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An FBP image reconstruction algorithm for x-ray differential phase contrast CT

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Abstract

Most recently, a novel data acquisition method has been proposed and experimentally implemented for x-ray differential phase contrast computed tomography (DPC-CT), in which a conventional x-ray tube and a Talbot-Lau type interferometer were utilized in data acquisition. The divergent nature of the data acquisition system requires a divergent-beam image reconstruction algorithm for DPC-CT. This paper focuses on addressing this image reconstruction issue. We developed a filtered backprojection algorithm to directly reconstruct the DPC-CT images from acquired projection data. The developed algorithm allows one to directly reconstruct the decrement of the real part of the refractive index from the measured data. In order to accurately reconstruct an image, the data need to be acquired over an angular range of at least 180° plus the fan-angle. Different from the parallel beam data acquisition and reconstruction methods, a 180° rotation angle for data acquisition system does not provide sufficient data for an accurate reconstruction of the entire field of view. Numerical simulations have been conducted to validate the image reconstruction algorithm.

Keywords

differential phase contrast CT; filtered backprojection

1. INTRODUCTION

In recent years, x-ray phase contrast imaging methods have attracted significant interest. Based upon the phase retrieval method, phase contrast imaging can be classified into three categories: interferometric methods,^{1, 2} analyzer crystal methods,^{3, 4} and free-space propagation methods.^{5–7} These methods normally require excellent spatial coherence and/or excellent spatial resolution of the detectors. Phase contrast imaging is thus normally conducted using high-brilliance synchrotron beam lines or micro-focus x-ray tubes. It is difficult to adapt these phase retrieval methods to a clinical setting for medical imaging.

Shear interferometry method has been introduced to measure the gradient of the local phase shift distribution using synchrotron beam lines.^{8–10} The key theoretical foundation is the Talbot effect^{11–14} and Moiré interferometry.¹⁵ Computed tomography imaging methods have been developed^{10, 11, 16} based on the phase contrast projection images.

The shear interferometry method only requires a mild spatial coherence length, which opens up a new opportunity to implement phase contrast imaging using a conventional X-ray tube and the shear interferometry method. As a matter of fact, most recently, an ingenious scheme was proposed and experimentally implemented for x-ray differential phase contrast imaging¹⁷ and differential phase contrast computed tomography (DPC-CT).^{18, 19} In this scheme, Pfeiffer et al^{17–19} proposed to mount an absorption grating in front of a low-brilliance x-ray source to generate coherent line sources (Figure 1). In this proposal, the coherence length is determined by the width of grating openings, not the x-ray focal spot size. Thus, a conventional x-ray tube with high x-ray output can be turned into many mutually incoherent line sources. The coherence length of the beam is determined by the width of the opening of the absorption grating, which is about 20–50 μm or even less. In this paper, we refer to this grating as a beam slicer. Its role is to slice the cone beam x-rays emanated from the focal spot with a size of 300–1000 μm into many fan beam x-rays with good spatial coherence. Using this method, for each fan beam x-rays, the spatial coherence length has been improved by a factor of ten or more. Thus, it enables to significantly reduce the distance between the x-ray source and the image object. As a consequence, this method allows to construct an x-ray phase contrast imaging system with compact size which is often required in medical application and non-destructive inspection application.

For a given x-ray detector with square shape, the degree of divergence is determined by the spanned fan-angle/cone-angle which is given by the following formula:

$$\gamma_m = 2 \arctan\left(\frac{H}{2D}\right) \quad (1)$$

where H is the length of the detector and D is the distance from the x-ray focal spot to the detector. Therefore, when the source to detector distance D is very long, the diverging effect of the x-ray beams are negligible, this is often the case for synchrotron beam lines and also for in-line holography phase contrast imaging method. In this case, the beams are well approximated as plane wave and a parallel beam image reconstruction method is sufficiently good for image reconstruction in DPC-CT. However, using the new phase retrieval method by Pfeiffer et al,^{17–19} the distance D is significantly reduced ($D \approx 160\text{cm}$ ¹⁷). Note that the effective detector size is limited by the available size of the third grating in front of the detector, it is about 6cm.¹⁹ Using these parameters, the divergence angle γ_m ¹⁹ can be estimated as

$$\gamma_m = 2 \arctan\left(\frac{H}{2D}\right) \approx 2^\circ \quad (2)$$

In this case, the parallel beam approximation is still acceptable as we will present later (See Section 3). Thus, the well-known parallel-beam image reconstruction method²³ can be adapted to reconstruct DPC-CT images.^{18, 20, 21}

