Published in final edited form as:

IEEE Trans Ultrason Ferroelectr Freq Control. 2009 January ; 56(1): 77–90. doi:10.1109/TUFFC.

2009.1007

Generalized Cystic Resolution: A Metric for Assessing the Fundamental Limits on Beamformer Performance

Drake A. Guenther and William F. Walker

The authors are with the Department of Biomedical Engineering, University of Virginia, Charlottesville (e-mail: dag2m@virginia.edu).

Abstract

Existing methods for characterizing the imaging performance of ultrasound systems do not clearly quantify the impact of contrast, spatial resolution, and signal-to-noise ratio (SNR). Although the beamplot, contrast resolution metrics, SNR measurements, ideal observer methods, and contrast-detail analysis provide useful information, it remains difficult to discern how changes in system parameters affect these metrics and clinical imaging performance. In this paper, we present a rigorous methodology for characterizing the pulse-echo imaging performance of arbitrary ultrasound systems. Our metric incorporates the 4-D spatio-temporal system response, which is defined as a function of the individual beamformer channel weights. The metric also incorporates the individual beamformer channel electronic SNR. Whereas earlier performance measures dealt solely with contrast resolution or echo signal-to-noise ratio, our metric combines them so that tradeoffs between these parameters are easily distinguishable. The new metric quantifies an arbitrary system's contrast resolution and SNR performance as a function of cyst size, beamformer channel weights, and beamformer channel SNR.

We present a theoretical derivation of the unified performance metric and provide simulation and experimental results highlighting the metric's utility. We compare the fundamental performance limits of 2 beamforming strategies: the dynamic focus finite impulse response (FIR) filter beamformer and the spatial matched filter (SMF) beamformer to the performance of the conventional delay-and-sum (DAS) beamformer. Results from this study show that the SMF beamformer and the FIR beamformer offer significant gains in beamformer SNR and contrast resolution compared with the DAS beamformer, respectively. The metric clearly distinguishes the performance of the SMF beamformer, which enhances system sensitivity, from the FIR beamformer, which optimizes system contrast resolution. Finally, the metric provides one quantitative goal for optimizing a broadband beamformer's contrast resolution performance.

I. INTRODUCTION

THE ability to meaningfully quantify the imaging performance of ultrasound systems is critical, both to characterize the fundamental imaging limits of the system and to optimize image quality. Accurate quantitative evaluation of different beamforming strategies or system operating characteristics is essential during system development and can significantly reduce the time and cost associated with design. Many factors must be considered during beamformer design. Although cost, speed, and hardware specifications affect development to varying degrees, beamformer resolution, contrast, sensitivity and signal-to-noise ratio (SNR) receive a great deal of attention because they significantly affect image quality and the fidelity of the clinically relevant information.

Perhaps the most common metric used to characterize scanner resolution is the -6 dB beamwidth, or full-width at half-maximum (FWHM), of the beamplot [1]. This metric has been adapted from radar, yet sometimes provides misleading information about ultrasound scanner

resolution because the FWHM neglects the effects of sidelobes and grating lobes on image contrast. In addition, the targets of interest in medical ultrasound are normally weakly reflecting tissues in an inhomogeneous scattering medium, whereas radar targets are brightly reflecting objects in a nonscattering background.

Historically, estimating the contrast performance of existing imaging systems was accomplished by imaging phantoms or human subjects. Studies such as those presented in [2] and [3] used tissue-mimicking phantoms containing lesions of varying contrast and sizes to compare different lesion detection algorithms and the contrast resolution of different imaging systems. In a series of seminal works, Wagner, Smith, and colleagues developed a rigorous ideal observer model for lesion detectability based on speckle size in envelope detected B-mode images [2], [4], [5]. Using pre-envelope detected radio-frequency images, Abbey Zemp, and coworkers further developed the model for a variety of discrimination tasks [6], [7]. The application of the ideal observer methodology to task-based assessment of ultrasound images has provided numerous insights and improvements to the ultrasound field.

We have been investigating a similar approach to quantifying the performance of ultrasound systems using a cystic resolution metric originally developed by Vilkomerson *et al.* [8]. Whereas the ideal observer models are derived using the entire image data set, the cystic resolution metric focuses on the point contrast at the center of the cyst (one pixel in the pre-envelope detected image). Using Vilkomerson's metric, scanner performance is quantified as the size of a void that produces a given level of point contrast. Johnson [9] extended the metric to a 3-D broadband model using circular apertures and compared different system operating conditions using contrast versus cyst diameter curves. Üstüner and Holley [10] presented a similar concept using arbitrary apertures but did not provide a theoretical foundation, resulting in a limited understanding of the metric as well as its utility. Our group previously presented a cystic resolution metric that uses the system's 4-D spatio-temporal response and accounts for electronic noise [11]. Unlike the beamplot, the cystic resolution metric identifies specific points in the system point spread function (PSF) that most affect image quality. Results of the metric are easily understood by plotting cystic contrast versus cyst size, producing curves similar to those developed by Vilkomerson [8] and Johnson [9]. Furthermore, the metric is useful in guiding the design and optimization of ultrasound systems by allowing for easy comparison of performance tradeoffs when changing system parameters.

