWHM is a linear function of the distance to the detector surface, and increases by a factor of $\tan(\alpha/2)$, where α is the acceptance angle of the collimator holes.

The collimator rotation radius was 24 cm. The phantom was filled with water and 25 mCi of Tc-99m. The data acquisition time was 1 hour, with 180 view angles using all 3 detectors. The detector pixel size was 0.23 cm. The image was reconstructed in a 128×128 array, and the image pixel size was 0.23 cm. The ML-EM algorithm with 150 iterations was used for raw image reconstruction and attenuation correction.

In this phantom experiment, the further-blurred image $\hat{q}$ was obtained by rotational convolution:

$$\hat{q} = \sqrt{\frac{1.134 - 0.007r}{1.134}} \cdot \frac{g + 0.5g_{1.5^\circ} + 0.5g_{-1.5^\circ}}{2} \quad (13)$$

where r has the unit of pixels.

Since this particular phantom was constant in the axial direction, no axial further filtering was applied. After rotational convolution, a further blurred image $\hat{q}$ was obtained.

An inverse Gaussian filter

$$H(u, v) = \exp \left(\frac{u^2 + v^2}{2\sigma^2} \right) \text{ if } u^2 + v^2 < \Omega^2, \text{ and } H(u, v) = 0 \text{ otherwise} \quad (14)$$

(with a variance of $\sigma^2 = 100$ (cycles/pixel)², and cutoff frequency $\Omega = 19.5$ cycles/pixel) was chosen to deblur the further-blurred image $\hat{q}$ in the frequency domain. In fact, the design of an inverse filter (14) has many choices. For example, a smooth transition at the cutoff frequency Ω can be used to reduce the Gibbs ringings. We only present a basic version here in (14). Here the unit is the image pixel size.

B. Hoffman Brain phantom

A Hoffman brain phantom was used in another experiment, imaged with a Philips' IRIX three-detector system. Three low-energy high-resolution convergent collimators were used during data acquisition. A fan-beam collimator was mounted on detector 1, and two cone-beam collimators were mounted on detectors 2 and 3 [24]. The three convergent collimators had a full-width at half-maximum (FWHM) of 0.8 cm at a distance of 10 cm and the focal-length was 65 cm. The collimator rotation radius was 26 cm. The phantom was filled with water and 22 mCi of Tc-99m. The data acquisition time was 21 minutes, with 120 view angles over 360°. The detector pixel size was 0.467 cm. The image was reconstructed in a 64×64 array, and the image pixel size was 0.467 cm. The ML-EM algorithm with 30 iterations was used for raw image reconstruction without attenuation correction.

In this phantom experiment, the further-blurred image $\hat{q}$ was obtained by rotational convolution:

$$\hat{q} = \sqrt{\frac{1.456 - 0.019r}{1.456}} \cdot \frac{g + 0.5g_{2^\circ} + 0.5g_{-2^\circ}}{2}, \quad (15)$$

where r has the unit of pixels.

The axial further blurring was achieved by z-direction convolution of the image $\hat{q}$ with a Gaussian kernel $ker_r(z)$. The standard deviation of the kernel was chosen as $0.019r$, where r is the distance from an image voxel to the axis of rotation with a unit of pixels. After rotational convolution and axial convolution, a further-blurred image q was obtained.

A 3D inverse Gaussian filter (14) with a variance of $\sigma^2 = 200$ (cycles/pixel)², and cutoff frequency $\Omega = 21$ cycles/pixel was chosen to deblur the further-blurred image q in the frequency domain. The reconstruction results are shown in Fig. 6.