When a larger size image object is scanned with the same detector-to-source distance D , the divergence angle γ_m is increased and parallel beam approximation becomes less accurate. When the divergence angle goes beyond 5° , it will be shown in Section 3 that the diverging nature of the data acquisition system is not negligible any more. In fact, if the size of the

image object is doubled in the above estimation, the divergence angle is almost 5° . Therefore, a divergent beam image reconstruction algorithm is desirable for this new DPC-CT data acquisition method. In this paper, we present an image reconstruction formula for fan-beam DPC-CT which can be utilized to accurately reconstruct DPC-CT images at any fan-angle provided that the data is acquired within the angular range of 180° plus fan-angle.

2. METHODS

2.1 Parallel beam data acquisition geometry for differential phase retrieval method

As explained by Pfeiffer et al,¹⁸ the fundamental idea in differential phase retrieval method is to measure the angular change $\Theta(\rho, \theta)$ of the incident x-ray beam. As shown in Figure 2, the measured data is labeled by the distance ρ , which is measured from the origin of the coordinate system to the incident ray direction and the angle θ , which specifies the direction of the incident ray. The quantity $\Theta(\rho, \theta)$ is related to the local gradient of the object's phase shift as follows:^{15, 22}

$$\Theta(\rho, \theta) = \frac{\partial}{\partial \rho} \int_l dl \delta(x, y) \quad (3)$$

where $\delta(x, y)$ is the decrement of the real part of the object's refractive index $n = 1 - \delta + i\beta$. The letter l labels the incident ray. Namely, the above line integral is performed along the incident ray direction. Note that the integral in Equation (3) is nothing but the Radon transform $R_\delta(\rho, \theta)$ of the target function $\delta(x, y)$. Thus, Equation (3) can be rewritten as

$$\Theta(\rho, \theta) = \frac{\partial}{\partial \rho} R_\delta(\rho, \theta) \quad (4)$$

After a Fourier transform with respect to the variable ρ , Equation (4) is recast into the following form:

$$\tilde{\Theta}(\omega, \theta) = -i2\pi\omega \tilde{R}_\delta(\omega, \theta) \quad (5)$$

Using the inverse Radon transform in Fourier space, one can easily obtain the following image reconstruction formula:^{19–21}

$$\begin{aligned} \delta(x, y) &= \frac{1}{2\pi^2} \int_0^\pi d\theta \int_{-\infty}^{+\infty} d\omega [i\pi \text{sgn}(\omega)] \tilde{\Theta}(\omega, \theta) \\ &\times e^{i2\pi\omega(x \cos \theta + y \sin \theta)} \end{aligned} \quad (6)$$

Therefore, after the data are acquired from view angles in the angular range $[0, \pi]$, a one-dimensional Fourier transform is performed with respect to the Radon distance ρ to obtain $\tilde{\Theta}(\omega, \theta)$. A filtering kernel $i\pi \text{sgn}(\omega)$ is used to filter the data. Finally, an integral over the view angles is performed to reconstruct images.

2.2 Fan-beam data acquisition for differential phase retrieval method

Note that the image reconstruction formula (6) makes sense only when the acquired data are written in terms of the Radon distance ρ and the view angle θ . Namely, it is only convenient for the parallel beam data acquisition geometry.

For the fan beam data acquisition geometry as shown in Figure 3, the x-ray tube and detector are assumed to rotate around the image object along a circle with radius R and source-to-detector distance D . The acquired data are naturally labeled by the angular position, t , of the x-ray focal spot and the detector position which is labeled by a projection distance u . We referred to this measured data as $\Theta(t, u)$ in this paper. The half-width of the detector is u_m , which implies that the fan angle γ_m is $\gamma_m = 2 \arctan(u_m/D)$.