Cystic resolution combines spatial resolution and contrast in a meaningful way, and the concept is extremely useful for ultrasound imaging systems. However, one serious limitation of the metric [11] is that it neglects beamformer sensitivity, only allowing for analyzing cystic resolution as a function of output SNR. It is possible that a system maximizing contrast resolution may achieve such a low array gain [12] that any theoretical improvements in contrast will be negated by the loss in detection sensitivity. Recently, Liu *et al.* defined a rigorous measure of beamformer echo signal-to-noise ratio for comparing the sensitivities of various beamforming strategies such as the conventional delay-and-sum (DAS) beamformer and the spatial matched filter (SMF) beamformer [13]. Their results are intriguing and highlight the possible improvements in SNR achievable with novel beamforming strategies, but their analysis focused on beamformer output SNR and neglected contrast resolution performance. We aim to clarify the relationship between beamformer output SNR, beamformer array gain, and contrast resolution, so that tradeoffs in system operating conditions can be readily compared and different beamforming strategies can be optimized.

This paper develops a unified framework for analyzing the contrast resolution and SNR performance of various beamforming strategies. The new performance metric extends the previous cystic resolution metric [11] by making system contrast resolution a function of the individual transducer element spatio-temporal responses and the individual beamformer

channel weights, or apodization. In addition, we generalize electronic SNR as individual channel SNR, not post beamsum electronic SNR as in [11]. Generalizing SNR allows us to analyze an arbitrary beamformer's clinical imaging performance as a function of input channel SNR, post beamsum SNR, or array gain. The new metric extends the work presented in [13] by combining cystic resolution and beamformer sensitivity so that the intrinsic relationship between the 2 can be explored. We show how using this new metric allows for straightforward system optimization with one quantitative goal, aids theoretical system design, and characterizes the fundamental performance limits of a beamformer. We also present simulation and experimental results wherein the metric is used to compare the conventional delay-and-sum (DAS) beamformer's imaging performance with the FIR beamformer's [14] and SMF beamformer's [15], [16] imaging performance.

II. THEORY

A. Derivation of Echo Signal-to-Noise Ratio, Input Channel Signal-to-Noise Ratio, Array Gain, and Cystic Resolution

Our goal is to derive a metric that quantifies the beamformer signal-to-noise ratio and contrast resolution of an arbitrary broadband imaging system. Although both these metrics have been derived before (cystic resolution in [11] and SNR in [11], [13]), we generalize the expressions to be functions of cyst size, the individual element responses, the beamformer channel weights, and the beamformer channel SNR. Doing so allows for greater flexibility when analyzing beamformer performance and leads to straightforward system optimization.

We present a 2-way broadband imaging formulation in this paper. The one-way broadband imaging formulation can be expressed in a similar manner; but we note that, in most ultrasonic imaging applications, the 2-way pulse-echo response is of greater interest. A simplified version of a conventional delay-and-sum beamformer is depicted in Fig. 1. The receive delays and apodization values are updated dynamically and applied before beamsummation. Mathematically, each A-line sample is the superposition of a sample on each individual receive channel multiplied by a real weight. The signal on each receive channel will be a function of the system's transmit spatio-temporal response, that individual receive element's spatio-temporal response, the scattering of the target, and the electronic noise. Let the pulse-echo 4-D spatio-temporal point spread function (PSF) of the ultrasound system be defined as:

$$P_{\text{TOT}}(\vec{x}, t) = \sum_{k=1}^N w_k P_k(\vec{x}, t), \quad (1)$$

where $P_k(\vec{x}, t)$, a function of 3-D space ($\vec{x}$) and time (t), is the 2-way, spatio-temporal response for the k th receive element and w_k is the weight applied to the k th receive element. Notice that this formulation allows for modeling the spatial and temporal shift variance of the PSF. It should also be noted that the transmit conditions are incorporated into this formulation including excitation pulse, transmit apodization, transmit focusing, element electromechanical impulse response, element spatial impulse response, and so forth. Time dependence of the receive channel weights, w_k , as well as the transmit and receive focal delays for the spatio-temporal channel responses, $P_k(\vec{x}, t)$, are not explicitly included in (1) for clarity. Our model only assumes linearity on receive, so nonlinear propagation on transmit can also be included.

We model the medium scattering function, $N(\vec{k})$, as a stochastic process and neglect tissue motion during reception; the function, therefore, is constant with time. The electronic noise on each receive channel, $E_k(t)$, is also modeled as a stochastic process. Assuming that the

electronic noise is purely additive, the received signal at the output of the beamformer as a function of time is:

$$y(t) = \int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}, t) N(\vec{x}) d\vec{x} + \sum_{k=1}^N w_k E_k(t). \quad (2)$$

where the integration is performed over 3-D space.

The mean squared received signal, $\langle y^2(t) \rangle$, where $\langle \cdot \rangle$ denotes the expected value operator, is given by

$$\begin{aligned} \langle y^2(t) \rangle &= \left\langle \left[\int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}_1, t) N(\vec{x}_1) d\vec{x}_1 + \sum_{k=1}^N w_k E_k(t) \right] \right. \\ &\quad \times \left. \left[\int_{-\infty}^{+\infty} \sum_{l=1}^N w_l P_l(\vec{x}_2, t) N(\vec{x}_2) d\vec{x}_2 + \sum_{l=1}^N w_l E_l(t) \right] \right\rangle \\ &= \left\langle \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}_1, t) N(\vec{x}_1) \sum_{l=1}^N w_l P_l(\vec{x}_2, t) N(\vec{x}_2) d\vec{x}_1 d\vec{x}_2 \right\rangle \\ &\quad + \left\langle \int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}_1, t) N(\vec{x}_1) \sum_{l=1}^N w_l E_l(t) d\vec{x}_1 \right\rangle \\ &\quad + \left\langle \int_{-\infty}^{+\infty} \sum_{l=1}^N w_l P_l(\vec{x}_2, t) N(\vec{x}_2) \sum_{k=1}^N w_k E_k(t) d\vec{x}_2 \right\rangle \\ &\quad + \left\langle \sum_{k=1}^N w_k E_k(t) \sum_{l=1}^N w_l E_l(t) \right\rangle. \end{aligned} \quad (3)$$

