Published in final edited form as:

Mech Res Commun. 2009 January 1; 36(1): 10–21. doi:10.1016/j.mechrescom.2008.06.013.

Magneto-Hydrodynamics Based Microfluidics

Shizhi Qian^a and Haim H. Bau^{b,*}

^a Department of Aerospace Engineering, Old Dominion University, Norfolk, VA 23529-0247, USA

^b Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, PA 19104-6315, USA

Abstract

In microfluidic devices, it is necessary to propel samples and reagents from one part of the device to another, stir fluids, and detect the presence of chemical and biological targets. Given the small size of these devices, the above tasks are far from trivial. Magneto-hydrodynamics (MHD) offers an elegant means to control fluid flow in microdevices without a need for mechanical components. In this paper, we review the theory of MHD for low conductivity fluids and describe various applications of MHD such as fluid pumping, flow control in fluidic networks, fluid stirring and mixing, circular liquid chromatography, thermal reactors, and microcoolers.

Keywords

Microfluidics; Lab-on-a-chip; Magneto-hydrodynamics; MHD; Lorentz force; Micro-pump; Chaotic Stirrer; Fluid Manipulation; Mixing; Chaos

1. Introduction

In recent years, there has been a growing interest in developing lab-on-a-chip (LOC) systems for bio-detection, biotechnology, chemical reactors, and medical, pharmaceutical, and environmental monitoring. LOC is a minute chemical processing plant that integrates common laboratory procedures ranging from filtration and mixing to separation and detection. The various operations are done automatically within a single platform. To achieve these tasks, it is necessary to propel, stir, and control fluids. Since in many applications, one uses buffers and solutions that are electrically conductive, one can transmit electric currents through the solutions. In the presence of an external magnetic field, the interaction between the electric currents and magnetic fields results in Lorentz body forces, which, in turn, can be used to propel and manipulate fluids. This is the domain of magneto-hydrodynamics (MHD).

The application of MHD to pump, confine, and control liquid metals and ionized gases is well-known (Woodson and Melcher, 1969; Davidson, 2001). The application of MHD to weakly conductive electrolyte solutions is somewhat more complicated due to electrodes' electrochemistry. Recently, various MHD-based microfluidic devices including micro-pumps (Jang and Lee, 2000; Lemoff and Lee, 2000; Huang et al., 2000; Bau, 2001; Zhong et al., 2002; Bau et al., 2002, ²⁰⁰³; Sawaya et al., 2002; West et al., 2002, ²⁰⁰³; Ghaddar and Sawaya,

* Corresponding author. bau@seas.upenn.edu.

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2003; Bao and Harrison, 2003a, 2003b; Eijkel et al., 2004; Wang et al, 2004; Arumugam et al., 2004, ²⁰⁰⁶; Qian and Bau, 2005b; Homsy et al., 2005, ²⁰⁰⁷; Affanni and Chiorboli, 2006; Aguilar et al., 2006; Kabbani et al., 2007; Patel and Kassegne, 2007; Duwairi and Abdullah, 2007; Ho, 2007), stirrers (Bau et al., 2001; Yi et al., 2002; Qian et al., 2002; Gleeson and West, 2002; Xiang and Bau, 2003; Gleeson et al., 2004; Qian and Bau, 2005a), networks (Bau et al., 2002, 2003), heat exchangers (Sviridov et al., 2003; Singhal et al., 2004; Duwairi and Abdullah, 2007), and analytical and biomedical devices (Leventis and Gao, 2001; West et al., 2002, 2003; Bao and Harrison, 2003a; Lemoff and Lee, 2003; Eijkel et al., 2004; Clark and Fritsch, 2004; Homsy et al., 2007; Gao et al., 2007; Panta et al., 2008) operating under either DC or AC electric fields have been designed, modeled, constructed, and tested. The DC operation is often adversely impacted by the electrodes' electrochemistry leading to bubble formation and electrode corrosion. These problems are partially solved with the use of AC fields. AC operation requires, however, the use of electromagnets instead of the permanent magnets that are used in DC operation, which increases power consumption. Moreover, AC operation induces parasitic eddy currents that may lead to excessive heating. DC operation with RedOx species that undergo reversible electrochemical reactions alleviates many of the disadvantages of DC MHD (Qian and Bau, 2005b; Arumugam et al., 2006; Kabbani et al., 2007).

The advantage of MHD compared to electroosmosis is operation at relatively small electrode potentials, typically below 1 V, and much higher flow rates as long as the conduit's dimensions are not too small. The disadvantage of MHD is that it is a volumetric body force which scales unfavorably as the conduit's dimensions are reduced. Thus, MHD is appropriate mostly for moderate conduit sizes with characteristic dimensions on the order of 100 μm or larger.

In this paper, we review the basic theory of MHD as applied to low conductivity solutions and describe various applications of MHD such as pumps, integrated fluidic networks, stirrers, liquid chromatographs, thermal cyclers, and microcoolers.