IV. CONCLUSION

This paper presented a new idea of using an efficient method to deblur an image that has a non-stationary point spread function (PSF). The main concept is to further blur the raw image so that the resultant image has a stationary PSF. There exist many efficient methods to deblur an image that has a stationary PSF. This paper also provided some preliminary computer simulation and experimental phantom studies to prove the feasibility of this new concept. As an application, we considered the collimator blurring problem in circular-orbit SPECT. In this case, the further-blurring can be accomplished by using rotational convolution and axial convolution, taking advantage of the fact that the PSF in the raw image has a strong rotational symmetry. In fact, this further-blurring idea can be extended to other applications of image deblurring, where the non-stationary PSF in the image is readily estimated.

As in all other new technology development, there are many challenges in fully developing this concept. First, it is not easy to estimate the non-stationary PSF in the raw image, especially when considering patient-dependent scattering, non-circular detector orbits, or

less-than-360° scans. Second, after the non-stationary PSF is obtained, it is not easy to develop a general further-blurring scheme to obtain an image with a shift-invariant PSF. We only have an approximate estimation of the PSF for circular-orbit SPECT. Third, the noise regularization needs special consideration in the final shift-invariant deblurring filter design. We will meet these challenges in our future development. After the new method is fully developed, we will compare the results from this method with the results from the current gold-standard iterative ML-EM algorithm that models the projection blurring in the projector/backprojector pair.

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REFERENCES

1. Gunter, DL. Collimator design for nuclear medicine, in Emission Tomography: The Fundamentals of PET and SPECT. Wernick, MN.; Aarsvold, JN., editors. Amsterdam: Elsevier Academic Press; 2004. p. 153-168.
2. Floyd CE, Jaszczak RJ, Manglos SH, Coleman RE. Compensation for collimator divergence in SPECT using inverse Monte Carlo reconstruction. IEEE Trans. Nucl. Sci. 1988; vol. 35:784–878.
3. Kamphuis C, Beekman FJ, Viergever MA. Evaluation of OS-EM vs. ML-EM for 1D, 2D and fully 3D SPECT reconstruction. IEEE Trans. Nucl. Sci. 1996; vol. 43:2018–2024.
4. Penney BC, King MA, Knesaurek K. A projector back-projector pair with accounts for the two-dimensional depth and distance dependent blurring in SPECT. IEEE Trans. Nucl. Sci. 1990; vol. 37:681–686.
5. Beekman FJ, Eijkman EGJ, Viergever MA, Born GF, Slijpen ETP. Object shape dependent PSF model for SPECT imaging. IEEE Trans. Nucl. Sci. 1993; vol. 40:31–39.
6. Tsui BMW, Hu H-B, Gilland DR, Gullberg GT. Implementation of simultaneous attenuation and detector response correction in SPECT. IEEE Trans. Nucl. Sci. 1988; vol. 35:778–783.
7. Formiconi AR. Geometrical response of multihole collimators. Phys. Med. Biol. 1998; vol. 43:3359–3379. [PubMed: 9832021]
8. Liang Z. Compensation for attenuation, scatter, and detector response in SPECT reconstruction via iterative FBP methods. Med. Phys. 1993; vol. 20:1097–1106. [PubMed: 8413018]
9. Zeng GL, Gullberg GT, Tsui BMW, Terry JA. Three-dimensional iterative reconstruction algorithms with attenuation and geometric point response correction. IEEE Trans. Nucl. Sci. 1991; vol. 38:693–702.
10. Zeng GL, Gullberg GT. Frequency domain correction of the three-dimensional geometric point response function in SPECT imaging. IEEE Trans. Nucl. Sci. 1992; vol. 39:1444–1453.
11. Edholm PR, Lewitt RM, Lindholm B. Novel properties of the Fourier decomposition of the sonogram. International Workshop on Physics and Engineering of Computerized Multidimensional Imaging and Processing, Proceedings of SPIE. 1986; vol. 671:8–18.
12. Lewitt RM, Edholm PR, Xia W. Fourier method for correction of depth-dependent collimator blurring. Medical Imaging III: Image Processing, SPIE. 1989; vol. 1092:232–243.
13. Hawkins WG, Yang N-C, Lechner PK. Validation of the circular harmonic transform (CHT) algorithm for quantitative SPECT. J. Nucl. Med. 1991; vol. 32:141–150. [PubMed: 1988620]
14. Glick SJ, Penney BC, King MA, Byrne CL. Noniterative compensation for the distance-dependent detector response and photon attenuation in SPECT imaging. IEEE Trans. Med. Imag. 1994; vol. 13:363–374.
15. Xia W, Lewitt RM, Edholm PR. Fourier correction for spatially variant collimator blurring in SPECT. IEEE Trans. Nucl. Sci. 1995; vol. 42:100–115.