Rearranging the expected value operator to account for the fact that the PSF is deterministic while the scattering function, $N(\vec{x})$, and the electronic noise, $E_k(t)$, are stochastic yields:

$$\begin{aligned} \langle y^2(t) \rangle &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}_1, t) \sum_{l=1}^N w_l P_l(\vec{x}_2, t) \langle N(\vec{x}_1) N(\vec{x}_2) \rangle d\vec{x}_1 d\vec{x}_2 \\ &\quad + \int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}_1, t) \left\langle \sum_{l=1}^N w_l E_l(t) N(\vec{x}_1) \right\rangle d\vec{x}_1 \\ &\quad + \int_{-\infty}^{+\infty} \sum_{l=1}^N w_l P_l(\vec{x}_2, t) \left\langle \sum_{k=1}^N w_k E_k(t) N(\vec{x}_2) \right\rangle d\vec{x}_2 \\ &\quad + \sum_{k=1}^N \sum_{l=1}^N w_k w_l \langle E_k(t) E_l(t) \rangle. \end{aligned} \quad (4)$$

We assume that the scattering function, $N(\vec{x})$, and the electronic noise, $E_k(t)$, are uncorrelated, which simplifies expression (4) to:

$$\begin{aligned}
\langle y^2(t) \rangle &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}_1, t) \sum_{l=1}^N w_l P_l(\vec{x}_2, t) \langle N(\vec{x}_1) N(\vec{x}_2) \rangle d\vec{x}_1 d\vec{x}_2 \\
&\quad + \sum_{k=1}^N \sum_{l=1}^N w_k w_l \langle E_k(t) E_l(t) \rangle.
\end{aligned} \tag{5}$$

The target scattering function is modeled as a stationary, zero-mean, multivariate normal (MVN) stochastic process: $N(\vec{x}) \sim \text{MVN}(0, \sigma_N^2 I)$, where I is an identity matrix. The autocorrelation of the scattering function is a delta function so that (5) becomes:

$$\begin{aligned}
\langle y^2(t) \rangle &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}_1, t) \sum_{l=1}^N w_l P_l(\vec{x}_2, t) \sigma_N^2 \delta(\vec{x}_1 - \vec{x}_2) d\vec{x}_1 d\vec{x}_2 \\
&\quad + \sum_{k=1}^N \sum_{l=1}^N w_k w_l \langle E_k(t) E_l(t) \rangle,
\end{aligned} \tag{6}$$

where σ_N^2 , the variance of the medium scattering function, scales the delta function. We now perform the spatial integration over $\vec{x}_2$:

$$\begin{aligned}
\langle y^2(t) \rangle &= \sigma_N^2 \int_{-\infty}^{+\infty} \sum_{k=1}^N w_k P_k(\vec{x}_1, t) \sum_{l=1}^N w_l P_l(\vec{x}_1, t) d\vec{x}_1 \\
&\quad + \sum_{k=1}^N \sum_{l=1}^N w_k w_l \langle E_k(t) E_l(t) \rangle.
\end{aligned} \tag{7}$$

We can simplify the first term using just one index, and we define the noise autocovariance: $R_E(t) = \langle E_k(t) E_l(t) \rangle$. Eq. (7) simplifies to:

$$\langle y^2(t) \rangle = \sigma_N^2 \int_{-\infty}^{+\infty} \left[\sum_{k=1}^N w_k P_k(\vec{x}_1, t) \right]^2 d\vec{x}_1 + \sum_{k=1}^N \sum_{l=1}^N w_k R_E(t) w_l. \tag{8}$$

Assuming that the electronic noise is uncorrelated between receive channels, the noise autocovariance matrix becomes purely diagonal such that $R_E(t) = 0$ whenever $k \neq l$. The expression further simplifies to:

$$\langle y^2(t) \rangle = \sigma_N^2 \int_{-\infty}^{+\infty} \left[\sum_{k=1}^N w_k P_k(\vec{x}_1, t) \right]^2 d\vec{x}_1 + \sum_{k=1}^N w_k^2 R_E(t), \tag{9}$$

The first term in (9) is the true signal component and the second term is the noise component of the mean squared output signal. We define the beamformer's output signal-to-noise ratio:

$$\text{SNR}(t) = \frac{\sigma_{\text{signal}}(t)}{\sigma_{\text{noise}}(t)}, \quad (10)$$

where $\sigma_{\text{signal}}(t)$ and $\sigma_{\text{noise}}(t)$ are the standard deviations of the signal and the noise components, respectively. Note that we define these standard deviations over an ensemble of signal and noise realizations and not in time. Using the terms in (9), we arrive at the following expression for the beamformer's output SNR:

$$\text{SNR}_{\text{out}}(t) = \frac{\sigma_{\text{signal}}(t)}{\sigma_{\text{noise}}(t)} = \sqrt{\frac{\sigma_N^2 \int_{-\infty}^{+\infty} \left[\sum_{k=1}^N w_k P_k(\vec{x}, t) \right]^2 d\vec{x}}{\sum_{k=1}^N w_k^2 R_E(t)}} \quad (11)$$

This expression shows that the echo signal-to-noise ratio at the output of the beamformer is a ratio between the energy of the entire PSF and the beamformer's noise gain. This is similar to the expressions derived in [11] and [13], but we now express SNR as a function of the receive beamformer weights, the individual receive element pulse-echo, spatio-temporal responses, and the individual receive channel electronic noise.