2. Theory

We consider an incompressible, viscous fluid. The velocity vector $\mathbf{u}$ satisfies the continuity equation

$$\nabla \cdot \mathbf{u} = 0. \quad (1)$$

We adopt here the notation that bold letters represent vectors. The momentum equation is

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{u}, \quad (2)$$

where ρ and μ are, respectively, the liquid's density and viscosity; t is time; $\mathbf{J}$ is the electric current flux; p is pressure; and $\mathbf{B}$ is the magnetic field intensity. In the above, we assume that the liquid's magnetic permeability is sufficiently small so that the magnetic field inside the fluid can be approximated with $\mathbf{B}$. The Ohm's law,

$$\mathbf{J} = \sigma(-\nabla V + \mathbf{u} \times \mathbf{B}), \quad (3)$$

provides us with a relationship between the current flux and the electric potential V . In the above, σ is the electric conductivity of the solution. The second term in RHS of equation (3)

represents current induction caused by the motion of a conductor in an electric field. Strictly speaking, equation (3) applies only to liquid metals in which the current is transported by electrons.

In the case of electrolyte solutions, a more accurate model for the current flux consists of the Nernst-Planck (**NP**) equations for the various ionic species. The ionic flux density of species k is:

$$\mathbf{N}_k = \mathbf{u} c_k - D_k \nabla c_k - z_k \frac{D_k}{RT} F c_k (\nabla V + \mathbf{u} \times \mathbf{B}), \quad (k=1, \dots, N), \quad (4)$$

where c_k is the molar concentration, D_k is the diffusion coefficient, and z_k is the valance of the k^{th} ionic species. F is Faraday's constant ($F=96484.6 \text{ C/mol}$), R is the universal gas constant, T is the absolute temperature of the electrolyte solution, N is the total number of species present in the electrolyte solution, and $\mathbf{u} \times \mathbf{B}$ is the induction term.

Under steady state conditions,

$$\nabla \cdot \mathbf{N}_k = 0, \quad (k=1, \dots, N), \quad (5)$$

and the current flux

$$\mathbf{J} = F \sum_{k=1}^N z_k \mathbf{N}_k. \quad (6)$$

The potential in the electrolyte solution is governed by the local electroneutrality condition:

$$\sum_{k=1}^N z_k c_k = 0. \quad (7)$$

Electroneutrality holds everywhere except in the thin Debye screening layer next to solid surfaces. Although the Debye screening layer is only a few nanometers in thickness, the potential drop across this layer can be significant.

The boundary conditions associated with equations (5) and (7) consist of zero normal flux of each of the species at insulating walls; given ionic concentrations at the conduit's inlet; normal flux dominated by convective flux at the outlet of the conduit (outflow boundary condition); and Butler-Volmer equation (Bard and Faulkner, 2000) at the surfaces of the electrodes.

The **NP** equations (5) and the local electroneutrality condition (7) constitute a well-defined and widely used approximation for electrochemical transport phenomena. Witness that the models for the fluid motion (equations 1-2) and the ionic mass transport (equations 5 and 7) are strongly coupled. The flow field affects the mass transport due to the presence of the convective flux in expression (4). On the other hand, the ionic mass transport affects the current density $\mathbf{J}$, which, in turn, affects the flow field through the Lorentz body force $\mathbf{J} \times \mathbf{B}$. Therefore, one needs to solve simultaneously the full mathematical model, which consists of the continuity and Navier-Stokes equations, the set of NP equations, and the local electroneutrality condition with the appropriate boundary conditions, to obtain the flow field, the ionic species'

concentrations, and the potential of the electrolyte solution. Due to space limitations, we will not be able to discuss here the solution of the full mathematical model in any detail. The numerical solutions of the 2D and 3D systems were reported, respectively, in Qian and Bau (2005b) and Kabbani et al. (2007).

The energy equation is

$$\rho c_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \kappa \nabla^2 T + \frac{\mathbf{J} \cdot \mathbf{J}}{\sigma} + \Phi, \quad (8)$$

where κ and c_p are, respectively, the thermal conductivity and the heat capacity of the fluid.

$\Phi = 2\mu \mathbf{e} \cdot \mathbf{e}$ is the viscous dissipation, where $\mathbf{e} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ is the strain rate tensor.