For fan-beam data, one way to reconstruct an image is to rebin the acquired data into parallel-beam projection data and then utilize Eq. (6) to reconstruct the image. In this paper, we develop new image reconstruction formulae which directly reconstruct images from the acquired data $\Theta(t, u)$ without requiring the rebinning step.

2.3 Fan-beam reconstruction formula for full scan case

In order to achieve this goal, we rewrite Equation (6) in the real domain:

$$\delta(x, y) = \frac{1}{4\pi^2} \int_0^{2\pi} d\theta \int_{-\infty}^{+\infty} d\rho \frac{1}{x \cos \theta + y \sin \theta - \rho} \Theta(\rho, \theta). \quad (7)$$

This formula dictates how each individual projection data $\Theta(\rho, \theta)$ contributes to the final image. Note that we have extended the integral limit from $[0, \pi]$ to $[0, 2\pi]$ and introduced an extra factor to account for the redundancy.

In the following, we derive image reconstruction formulae for fan-beam data acquisition geometry. It is easy to see that each individual projection datum $\Theta(\rho, \theta)$ can also be relabeled in terms of the Radon distance ρ and angle θ by the following nonlinear coordinate transform:

$$\rho = \frac{-Ru}{\sqrt{u^2 + D^2}}, \quad \theta = \frac{\pi}{2} + t + \arctan \frac{u}{D}. \quad (8)$$

This transform introduces an extra Jacobian factor:

$$d\rho d\theta = \frac{RD^2}{(u^2 + D^2)^{3/2}} du dt, \quad (9)$$

$$x \cos \theta + y \sin \theta - \rho = \frac{L(x, y; t)}{\sqrt{u^2 + D^2}} [u - U(x, y; t)] \quad (10)$$

where the functions $L(x, y; t)$ and $U(x, y; t)$ is given by

$$L(x, y; t) = R - x \cos t - y \sin t, \quad (11)$$

$$U(x, y; t) = D \frac{x \sin t - y \cos t}{R - x \cos t - y \sin t}. \quad (12)$$

Therefore, the equation (7) can now be written as

$$\delta(x, y) = \frac{1}{4\pi^2} \int_0^{2\pi} dt \frac{R}{L(x, y; t)} F[t, U(x, y)], \quad (13)$$

where the filtered function $F[t, U(x, y)]$ is given by

$$F(t, U) = \int_{-\infty}^{+\infty} du \frac{1}{u - U} \bar{\Theta}(t, u). \quad (14)$$

In this equation, the pre-weighted projection data $\bar{\Theta}(t, u)$ is defined as:

$$\bar{\Theta}(t, u) = \frac{D^2}{u^2 + D^2} \Theta(t, u) \quad (15)$$

Equations (13,14,15) provide a direct image reconstruction from the measured projection data $\bar{\Theta}(t, u)$. The algorithm is summarized in the following steps:

1. Pre-weight the acquired projection data using Eq. (15).
2. Filter the pre-weighted data using the Hilbert filtering kernel in Eq. (14).
3. Back-project the filtered data to reconstruct an image using Eq. (13).

These three steps represent the image reconstruction algorithm for fan-beam DPC-CT. Note that this formula requires that the data are acquired along a full circle. This is referred to as the full scan data acquisition mode. In the next subsection, we extend the above result to the short scan case where the data are acquired from less than a full circle.

2.4 Fan-beam reconstruction formula for short scan case

In absorption X-ray CT, it has been proven that the data are sufficient to reconstruct the entire image object when the angular range is longer than $180^\circ + \text{fan angle}$, where the fan angle is defined as the entire angle subtended by the detector from the source position. The same condition is also true for the differential phase contrast CT case. We refer to this case as the short scan mode. When the short scan data acquisition mode is utilized, some incident rays have a conjugate ray, but some other rays are only measured once. Therefore, a weighting function is needed to weight the redundantly measured data. In fact, it is easy to demonstrate that the following two data points should be considered the same:

$$(t, u) \iff (t' = t + \pi - 2 \arctan \frac{u}{D}, u' = -u) \quad (16)$$

Thus, similar to absorption CT, the following weighting function can be utilized to weight the data before the filtering step is conducted.