A similar analysis can be performed for the input SNR at one receive channel, k . This expression is:

$$\begin{aligned} \text{SNR}_{\text{in}}(t) &= \frac{\sigma_{\text{signal}}(t)}{\sigma_{\text{noise}}(t)} = \sqrt{\frac{\sigma_N^2 \int_{-\infty}^{+\infty} [w_k P_k(\vec{x}, t)]^2 d\vec{x}}{w_k \langle E_k(t) E_k(t) \rangle w_k}} \\ &= \sqrt{\frac{\sigma_N^2 \int_{-\infty}^{+\infty} [P_k(\vec{x}, t)]^2 d\vec{x}}{R_{E_k}(t)}}. \end{aligned} \quad (12)$$

Note that the noise autocovariance, $R_{E_k}(t) = \langle E_k(t) E_k(t) \rangle$, is computed for one channel.

Given the above expressions for input and output SNR, we can now quantify the beamformer's array gain as the ratio of output SNR to input SNR [12]:

$$\begin{aligned} \text{AG}(t) &= \frac{\text{SNR}_{\text{out}}(t)}{\text{SNR}_{\text{in}}(t)} \\ &= \sqrt{\frac{\sigma_N^2 \int_{-\infty}^{+\infty} \left[\sum_{k=1}^N w_k P_k(\vec{x}, t) \right]^2 d\vec{x}}{\sum_{k=1}^N w_k^2 R_{E_k}(t)}} / \sqrt{\frac{\sigma_N^2 \int_{-\infty}^{+\infty} [P_k(\vec{x}, t)]^2 d\vec{x}}{R_{E_k}(t)}}. \end{aligned} \quad (13)$$

$$AG(t) = \sqrt{\frac{R_{E_k}(t) \int_{-\infty}^{+\infty} \left[\sum_{k=1}^N w_k P_k(\vec{x}, t) \right]^2 d\vec{x}}{\sum_{k=1}^N w_k^2 R_E(t) \int_{-\infty}^{+\infty} [P_k(\vec{x}, t)]^2 d\vec{x}}}. \quad (14)$$

The array gain, $AG(t)$, quantifies the improvement in SNR obtained by using the beamformer.

The contrast resolution metric presented in [11] quantifies the contrast achieved by a given imaging system at the center of an anechoic cyst embedded in a speckle-generating medium. Cystic resolution is a simple ratio between the RMS energy of the signal when the cyst is present and when the cyst is not present. Using our receive beamformer formulation cystic resolution (CR) is expressed as:

$$\begin{aligned} CR(t) &= \frac{\sigma_{\text{cyst}}(t)}{\sigma_{\text{no cyst}}(t)} \\ &= \sqrt{\frac{\sigma_N^2 \int_{-\infty}^{+\infty} \left[\sum_{k=1}^N w_k P_k^{\text{out}}(\vec{x}, t) \right]^2 d\vec{x} + \sum_{k=1}^N w_k^2 R_E(t)}{\sigma_N^2 \int_{-\infty}^{+\infty} \left[\sum_{k=1}^N w_k P_k^{\text{tot}}(\vec{x}, t) \right]^2 d\vec{x} + \sum_{k=1}^N w_k^2 R_E(t)}} \end{aligned} \quad (15)$$

where $P_k^{\text{out}}(\vec{x}, t)$ is the 4-D spatio-temporal PSF of the k th receive element evaluated at every spatial point outside the cyst, and $P_k^{\text{tot}}(\vec{x}, t)$ is the entire k th receive element's response. Cystic resolution is now a function of the receive beamformer weights, the individual receive element responses, and the individual receive channel electronic noise defined statistically by $R_E(t)$. Note that system performance can be evaluated as a function of cyst size as was done in [11], [14], [17], [18].

B. Linear Algebra Formulation for SNR_{out}, SNR_{in}, AG, and CR

The analysis presented in [14], [19] used the cystic resolution metric as a cost function to compute apodization profiles that optimized cystic contrast. That particular algorithm used a linear algebra formulation of the system's PSF and ignored electronic noise. Throughout the remainder of this paper, we will use the same linear algebra formulation of the PSF to simplify the mathematics and to further develop the relationship between beamformer cystic resolution (CR) and beamformer SNR. We also constrain our analysis to a specific instant in time, when the PSF is centered inside the cyst, because analysis at additional times does not yield further insight. As a result, the SNR and CR equations in (11), (12), (14), and (15), are only evaluated at one time instant. Such an analysis requires using the spatio-temporal PSFs of the individual receive elements, $P_k(\vec{x}, t)$, also at an instant in time. To avoid confusion between the purely spatial response of an element and the full 4-D PSF of the element, we call the purely spatial response the instantaneous spatial response (ISR), $P_k(\vec{x}, t_0)$, where t_0 denotes an instant in time. Note that the ISR is inherently different than what is commonly accepted as the PSF of the system. The ISR measures the sensitivity of the system, including all aspects of beamforming, at all points in space at an instant in time. The PSF would be the time record of all the system ISRs.