We non-dimensionalize the equations of motion using the conduit's height H as the length scale in the x and z directions and the width W as the length scale in the y -direction (Fig. 1). The magnetic field intensity scale is the maximum field intensity B . The electric potential scale is the maximum voltage difference in the solution V . It is perhaps important to point out that in the case of an electrolyte solution, there may be a significant potential difference across the electric double layer. The electric current flux, velocity, pressure, temperature, and time scales are, respectively, $\sigma V/W$, $U = \sigma V B H^2 / (\mu W)$, $\mu U/H$, $\Delta T = (\sigma V/W)^2 H / (\sigma \rho c_p U)$, and H/U . The dimensionless equations are:

$$\text{Re} \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla^2 \mathbf{u}, \quad (9)$$

$$\mathbf{J} = -\nabla V + H_a^2 \mathbf{u} \times \mathbf{B}, \quad (10)$$

and

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{\text{Pe}} \nabla^2 T + \mathbf{J} \cdot \mathbf{J} + \frac{\text{Ec}}{\text{Pe}} \nabla \mathbf{u} \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T). \quad (11)$$

In the above, we used the same symbols for the dimensionless quantities as for their

dimensional counterparts. $\nabla = \hat{e}_x \frac{\partial}{\partial x} + \varepsilon \hat{e}_y \frac{\partial}{\partial y} + \hat{e}_z \frac{\partial}{\partial z}$; $\hat{e}_z$ is a unit vector in the z -direction; $\varepsilon = H/W$ is the conduit's aspect ratio; $H_a = B H \sqrt{\sigma/\mu}$ is the Hartman number; $\text{Re} = UH/\nu$ is the Reynolds number; $\text{Pe} = UH \rho c_p / \kappa$ is the Peclet number; and $\text{Ec} = \mu U^2 / \kappa \Delta T$ is the Eckert number.

It is instructive to examine the simple case of a conduit with a rectangular cross-section in which two parallel electrodes cover two opposing walls (Fig. 1). We focus on the conduit's section equipped with the electrodes and assume $H \ll W \ll L$ ($\varepsilon \ll 1$). In other words, we investigate fully developed flow. The magnetic field $\mathbf{B} = \hat{e}_z$ is uniform; and $\mathbf{J} = \hat{e}_y$. Away from side boundaries $y = \pm 1/2$, $\mathbf{u} = u(z)\hat{e}_x$. Assuming nearly isothermal flow and solving equations (9) and (10), we obtain

$$u_x(z) = -H_a^{-2} \left(\varepsilon \frac{dV}{dy} + \frac{dp}{dx} \right) \left[1 - \frac{\cosh(H_a z)}{\cosh(\frac{H_a}{2})} \right] \quad (-\frac{1}{2} < z < \frac{1}{2}). \quad (12)$$

In the above, we assumed that $\frac{dV}{dy}$ is constant along most of the conduit's width.

When the Hartman number is large, the velocity scales like H_a^{-2} . In other words, the dimensional velocity is on the order of $V/(BW)$. The magnitude of the velocity is dictated by a balance between the applied and induced (Hartman break) electric currents. The velocity profile

$u_x(z) = -H_a^{-2} \left(\varepsilon \frac{dV}{dy} + \frac{dp}{dx} \right)$ is nearly flat along most of the conduit's height with two boundary (Hartman) layers of thickness H_a^{-1} next to the solid surfaces ($z = \pm 1/2$). High Hartman numbers are typically encountered when operating with liquid metals. When the working fluid is mercury ($\sigma = 10^6/\text{m}\Omega$ and $\mu = 1.5 \times 10^{-3} \text{N}\cdot\text{s}/\text{m}^2$), $B = 0.4 \text{T}$, and $H = 200 \mu\text{m}$, the Hartman number $H_a \sim 2$.

When the Hartman number is small, we can expand the hyperbolic *cosh* terms into their corresponding Taylor series and obtain the parabolic profile:

$$u_x(z) = -\frac{1}{2} \left(\varepsilon \frac{dV}{dy} + \frac{dp}{dx} \right) \left[\frac{1}{4} - z^2 \right] \quad (-\frac{1}{2} < z < \frac{1}{2}). \quad (13)$$

In this case, the magnitude of the velocity is dictated by a balance between the Lorentz force and the viscous force. The Lorentz force acts as a velocity-independent, body force similar to pressure gradient. When the working fluid is an electrolyte solution, one is likely to operate in the low Hartman number regime. For example, when the working fluid is a 100mM salt solution ($\sigma \sim 10(\text{m}\Omega)^{-1}$ and $\mu = 10^{-3} \text{N}\cdot\text{s}/\text{m}^2$), $B = 0.4 \text{T}$, and $H = 200 \mu\text{m}$, the Hartman number $H_a \sim 10^{-2}$.

In most of this manuscript, we will focus on flows with $H_a \ll 1$. In this case, due to the relatively large potential drop across the electric double layers, it is usually beneficial to scale the velocity with the electric current rather than the potential.