$$W(t, u) = \begin{cases} \sin^2 \left(\frac{\pi}{2} \frac{t}{\gamma_m - 2\gamma} \right), & t \in [0, \gamma_m - 2\gamma) \\ 1, & t \in [\gamma_m - 2\gamma, \pi - 2\gamma) \\ \sin^2 \left(\frac{\pi}{2} \frac{\pi - t + \gamma_m}{\gamma_m + 2\gamma} \right), & t \in [\pi - 2\gamma, \pi + \gamma_m] \end{cases} \quad (17)$$

where the variable γ is defined as $\gamma = \arctan(u/D)$. Thus, by incorporating the normalized weighting function $W(t, u)$ into Eq. (15), the pre-weighted projection data read as:

$$\bar{\Theta}(t, u) = W(t, u) \frac{D^2}{u^2 + D^2} \Theta(t, u) \quad (18)$$

The image reconstruction formulae (13)–(15), together with Eq. (18) can be utilized to reconstruct image, except that the integration range of t in Eq. (13) is changed to $[0, \pi + \gamma_m]$ and the factor $1/4\pi^2$ is changed to $1/2\pi^2$.

3. RESULTS

Numerical simulations have been conducted using the fan beam data acquisition geometry as shown in Figure 3. The mathematical phantom which consists of a uniform and elliptical disk and two uniform circular disks was utilized. The projection data are numerically generated using the Snell's law.²² The decrement of the real part of the refractive index was assumed to be 1.0×10^{-6} and 0.5×10^{-6} for the circular and elliptical disks respectively. The distance from the source to the detector is selected to be 6. The detector is assumed to be collinear. The image matrices are 256×256 .

3.1 Full scan data acquisition mode

The radius of the source trajectory is assumed to be 4. The fan angle of the divergent beam is 30° in order to cover the entire image object. The view angle is sampled from 0° to 360° , with a view sampling rate $\Delta t = 0.5^\circ$. The detector is sampled from -3.46 to 3.46 , with a

detector sampling rate $\Delta u = \frac{6.92}{600} \approx 0.0115$. The results are presented in Figure 4. It is clear that the image was faithfully reconstructed using the new algorithm with very high accuracy.

3.2 Short scan data acquisition mode

All parameters are the same as the full scan data acquisition mode except: (1) the decrement of the real part of the refractive index of the circular objects is changed to 0; (2) the angular range of the data acquisition is reduced from 360° to 210° , which is sufficient for reconstruction according to the x-ray absorption CT image reconstruction theory.²³ The numerical results are presented in Figure 5. It can be clearly seen that the image is faithfully reconstructed with very high accuracy as indicated in image intensity plots.

3.3 Parallel beam approximation

In the first numerical experiment, we applied parallel-beam reconstruction algorithm¹⁹ to the case of 30° fan angle, which is the case presented in Figure 4. The parallel beam reconstruction algorithm generates significant image artifacts as shown in Figure 6 (a). In the second numerical experiment, the radius of the source trajectory is increased to 12 and

the fan angle of the divergent beam is reduced to 10° in order to cover the entire image object. As shown in Figure 6(b), the image quality is improved comparing with Figure 6(a), but apparently some blur still exists at the boundary. When the source to detector distance keeps increasing such that the fan angle is reduced to 5° as shown in figure 6(c), image artifacts are almost invisible. This indicates that the parallel beam approximation is acceptable. In the last numerical experiment, the fan angle of the divergent beam is reduced to 2.5° for the same image object. In this case, as shown in Figure 6(d), the parallel beam reconstruction method is sufficiently good to reconstruct DPC-CT images. Using the experimental data presented in Ref.,¹⁹ it is easy to estimate that the corresponding fan angle is smaller than 2.5° . Thus, in this case, the parallel beam approximation is sufficient to reconstruct almost artifacts free DPC-CT images.¹⁹

3.4 Super short scan data acquisition mode

In this subsection, we numerically demonstrate that the angular range for data acquisition should be longer than $\pi + \text{fan angle}$ for the presented new algorithm. When the scanning angular range is shorter than that, significant image artifacts may occur. In numerical simulations, all of the parameters are the same as the short scan case except that the source angle is reduced to from 210° to 180° , which is less than a short scan. As shown in Figure 7, image artifacts appear in the reconstructed image and signal drop is clearly observed in image intensity plots. This numerical experiment also indicates that the short scan angular range is necessary for the presented fan-beam DPC-CT reconstruction.