We have described a linear algebra formulation for the ISR in [19], and we present a brief review here. The instantaneous spatial sensitivity of a transducer during pulse-echo propagation can be expressed as a set of linear equations describing the contribution of each transducer element. We combine all the receive element ISRs to produce a propagation matrix, S . The transducer's spatial sensitivity field is then a matrix multiplication between this propagation matrix, S , and the set of aperture weights, w . For our formulation, S is a function of the transmit aperture weights, the excitation pulse, the electromechanical impulse response, and the individual element spatial impulse responses of the transmit-and-receive apertures. For the pulse-echo model, focusing on transmit can be included, while focusing on receive is accomplished by adjusting the time instant at which the ISR for an element is measured.

The 2-way pulse-echo propagation matrix, S , describing the ISR for a fixed transmit aperture and an n element focused receive aperture measured at a total number of p points in 3-D space is:

$$S = \begin{bmatrix} s_{1,1} & s_{1,2} & \cdots & s_{1,n} \\ s_{2,1} & \cdot & \cdots & \cdot \\ \vdots & \vdots & \ddots & \vdots \\ s_{p,1} & \cdot & \cdots & s_{p,n} \end{bmatrix}, \quad (16)$$

where $s_{i,j}$ is the response of the j th receive element at the i th point in space. Each column of the S matrix represents one receive element's ISR, which we previously referred to as $P_k(\vec{x}, t_0)$, discretely sampled at p points in 3-D space. Note that the receive aperture is focused at one point in space, and that the ISR is measured at p points in space. The receive aperture weighting function, w , for each of the n elements used on receive can be written in vector form as:

$$w = [w_1 \ w_2 \ w_3 \ \cdots \ w_n]^T, \quad (17)$$

where T denotes the vector transpose operation. Using (16) and (17), we can now write the 2-way pulse-echo system ISR, P , as follows:

$$P = S w. \quad (18)$$

Using the discrete linear algebra formulation for the system ISR, the signal at the output of the beamformer at an instant in time is:

$$y(t_o) = \Delta_{\vec{x}} P^T N + E_{t_o} w. \quad (19)$$

The $\Delta_{\vec{x}}$ term is a constant related to the volume of the discrete spatial samples and accounts for the approximation of the continuous spatial integral in (2) with the discrete summation of vectors P , the system ISR, and N , the spatially sampled target function. Note that vectors P and N are sampled on the same spatial grid and have the same length. E_{t_o} is the electronic noise on each channel measured at an instant in time arranged as a row vector.

From before, we assume that the electronic noise on each channel is a wide sense stationary, zero-mean, multivariate normal stochastic process: $E_k(t) \sim \text{MVN}(0, \sigma_E^2)$. We also assume that the noise is spatially uncorrelated across receive channel so the instantaneous noise

autocovariance matrix reduces to: $R_E(t_0) = \sigma_E^2 I$, where I is a n by n identity matrix where n corresponds to the number of receive elements and t_0 indicates an instant in time. We can now use our linear algebra expressions for the ISR and the electronic noise to express the instantaneous output SNR, instantaneous input SNR, and instantaneous array gain. The Δ_x^2 constant and the scaling factor σ_N^2 can be incorporated into the σ_E^2 term without loss of generality to produce a new constant, σ_{load} , which we call the loading constant.

$$\text{SNR}_{\text{out}} = \sqrt{\frac{\Delta_x^2 \sigma_N^2 w^T S_{\text{tot}}^T S_{\text{tot}} w}{w^T R_E w}} = \sqrt{\frac{w^T S_{\text{tot}}^T S_{\text{tot}} w}{\sigma_{\text{load}} w^T w}}, \quad (20)$$

$$\text{SNR}_{\text{in}} = \sqrt{\frac{\Delta_x^2 \sigma_N^2 w_k^T S_k^T S_k w_k}{w_k^T R_{E_k} w_k}} = \sqrt{\frac{S_k^T S_k}{\sigma_{\text{load}}}}, \quad (21)$$

$$\text{AG} = \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}} = \sqrt{\frac{w^T S_{\text{tot}}^T S_{\text{tot}} w}{\sigma_{\text{load}} w^T w} / \frac{S_k^T S_k}{\sigma_{\text{load}}}} = \sqrt{\frac{w^T S_{\text{tot}}^T S_{\text{tot}} w}{S_k^T S_k w^T w}}. \quad (22)$$

Note that S_k is a column vector corresponding to the ISR for one particular receive element measured at all points in space. From (22), it is observed that the beamformer array gain is independent of the electronic noise. In (22), the σ_{load} parameter cancels due to our assumptions. In general, this will not be the case, and array gain will be dependent on electronic noise as it is in (14). Output SNR and array gain (AG) are dependent on the beamformer weights, w . Given the single channel ISR, input SNR is purely a function of σ_{load} . For output SNR, input SNR, and array gain, larger values indicate better performance.

Using the same linear algebra formulations for the ISR and electronic noise, the instantaneous CR of the beamformer is:

$$\begin{aligned} \text{CR} &= \sqrt{\frac{\Delta_x^2 \sigma_N^2 w^T S_{\text{out}}^T S_{\text{out}} w + w^T R_E w}{\Delta_x^2 \sigma_N^2 w^T S_{\text{tot}}^T S_{\text{tot}} w + w^T R_E w}} \\ &= \sqrt{\frac{w^T (S_{\text{out}}^T S_{\text{out}} + \sigma_{\text{load}} I) w}{w^T (S_{\text{tot}}^T S_{\text{tot}} + \sigma_{\text{load}} I) w}}, \end{aligned} \quad (23)$$

where S_{out} is the propagation matrix associated with every spatial point of the ISR outside the cyst boundary and $\sigma_{\text{load}} = \sigma_E^2 / \Delta_x^2 \sigma_N^2$. We can express the inner product of the propagation matrices as one single matrix to further simplify the above expression:

$$\text{CR} = \sqrt{\frac{w^T (R_{\text{out}} + \sigma_{\text{load}} I) w}{w^T (R_{\text{tot}} + \sigma_{\text{load}} I) w}}. \quad (24)$$