The direction of the Lorentz body force can be controlled by judicious placement of the electrodes. Thus, magneto-hydrodynamics allows us to induce relatively complicated flow patterns. For example, by patterning electrodes on the conduit's bottom and/or top, transverse to the conduit's axis, and applying alternating potentials to adjacent electrodes (Fig. 2), one can induce cellular convection (Xiang and Bau, 2003). The conduit depicted in Fig. 2 is equipped with electrodes $A_{-n}, \dots, A_{-1}, A_0, A_1, \dots, A_n$ (where n is an integer) patterned transversely on the conduit's bottom. Only electrodes A_0 and A_{-1} are shown in the figure. The even and odd numbered electrodes are, respectively, subjected to potentials V and $-V$. The magnetic field is perpendicular to the image. The Lorentz force between electrodes A_{2n-1} and A_{2n} is directed upwards while the Lorentz force between electrodes A_{2n} and A_{2n+1} is directed downwards. The fluid moves in the direction of the Lorentz force, resulting in convective cells. The corresponding streamlines are shown in the figure. The arrows indicate the flow directions. For additional details on the analysis and corresponding experiments, see Xiang and Bau (2003).

Another intriguing feature of magneto-hydrodynamics is the forming of rotational flows. In general, the Lorentz body force $\mathbf{J} \times \mathbf{B}$ is rotational, $\nabla \times (\mathbf{J} \times \mathbf{B}) \neq 0$, and it cannot be balanced

with the pressure gradient (Moffatt, 1991). Thus, vortices are common in MHD flows, a feature that is advantageous for stirring.

As an example of rotational flow, we consider two-dimensional MHD flow in a long concentric annulus with inner radius r_1 and outer radius r_2 . The cylinder's surfaces act as electrodes. The Lorentz force A/r is directed azimuthally (in the $\hat{e}_\theta$ direction). In the above, r and θ are, respectively, the radial and azimuthal coordinates, $A=I'B/(2\pi\mu)$, and I' is the current per unit length. The azimuthal, dimensional velocity

$$u_\theta(r) = -\frac{A}{2r(r_2^2 - r_1^2)} \left(r_2^2 r_1^2 \ln\left(\frac{r_2}{r_1}\right) - r^2 r_1^2 \ln\left(\frac{r}{r_1}\right) - r^2 r_2^2 \ln\left(\frac{r_2}{r}\right) \right). \quad (14)$$

When the radius of the inner electrode is reduced to zero (while maintaining the current I' fixed), we obtain the rotator (agitator)

$$u_\theta(r) = \frac{Ar}{2} \ln\left(\frac{r_2}{r}\right). \quad (15)$$

A solution for a rotator eccentrically placed in a circular cavity was derived in Yi et al. (2002). Later in the paper, we will demonstrate that a few rotators (or agitators) can operate alternately to induce chaotic advection that is beneficial to mixing.

3. Practical Considerations

Many of the difficulties encountered when using electrolyte-based MHD devices are associated with the electrodes' chemistry. In a closed system, one must operate at sufficiently low potential differences between the electrodes, typically below $\sim 1.2\text{V}$, to avoid the electrolysis of water. The electrolysis of water would cause an accumulation of gas bubbles along the surface of the electrode. Such a gas blanket will shield the electrodes and prevent current transmission through the solution. Operation at high electrode potential differences would require special means to prevent the accumulation of vapor and remove the vapor bubbles. For example, Homsy et al. (2005) located the electrodes in separate conduits that are in liquid communication with the flow conduit. The bubbles formed in the electrodes' conduits never enter the flow conduit and are discharged through open reservoirs to the atmosphere.

Even in the absence of water's electrolysis, the electrodes' chemistry may lead to the accumulation of unwanted reaction products next to the electrodes and to the electrodes' corrosion, reducing the useful life of the device.

Many of the unwanted effects associated with electrode chemistry can be alleviated by operating with RedOx electrolytes such as $\text{FeCl}_2/\text{FeCl}_3$ that undergo reversible reactions at the electrodes (Leventis and Gao, 2001; Clark and Fritsch, 2004; Qian et al., 2006). The RedOx electrolytes facilitate relatively high current fluxes at low electrode potential differences, do not form any reaction byproducts at the electrodes' surfaces, and do not cause electrode corrosion (when inert electrodes are used).

Another remedy is to operate with AC electric fields (Lemoff and Lee, 2000; West et al., 2003; Eijkel et al., 2004). Of course, change in the direction of the electric field would result in a change in the direction of the flow. When unidirectional flow is needed, one must alternate the magnetic field in synch with the alternations in the electric field. When AC electric and

magnetic fields are used (AC-MHD), the magnetic flux density and the current flux are, respectively, $\mathbf{B} \sin(\omega t)$ and $\mathbf{J} \sin(\omega t + \varphi)$, where ω is the angular frequency of the fields and φ is the phase angle between the electric and magnetic fields. The linearized (low Reynolds number) momentum equation for AC-MHD flow is

$$\rho \frac{\partial \mathbf{u}}{\partial t} = \mathbf{J} \times \mathbf{B} \sin(\omega t) \sin(\omega t + \varphi) - \nabla p + \mu \nabla^2 \mathbf{u}. \quad (16)$$

The time-averaged velocity and pressure are, respectively, $\langle u \rangle = \int_0^{2\pi} u d\omega t$ and $\langle p \rangle = \int_0^{2\pi} p d\omega t$. Integrating equation (16) with respect to ωt from $\omega t = 0$ to 2π , we obtain

$$0 = \mathbf{J} \times \mathbf{B} \frac{\cos(\varphi)}{2} - \nabla \langle p \rangle + \mu \nabla^2 \langle \mathbf{u} \rangle. \quad (17)$$

Some of the disadvantages of AC operation are the need to use electromagnets (instead of zero-power consumption permanent magnets) and the induction of parasitic currents and heating (in particular at high frequencies).