4. DISCUSSIONS

4.1 Difference from the fan beam absorption CT image reconstruction algorithm

In this paper, the fan beam image reconstruction algorithm was derived using a collinear detector geometry. It is easy to change to the curved detector case using the relation $\gamma = \arctan(u/D)$. For absorption CT, fan beam image reconstruction algorithm has been derived for both full scan and short scan cases.²³ The DPC-CT fan beam image reconstruction algorithm is different from its absorption counterpart in three aspects. First, the pre-weighting function is different. In DPC-CT, the preweighting factor is $D^2/(D^2 + u^2)$. In contrast, the pre-weighting factor is a square of root, i.e., $\sqrt{D^2/(D^2 + u^2)} = D/\sqrt{D^2 + u^2}$. Second, the distance weighting function in backprojection step is also different. In DPC-CT, the weight is $R/L(x, y; t)$. In contrast, this weighting function is squared in absorption CT, i.e., $R^2/L^2(x, y; t)$. Finally, the filtering kernel in the filtering step is also different. The Hilbert filter is used in the DPC-CT fan-beam reconstruction whereas the ramp filter is used in the absorption CT image reconstruction.

4.2 Importance when imaging large object compared to parallel beam DPC-CT reconstruction

When the source to detector distance is much larger than the size of the image object, one can expect that the difference between the parallel-beam and fan-beam image reconstruction methods is negligible. Our numerical results demonstrate that when the fan angle is smaller than 5° , the parallel-beam approximation is valid and parallel-beam image reconstruction

can be utilized to reconstruct DPC-CT images. However, for larger fan angles, the fan-beam image reconstruction algorithm such as the one presented in this paper is needed for an accurate image reconstruction.

5. CONCLUSION

In this paper, an image reconstruction formula for fan-beam DPC-CT was derived and validated. It has been demonstrated that for very large source to detector distance, the parallel beam approximation can be utilized to reconstruct images with acceptable image quality, such as in the most recent experiments.¹⁹ However, when the image object is relatively large, the fan angle is not small. In this case, parallel beam approximation can not be directly applied to reconstruct images. The fan-beam image reconstruction formula presented in this paper can be directly applied to accurately reconstruct DPC-CT images. The presented image reconstruction formula is also useful for other DPC-CT imaging method provided that the divergent beam data acquisition geometry is utilized.

The paper has not been submitted for publication or presentation elsewhere.

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Figure 1.
Geometrical model for fan beam DPC-CT data acquisition.

Figure 2.
A geometric model of the parallel beam data acquisition model.

Figure 3.
Geometrical model for fan beam DPC-CT data acquisition.

(a)

(b)

(c)

(d)

Figure 4.

Reconstruction results for a fan-beam full scan. (a) The reconstructed image. (b), (c) and (d) are image intensity plots corresponding to line 1, 2 and 3 in (a) respectively.

(a)

(b)

(c)

(d)

Figure 5.

Numerical results for short scan data acquisition mode. (a): Reconstructed image. (b), (c) and (d): Image intensity plots corresponding to the line 1, 2 and 3, respectively.

Figure 6. The parallel-beam reconstruction result for fan-beam projection data. (a), (b), (c), and (d) correspond to fan angle 30° , 10° , 5° , and 2.5° respectively.

Figure 7.

The reconstruction result for fan-beam less than a short scan. Figure 6(a) is the reconstructed image. (b), Figures (c) and (d) are profile plots corresponding to line 1, 2 and 3 respectively in Figure 6(a).