Now we see that the noise power term diagonally loads the ISR propagation matrices in the CR equation. Cystic resolution is a function of the input channel noise of the beamformer. When analyzing (24) in the absence of noise, if all the ISR energy lay within the cyst, $\text{CR} = 0$,

indicating optimal contrast. On the other hand, as cyst size decreases and more and more ISR energy lies outside the cyst, CR will approach 1. Therefore, when we present results using the new metric, a more negative dB value indicates better cystic resolution performance.

The σ_{load} parameter includes the spatial sampling constant Δ_x^2 , the beamformer channel noise power, σ_E^2 , and the constant σ_N^2 representing the variance of the target-scattering function. The loading parameter is essentially a scaled value of the RMS noise power: $\sigma_{\text{load}} = \sigma_E^2 / \Delta_x^2 \sigma_N^2$. For this work, we fix a value for σ_N^2 because measuring this value experimentally is intractable and in most imaging scenarios is not of great concern. The Δ_x^2 constant is determined from the volume (or area) of the spatial samples. The RMS noise power, σ_E^2 , is derived from second order statistics and may be determined experimentally. Although our formulation can include spatially colored noise, or different σ_E^2 values on each beamformer channel, in this derivation we assume that the noise is white and spatially uncorrelated.

C. Example Generalized Cystic Resolution Metric Curves

The generalized cystic resolution (GCR) performance metric derived in the previous sections can be used to describe the instantaneous contrast resolution and sensitivity performance of an arbitrary imaging system. The equations for beamformer input SNR, output SNR, array gain, and cystic resolution are functions of the individual beamformer channel weights, the individual beamformer channel noise power, and the individual channel instantaneous spatial responses. We stated earlier that, for a given beamformer and system ISR, input channel SNR is only a function of the electronic noise. Hence, we can investigate the CR performance of a beamformer to different input channel SNRs by adjusting σ_{load} . Cystic resolution is also a function of cyst size, implicitly defined by the R_{out} matrix. Therefore, we can plot cystic resolution as a function of cyst size AND input channel SNR. The GCR metric could also be used to plot cystic resolution as a function of output SNR using (20) as was done in [11]. Finally, the array gain for any beamformer can be computed using (22). The important thing to recognize is that the equation for cystic resolution, in (15), unifies spatial resolution, input channel SNR, output channel SNR, and point contrast at the center of the cyst.

Results from the new metric are easily understood by plotting CR curves. These curves form a surface relating beamformer input or output signal-to-noise ratio, cystic resolution, and cyst radius. To improve the metric's utility as a design tool, instead of using post beamsum SNR, we present the metric's CR results as a function of input SNR for a single receive channel. In situations where modeling the channel SNR across the aperture or post beamsum SNR is of more importance, the metric can be readily adapted using (20).

An example GCR surface for an arbitrary beamformer is shown in Fig. 2. Receive channel input SNR, SNR_{in} from (21), is plotted along the x-axis, cyst radius is plotted along the y-axis, and cystic resolution, CR from (24), is plotted along the z-axis. In general, cystic resolution worsens as receive channel SNR and cyst radius decrease. Cystic resolution levels off for each cyst size as receive channel SNR increases. Receive channel SNR is varied by using different σ_{load} values. Taking a slice through this surface at a constant SNR (0 dB) results in the cystic resolution versus cyst radius plot on the right in Fig. 2. These curves correspond to the same cystic resolution versus cyst radius curves presented by Ranganathan [11] but are a function of input SNR not post beamsum SNR. Taking a slice at a constant cyst radius (0.6 mm), results in the cystic resolution versus receive channel SNR plot seen on the left of Fig. 2. This plot shows that CR decreases with decreasing channel SNR. Cystic resolution versus receive channel SNR curves describe the robustness of the beamformer in the presence of noise. This point will be emphasized later in this manuscript when comparing the GCR curves for the FIR

and SMF beamformers. Note that an ideal beamformer would have a low value in the back left corner of the 3-D SNR-cyst radius-CR space: good cystic resolution at small cyst radii and low channel SNRs.

D. Application of the Unified Performance Metric to Novel Beamformers

The generalized cystic resolution performance metric, which was derived considering the conventional receive beamformer, can also be applied to the novel finite impulse response (FIR) filter receive beamformer described in [14], [18] as well as the spatial matched filter (SMF) beamformer [15], [16].