4. Applications

4.1 MHD-Based Micro-Pumps

The best known application of MHD is fluid pumping. One possible embodiment of a MHD pump is depicted in Fig. 1. The device consists of a conduit with two electrodes deposited along its opposing walls. The conduit is filled with a conductive medium (either electrolyte solution or liquid metal). When a potential difference is applied across the two opposing electrodes, current flux $\mathbf{J}$ flows through the solution. In the presence of a magnetic field $\mathbf{B}$, the electric current $\mathbf{J}$ interacts with the magnetic field $\mathbf{B}$ to generate a Lorentz body force $\mathbf{J} \times \mathbf{B}$ which drives fluid motion. The magnetic field can be applied either with a permanent magnet or an electromagnet.

When the conduit and electrodes are sufficiently long, the flow will be fully-developed along most of the length of the conduit. Recent 3-D numerical computations provide information on the development length (Kabbani et al., 2007). The fully-developed velocity profile of MHD flow with uniform current flux $\mathbf{J}$ and uniform magnetic field $\mathbf{B}$ is identical to the velocity profile of fully-developed, pressure-driven flow (White, 2006; Kabbani et al., 2008):

$$u_x(y, z) = A_1 \bar{U} \sum_{i=1,3,5,\dots}^{\infty} (-1)^{\frac{i-1}{2}} \left[1 - \frac{\cosh\left(\frac{i\pi z}{2a}\right)}{\cosh\left(\frac{i\pi b}{2a}\right)} \right] \frac{\cos\left(\frac{i\pi y}{2a}\right)}{i^3}, \quad (18)$$

and

$$A_1 = \frac{48}{\left[\pi^3 - \frac{192a}{\pi^2 b} \left(\tanh\left(\frac{\pi b}{2a}\right) + \frac{31}{32} \zeta(5) - 1 \right) \right]}. \quad (19)$$

In the above, $a=\min(H/2, W/2)$; $b=\max(H/2, W/2)$; $\zeta(\alpha)=\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$ is the Riemann zeta function; A_I is only a function of the conduit's aspect ratio b/a ;

$$\bar{U} = \frac{16a^2}{\pi^3 A_I \mu} \frac{\Delta p}{L} + \frac{2caWB}{\pi^3 b A_I \mu L} I \quad (20)$$

is the cross-sectional and time averaged velocity; $I(=J \times L_E \times H)$ is the electric current through the conduit; $c=2$ when the electric and magnetic fields are time-independent (DC); and $c=\cos(\varphi)$ when AC electric and magnetic fields are used.

More generally, the volumetric flow rate

$$Q_i = H_{p,i} \Delta p_i + H_{MHD,i} I_i. \quad (21)$$

The above equation can be viewed as a constitutive relationship for the MHD pump with H_p and H_{MHD} representing, respectively, the “hydraulic conductivity” and the “MHD conductivity.” We added a subscript i to facilitate the use of equation (21) in network setting (see section 4.2). The hydraulic and MHD conductivities depend on the conduit's geometry and the arrangement of the electrodes. For the case of a rectangular conduit with the electrodes

$$\text{covering the entire surface of the opposing walls, } H_{p,i} = \frac{\partial Q_i}{\partial \Delta p_i} = \frac{16a_i^2 W_i H_i}{\pi^3 A_{1,\mu} L_i} \text{ and}$$

$$H_{MHD,i} = \frac{\partial Q_i}{\partial I_i} = \frac{c W_i B}{8a_i b_i} H_{p,i}.$$

The stagnation pressure (Δp_s) of the pump

$$\Delta p_s = \frac{c}{2} J B L_E. \quad (22)$$

Over the years, various researchers have constructed MHD pumps. Table 1 summarizes some of the literature.

Figs. 3 and 4 compare, respectively, the experimental data (symbols) of Ho (2007) (flow rate as a function of conduit's width) and of Lemoff and Lee (2000) (average velocity as a function of solvent concentration) with the theoretical predictions (solid line) of equation (21) in the absence of the pressure gradient ($\Delta p=0$). At low solute concentrations, the theoretical predictions agree reasonably well with the experimental data. At high solute concentrations (Fig. 4), the theory over-predicts the flow rate. This is perhaps because the theory (equation 21) does not account for the Hartman breaking.