16. Kohli V, King MA, Glick SJ, Pan T-S. Comparison of frequency-distance relationship and Gaussian-diffusion based methods of compensation for distance-dependent spatial resolution in SPECT imaging. *Phys. Med. Biol.* 1998; vol. 43:1025–1037. [PubMed: 9572525]
17. Soares EJ, Glick SJ, King MA MA. Noise characterization of combined Bellini-type attenuation and frequency-distance principle restoration filtering. *IEEE Trans. Nucl. Sci.* 1996; vol. 43:3278–3290.
18. Lee K, Kinahan PE, Fessler JA, Miyaoka RS, Janes M, Lewellen TK. Pragmatic fully 3D image reconstruction for MiCES mouse imaging PET scanner. *Phys. Med. Biol.* 2004; vol. 49:4563–4578. [PubMed: 15552417]
19. Novikov RG. An inversion formula for the attenuated x-ray transformation. *Ark. Math.* 2002; vol. 40:145–167.
20. Natterer F. An inversion formula for the attenuated Radon transform. *Inverse Problems.* 2001; vol. 17:113–119.
21. You J, Zeng GL, Liang Z. FBP algorithms for non-uniform attenuated fan-beam projections. *Inverse Problems.* 2005; vol. 21:1179–1192.
22. Huang Q, Zeng GL, You J, Gullberg GT. An FDK-like cone-beam reconstruction algorithm for nonuniform attenuated projections acquired using a circular trajectory. *Phys. Med. Biol.* 2005; vol. 50:2329–2339. [PubMed: 15876670]
23. Tang Q, Zeng GL, Gullberg GT. Analytical fan-beam and cone-beam reconstruction algorithms with uniform attenuation correction. *Phys. Med. Biol.* 2005; vol. 50:3153–3170. [PubMed: 15972987]
24. Gullberg GT, Zeng GL. Cardiac single-photon emission-computed tomography using combined cone-beam/fan-beam collimation. *IEEE Trans. Nucl. Sci.* 2005; vol. 52:143–153.

Figure 1.

Image domain PSF has a systematic elongation in the radial direction. The elongation effect worsens as the location is farther away from the axis of rotation.

- (a) True phantom
- (b) Reconstruction using attenuation-free data (the conventional FBP reconstruction)
- (c) Reconstruction using attenuated data (Novikov's FBP reconstruction algorithm corrects for the attenuation effect)

Figure 2.
Estimation of the image domain PSF h by considering the locations (marked as #0, #1, and #2) on the x -axis.

Figure 3.

The point source object and the blurred reconstruction are first rotated to the x -axis. The PSF is then estimated by the relation that the image is the 2D convolution of the object with the PSF h_{det} .

Figure 4.

(A) The original true (unblurred) image f consisting of three small dots. (B) The raw reconstruction g from the SPECT projections of the image shown in (A). (C) The image g is rotated clockwise and counter-clockwise for a few small angles, obtaining a few rotated versions of g . (D) Weighted sum $\hat{q}$ of the versions from (C). This summed image has a shift-invariant PSF.

Figure 5.
Left: Jaszczak phantom, Middle: Raw reconstruction, and Right: Deblurred image using proposed method.

Figure 6. Hoffman brain phantom reconstructions. Top: Three orthogonal views of the raw reconstruction, and Bottom: Same three orthogonal views of the deblurred image using proposed method.