The FIR beamformer has a similar architecture to the conventional DAS beamformer, but the apodization weights are replaced with channel unique weights at multiple delays. This architecture is well known in the radar literature [12], [20], [21] but is yet to be implemented in a conventional ultrasound system, to the knowledge of the authors. A simplified depiction of the FIR beamformer architecture is shown in Fig. 1. The FIR filters are spatially and temporally variant, and must be updated for each output pixel in the final image. It was shown in [14] that the ISR resulting from the FIR beamformer can still be modeled using our linear algebra formulation:

$$P_{\text{FIR}} = S_{\text{FIR}} W_{\text{FIR}}. \quad (25)$$

The S_{FIR} propagation matrix incorporates multiple ISRs for each receive channel. These channel ISRs are acquired at different times and are associated with different receive foci. To use the GCR performance metric for the FIR beamformer, we must revisit our assumptions about the noise statistics. In conventional ultrasound systems, bandpass filters are applied to each individual channel before focusing and summation. These filters will change the second order statistics of the noise introducing correlation in time. Assuming that the bandpass filters on each channel are identical with impulse response $h[n]$, the output signal, Y , from the filter given an input of a wide sense stationary random sequence, E , is modeled as the discrete temporal convolution:

$$Y[d] = \sum_{n=-\infty}^{+\infty} h[d-n]E[n]. \quad (26)$$

The autocorrelation function of Y , $R_{YY}[m]$, is well known in the signal processing literature as [22]:

$$R_{YY}[m] = (h[m] \otimes h^*[-m]) \otimes R_E[m] \quad (27)$$

where $\otimes$ denotes discrete convolution and $*$ denotes complex conjugation. In the previous sections of this paper, our analysis assumed that the electronic noise on each channel was *spatially* uncorrelated. We now assume that the noise is spatially and temporally uncorrelated on each channel, such that the temporal autocorrelation function of the noise, $R_E[m]$ in (27), is a scaled delta function. The output correlation function, $R_{YY}[m]$, is simply the autocorrelation of the impulse response of the bandpass filter multiplied by some positive constant. Therefore, the noise autocovariance matrix can be expanded to include temporally colored noise for the FIR beamformer. Instead of being purely diagonal (as when the noise is spatially and temporally uncorrelated), the noise autocovariance matrix will have values along the super- and subdiagonals due to the bandpass filter (BPF) of the system. The number of nonzero diagonals in the matrix will depend on the order of the FIR filters. The size of the noise

covariance matrix will be $(n \times k)$ by $(n \times k)$, where n is the number of receive elements in the aperture and k is the number of filter taps on each channel. As an example, for the 3-tap, 64-channel FIR beamformer, the noise autocovariance matrix will be a 5-diagonal matrix of size 192×192 where the 2 super- and 2 subdiagonals are 64 and 128 elements off the main diagonal:

$$R_E^{\text{BPF}}(t_0) = \sigma_E^2 \times \begin{bmatrix} H[0] & \cdots & H[1] & \cdots & H[2] & \cdots & 0 \\ \vdots & \ddots & 0 & \ddots & 0 & \ddots & \vdots \\ H[-1] & \cdots & H[0] & \cdots & H[1] & \cdots & H[2] \\ \vdots & \ddots & 0 & \ddots & 0 & \ddots & \vdots \\ H[-2] & \cdots & H[-1] & \cdots & H[0] & \cdots & H[1] \\ \vdots & \ddots & 0 & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & H[-2] & \cdots & H[-1] & \cdots & H[0] \end{bmatrix} \quad (28)$$

In (28) the values of $H[m]$ are given by the autocorrelation of the impulse response of the bandpass filter at lag m , and t_0 indicates that this is an instantaneous measure of the noise second order statistics. The effects of the bandpass filter should also be included in the system model.

Although the metric can account for a colored noise covariance matrix, we save such analysis for future research. For this paper, we assume that the noise is uncorrelated in space and time. Therefore, the noise autocovariance matrix for the FIR beamformer can still be considered an identity matrix with the RMS noise power along the main diagonal: $R_E = \sigma_E^2 I$.

The SMF beamformer described in [15], [16] has been shown to increase the output signal-to-noise ratio compared with conventional DAS beamforming by about 10 dB in the nearfield [13]. We can analyze the SMF beam-former with the proposed metric as well. The ISR resulting from the SMF beamformer can be modeled as a matrix multiplication between the element response propagation matrix and a weight vector of the receive spatial matched filters:

$$P_{\text{SMF}} = S_{\text{SMF}} W_{\text{SMF}}. \quad (29)$$

The SMF filter beamformer applies 2-D spatially and temporally variant filters to the unfocused RF data. One of the benefits of the SMF beamformer is that it can focus the RF data without any delay lines. However, this requires long, multitap filters to account for the pulse length as well as the delay curvature of the returning echoes. A simplified schematic of one possible implementation of the SMF beamformer is shown in Fig. 1.

The FIR and SMF beamformer are conceptually similar. The outputs of both beamformers are linear combinations of multiple samples of the receive channel RF data. In this paper, the filter sample rate, or tap spacing, for each beamformer is equal. The main discrepancies between the 2 beamformers are that the FIR beamformer operates on focused, pre-beamsum RF data, and the filter weights and tap length are tunable parameters. The weights for the FIR beamformer are computed to optimize cystic resolution, whereas the SMF beamformer weights theoretically maximize SNR in the presence of white noise [23]. Therefore, it will be interesting to compare the CR performance for these 2 beamformers, which are optimized for different performance criterion.

It is important to note that the 3 different beamformers investigated in this manuscript—the DAS, FIR, and SMF beamformers—have the same linear algebra formulations for their respective ISRs. All ISRs are formed by multiplying a propagation matrix (describing the

response of each receive element at each spatial location) with a receive weight vector. For the FIR and SMF beamformers, the propagation matrices include multiple ISRs associated with different instances in time but measured at the same spatial locations. Because each beamformer ISR has the same linear algebra formulation, the equations for cystic resolution and signal-to-noise ratio are the same for each beamformer.

The new metric incorporates all the information from our previous cystic resolution metric [11], while also including a measure describing the beamformer's output signal-to-noise ratio by explicitly dealing with input channel noise. In the next sections, we show how the new, unified metric can be used to compare the performance as well as investigate the fundamental limits of different beamforming strategies.