Since the direction of the electric current flux can be controlled by patterning the electrodes (i.e., the electrodes need not cover the entire side walls of the conduit), one can exercise some level of control on the velocity profile. This is something that cannot be done in pressure-driven flows.

Another unique feature of MHD flow is the feasibility of circulating a fluid continuously in a closed loop such as a toroidal conduit to form a conduit with a virtual “infinite length” (Zhong

et al., 2002). Such a toroidal pump was constructed by Zhong et al. (2002) and tested with NaCl solutions and with a mercury slug. When operating with mercury slugs, relatively high velocities (tens of cm/s) could be achieved in a 2mm wide and 700 μ m deep conduit.

4.2 MHD-based Microfluidic Networks

In lab-on-chip applications, it is often necessary to transport fluids and reagents across networks of conduits. The flow control typically requires the use of pumps and valves. It is usually cumbersome to implement mechanical valves and pumps in a lab-on-chip setting. MHD provides an elegant solution that does not require any mechanical components. The basic idea is to equip many of the network's conduits, if not all, with individually controlled electrodes. By judicious control of the electrodes' currents and in the presence of a magnetic field, it is possible to direct the fluid flow along any desired path without a need for mechanical valves (Bau et al., 2002, 2003). To make the ideas involved more concrete, Fig.5 shows a simple example of a MHD microfluidic network fabricated with low temperature co-fired ceramic tapes (LTCC). The MHD network contains two reservoirs; one stores reagent A, and the other contains reagent B. The solutions are pumped out from the two reservoirs, and flow through conduit 1 (which can double as a stirrer) into the torus (conduits 2-3). The electrodes are then re-programmed to circulate the fluid around the torus (2-3) any desired number of times. Various regions of the torus can be maintained at different temperatures, allowing thermal cycling and potentially PCR for DNA amplification. Subsequently, the electrodes are reprogrammed, and the fluid is pumped either back to conduit 1 or to conduit 4 for further processing. By changing the polarity of the electrodes, one can readily modulate the direction of the flow. Subsequently, the fluid can be split into two separate streams (conduits 5 and 6) in any desired proportions.

Using network theory, Bau et al. (2002, 2003) developed a framework to calculate the flow rates and pressure drops in the network's branches when the electrodes' currents are given and to determine the electrodes' currents when the flow rates in the various network conduits are specified. The algorithm was implemented in a computer routine MHDNets written in Matlab that contains a graphical user interface. The user sketches the network on the computer screen (i.e., Fig. 6) and specifies whether s/he desires to operate in design or control mode. When in control mode, the user needs to specify the desired flow rates in the various network branches and the code computes the electrodes' currents.

4.3 MHD-based Stirrers

Although the characteristic lengths associated with the microfluidic devices are small – typically on the order of 100 μ m – diffusion alone does not allow for sufficiently fast mixing. For example, at room temperature, myosin's diffusion coefficient in water is about 10^{-11} m²/s, and the diffusion time along a length of 100 μ m is intolerably large, about 10^3 seconds. Since the Reynolds numbers (Re) of flows in micro-devices are usually very small ($Re \ll 1$), the flows are laminar, well-organized, and provide poor mixing. In order to achieve a reasonable yield of chemical reactions and bioassays, micro-stirrers must be integrated into the lab-on-a-chip.

MHD provides us with a relatively easy means for mixing and stirring. Two different types of MHD stirrers have been reported in the literature. One type of stirrer relies on altering the flow direction to enhance dispersion (Gleeson and West, 2002; West et al., 2003; Gleeson et al., 2004) while the second type of stirrer takes advantage of the ease with which one can induce secondary flows.

West's group (Gleeson and West, 2002; West et al., 2003; Gleeson et al., 2004) describe a toroidal MHD stirrer, one half of which is filled with fluid **A** and the other half with fluid **B**. MHD is used to drive the flow in the torus. Due to the parabolic velocity profile, one fluid

penetrates into the other, increasing the interfacial surface between the two fluids and shortening the diffusion distance (Taylor dispersion). By altering periodically the polarity of the electrodes, one can reverse the direction of the flow and increase the complexity of the interface.

Alternatively, one can pattern electrodes of various shapes that induce electric fields in different directions such as the rotator described earlier (equation 15). The interaction of such electric fields with the magnetic field induces secondary flows that may benefit stirring and mixing (Bau et al., 2001). Although these secondary flows significantly enhance the mixing process, they are well-ordered and the mixing is still poor. One can do better, however. By periodically or aperiodically alternating among two or more different flow patterns, one can induce (Lagrangian) chaotic advection (Yi et al., 2002; Qian et al., 2002; Xiang and Bau, 2003; Qian and Bau, 2005a).

Below, we describe in some detail one particular implementation of a MHD stirrer. Yi et al. (2002) describe a MHD stirrer consisting of a closed cylindrical cavity with an electrode (denoted **C**) deposited around its periphery and two additional electrodes **A** and **B** placed eccentrically inside the cavity. The magnetic field is parallel to the cavity's axis. When a potential difference is applied across the electrode pair **A-C**, a circulatory flow pattern results with its center of rotation near the electrode **A**. Fig. 7 depicts the corresponding computed streamlines (a) and the experimental passive tracer's trajectories (b). A similar flow pattern results with its center of rotation next to the electrode **B** when a potential difference is applied across the electrode pair **B-C**.

When the electric potential differences are applied alternately across the electrode pairs **A-C** and **B-C** with a period T , the two different flow patterns are periodically switched on and off. At small periods T , the flow is regular and periodic in most of the cavity. At relatively large periods, the passive tracer experiences global chaotic advection. Fig. 8 depicts the Poincaré sections (stroboscopic images, left column) and flow visualization images (right column) at various periods. As the period T increases, so does the complexity of the flow.

From the foregoing, it is clear that MHD is very suitable for providing stirring in the microfluidic setting. Table 2 summarizes the various MHD stirrers that have been considered in the literature.

4.4 MHD-based Liquid Chromatography

Liquid chromatography (LC) is used for the separation, purification, and detection of various biochemicals. Typically, LC requires the pumping of a fluid sample containing different analytes unidirectionally in a fixed-length column equipped with a stationary phase. The various analytes in solution have different affinities with the stationary phase and, as the solution flows through the conduit, the various species are separated into bands. The column length that is needed for efficient separation depends, however, on the type of analytes that one needs to separate. For example, when separating proteins **A** and **B** that have very different affinities with the stationary phase, a short column would suffice. In contrast, when the affinities of **A** and **B** with the stationary phase are similar, a long column would be needed. When the analytes are not a priori known, the appropriate column length cannot be anticipated.

Unfortunately, existing LCs consist of a fixed length column that cannot adjust according to the separation task at hand. This shortcoming can be alleviated by using a column bent into a closed loop that has a virtual "infinite length." To make such a closed loop column a reality, one needs means for continuously circulating fluid in a closed loop. MHD is one of the few technologies that can continuously circulate the sample in a closed loop (Zhong et al., 2002;

Bao and Harrison, 2003a; Eijkel et al., 2004). Additionally, with judicious patterning of the electrodes, it is possible to control the velocity profile and affect dispersion.

Bao and Harrison (2003a, 2003b) and Eijkel et al. (2004) experimentally studied the suitability of a circular AC MHD-based micro-pump for chromatographic applications and concluded that a separation rate of 5.5 plates/s can be achieved in the AC MHD-based microfluidic device.

4.5 MHD-based Continuous PCR

The polymerase chain reaction (PCR) is a commonly used tool in biotechnology for nucleic acid amplification for genetic analysis and disease detection. To facilitate DNA amplification, one mixes the sample containing target DNA (amplicon) with the appropriate reagents and alters the suspension's temperature in a predetermined way. Most PCR processes are carried out with the sample maintained stationary in the reactor chamber while the temperature of the reactor is cycled. This arrangement necessitates the heating and cooling of both the reagents and the surrounding substrate, which is energy inefficient. Furthermore, the system has a relatively large thermal time constant. It would be desirable to maintain various regions at fixed temperatures and cycle the sample among these regions (continuous flow PCR). West et al. (2002) attempted to use the MHD to continuously circulate the PCR mix in a closed loop with three heaters being used to create three distinct regions at the temperatures needed for DNA denaturation ($\sim 94^{\circ}\text{C}$), annealing ($50\text{--}55^{\circ}\text{C}$), and extension (72°C). Unfortunately, no PCR amplification has been demonstrated. Similar ideas can, however, be applied to chemical reactors that require temperature alternations in their operation.

4.6 MHD-based Micro-Coolers

Since MHD can facilitate fluid circulation, it can be used to facilitate cooling. Liquid metals are particularly suitable for this purpose due to their high thermal conductivity, high boiling point temperature, and large electric conductivity. Since MHD propulsion is easy to implement, easy to miniaturize, and does not require mechanical components, it is ideal for microcooling applications, such as those required in microelectronics. Although various patents address MHD microcoolers, it is not known whether any products are in actual use.

5. Conclusions

In many microfluidic applications, it is necessary to propel fluids from one part of the device to another, control fluid motion, stir, and separate fluids. However, due to the small size of the devices and the desire to carry out a large number of operations, these tasks are far from trivial. MHD offers an elegant, inexpensive, flexible, customizable means of performing some of these functions.

The flow in MHD-based microfluidics is induced through the interaction between an external magnetic field and current flux transmitted through the solution. By judicious patterning of electrodes and the application of potential differences across electrode pairs, one can direct the liquid to flow along any desired path without a need for valves and pumps. Mathematical models and approximate solutions for both DC and AC MHD-based micro-pumps have been developed and validated, and can be used to design MHD-based microfluidic devices, including MHD-based microfluidic networks. MHD also provides a convenient means to stir liquids at very low Reynolds numbers.

MHD-based microfluidics operate at low voltages, can direct the liquid to flow along any desired path without a need for valves and pumps, and continuously circulate the sample in a closed loop, thereby forming a conduit with an “infinite length”, and can chaotically stir the

sample without any moving part. A few potential applications of the MHD-based microfluidics including liquid chromatography, thermal cycler, and electronics cooler are illustrated.

Acknowledgments

This work is supported, in part, by NIH STTR grant 545817 to Vegrandis (HHB).

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Fig.1.

A schematic diagram of the MHD “pump.” Two electrodes with a potential difference ΔV are deposited along the opposing walls of the conduit. The right figure depicts a cross-section of the conduit. The conduit is filled with an electrolyte solution and exposed to a uniform magnetic field of intensity B .

Fig. 2.

A conduit equipped with electrodes $A_{-n}, \dots, A_{-1}, A_0, A_1, \dots, A_n$ patterned transversely on the conduit's bottom. Only electrodes A_0 and A_1 are shown in the figure. The even and odd numbered electrodes are, respectively, subjected to potentials V and $-V$. The magnetic field is perpendicular to the image. The Lorentz force between electrodes A_{2n-1} and A_{2n} , where n is an integer, is directed upwards. The Lorentz force between electrodes A_{2n} and A_{2n+1} is directed downwards. The figure depicts the streamlines. The arrows indicate the direction of the flow.

Fig.3. Flow rate as a function of the conduit's width. The solid line and the triangles correspond, respectively, to theoretical results (equation 21) and the experimental data obtained from Ho (2007). $H=7\text{mm}$, $L=80\text{ mm}$, $L_E=35\text{mm}$, current $I=0.7\text{A}$, and $B=0.02\text{T}$.

Fig. 4.

The time-averaged velocity as a function of solute concentration. The current varied with solute concentration (1 M NaCl, 140 mA), (0.1 M NaCl, 100 mA), (0.01 M NaCl, 36 mA), (0.01 M NaOH, 24 mA), (PBS (pH 7.2), 12 mA), and (lambda DNA and 5mM NaCl, 10 mA). The solid line with solid squares, triangles ($\blacktriangle$), and circles ($\bullet$) correspond, respectively, to the predictions of equation (21), experimental data, and the predictions of Lemoff and Lee's theory (Lemoff and Lee, 2000). The conduit's dimensions are 20mm in length, 800 μ m in width, and 380 μ m in height. The electrodes are 4mm-long. $B=13$ mT.

Fig. 5.

A prototype of a MHD microfluidic network fabricated with low temperature, co-fired, ceramic tapes. The conduits are labeled with numbers to allow cross-reference with the text.

Fig. 6.

The user interface of the computer code MHDnets that computes the electrode currents needed to achieve desired flow rates in the branches of the network shown in Fig. 5.

Fig.7.

The flow field induced by one electrode pair A-C. (a) Streamlines. (b) A photograph of a flow visualization experiment (the figure appeared previously in Yi et al. (2002)).

Fig.8.

Poincaré sections (stroboscopic images, left column) and flow visualization photographs (right column) when the electric potential differences are applied alternately across the electrode pairs A-C and B-C with a period T . (a) $T=4$, (b) $T=10$, and (c) $T=40$. The figure appeared previously in Yi et al. (2002).

Table 1

A list of MHD micro-pumps reported in the literature

Authors	Substrate material	Comments
Jang & Lee (2000)	Silicon	DC, Saline solution (sea water), significant bubble formation
Zhong et al.(2002)	Low temperature co-fired ceramic (LTCC) tapes with screen-printed electrodes.	DC, toroidal (closed loop) operated with mercury slugs and NaCl solution
Homsy et al. (2005, 2007)	Pyrex wafer	DC, high current density, vapor removal
Leventis & Gao (2001) Fritsch's group(Arumugam et al., 2005, 2006; Aguilar et al., 2006)	LTCC	DC, RedOx solution
Lemoff & Lee (2000) West et al. (2002, 2003) Eijkel et al. (2004) Bao & Harrison (2003a, 2003b)	Silicon	AC. Significant power dissipation and heat generation due to the induction of eddy currents

Table 2

A summary of a few MHD stirrers reported in the literature

Authors	Principle of Operation	Comments
West's group (Gleeson and West, 2002; West et al., 2003; Gleeson et al., 2004)	Taylor dispersion	
Bau's group (Yi et al., 2002; Qian et al., 2002; Xiang and Bau, 2003; Qian and Bau, 2005a)	Chaotic advection	A number of "agitators" are actuated alternately