III. SIMULATION RESULTS

We simulated a 64-element linear array operating at 6.5 MHz and 75% fractional bandwidth in DELFI [24], a custom ultrasound simulation tool that can be downloaded from the MATLAB (Mathworks, Natick, MA) file exchange Web site (www.mathworks.com/matlabcentral). All calculations were performed on an IBM Intellistation Z Pro (Processor speed 2.80 GHz, 4.00 Gb RAM; IBM Corporation, Armonk, NY). The system ISRs were calculated in azimuth and range on a 20 μm by 20 μm sampling grid. Care was taken to sample the entire spatial extent of each response. The time at which the ISRs were captured depended on which beamformer was being implemented. For the DAS beamformer, the ISRs were calculated at the instant in time corresponding to a particular receive focus. To investigate the FIR beamformer with multiple taps on each channel we had to acquire multiple ISRs for each receive channel. The receive focus of every ISR coming into the FIR beamformer is separated by 19.3 μm in the axial direction (equivalent to a 40 MHz filter sampling rate assuming a sound speed of 1540 m/s). We acquired 7 ISRs with different receive foci centered around one predetermined focal depth to have the spatial responses required to calculate the unique 1, 3, 5, and 7-tap FIR filters for the FIR beamformer. For instance to calculate the 3-tap FIR filters for the FIR beamformer with a receive focus at 1.0 cm, we used the receive element ISRs measured over an area in azimuth and range that had receive foci at 0.99807 cm, 1.00000 cm, and 1.00193 cm. We constructed the FIR filters using the QCLS apodization design algorithm described in [14], [19]. These filters optimize the CR of the beamformer in the absence of electronic noise. Note that optimal FIR-QCLS filters can be computed for any cyst size. We investigated a range of FIR-QCLS filter design cyst sizes from 0.1 mm to 1.0 mm. We can also use the metric to see how one set of filters performs over a range of cyst sizes.

To produce the SMF beamformer receive element ISRs, we had to acquire multiple spatial responses for each receive channel. Assuming a sampling frequency of 40 MHz, the temporal pulse-echo acoustic pulse was 47 samples in length. Therefore, we acquired 47 spatial responses for each receive channel separated in time by 25 ns. These ISRs all had a receive focus of 1.0 cm but were measured at different instants in time. A spatial matched filtered single element response was formed by multiplying each receive channel ISR with the associated temporal acoustic pulse sample and then summing all the 47 element spatial responses in time. Note that we "prefocus" the ISRs for the FIR and SMF beamformers. In the original SMF beamformer implementation, the spatial matched filters do the focusing themselves [15]. Prefocusing saves computational resources in our simulations and does not change the output of the SMF beamformer.

Fig. 3 shows the contrast curves for the investigated beamformers using Ranganathan's cystic resolution metric [11]. These results are for the simulated 64-element linear array with a transmit focus at 2.0 cm ($f/2$) and a receive focus at 1.0 cm ($f/1$). All of these curves are computed assuming infinite output SNR ($\sigma_{\text{load}} = 0$), hence, they represent the fundamental CR

performance limits for each beamformer. Although we investigated different apodization functions for the DAS beamformer, such as the Hamming and Nuttall window, we only show the rectangular window results for clarity. All FIR-QCLS filters were computed for a 0.4 mm design cyst radius, indicated by the dashed vertical line. Fig. 3 shows the dramatic increases in cystic resolution possible when using the multitap FIR beamformer. The 5-tap and 7-tap FIR beamformers improve contrast by more than 25 dB for a range of cyst sizes. The SMF beamformer increases CR by 1 to 2 dB over the rectangular DAS and 1-tap FIR beamformer. Keep in mind the difference in tap length between the FIR (1–7 taps) and SMF (47 taps) beamformers and the differences in their design goals. Also shown in Fig. 3 are the integrated lateral beamplots for the rectangular DAS, SMF, 3-tap, and 7-tap FIR beamformers. The 1-tap and 5-tap FIR beamformers are left out for clarity. These beamplots show how the optimal contrast resolution FIR beamformer is able to dramatically reduce sidelobe levels in the ISR.

The GCR surface for the rectangular DAS beamformer and the 3-tap FIR beamformer is shown in Fig. 4. This figure corresponds to the example GCR surface described in Fig. 2. The receive channel SNR along the x-axis is measured relative to the middle element of the DAS beamformer. The solid lines detached from the GCR surfaces plot the cystic resolution for infinite channel SNR. These curves correspond to the rectangular and 3-tap curves in Fig. 3, respectively. The GCR surfaces show that cystic resolution decreases with decreasing channel SNR and decreasing cyst radius. Note that the 3-tap FIR beamformer's CR performance degrades as the receive channel SNR approaches 30 dB. In fact, as the receive channel SNR reaches 0 dB, the cystic resolution performance of the 3-tap FIR beamformer approaches 0 dB (no contrast). The rectangular DAS beamformer's cystic resolution remains relatively constant until the receive channel SNR approaches 0 dB. These GCR surfaces highlight performance aspects that are not apparent with Ranganathan's metric [11] or the beamplots in Fig. 3. Fig. 4 shows how the new metric can be used to compare the robustness of different beamformers in the presence of noise.

Although we can produce GCR surfaces for every investigated beamformer, the figures quickly become cluttered. Taking a slice through the surfaces at a constant cyst radius can help clarify the data and show interesting trends. In Fig. 5 we show a slice through the GCR surfaces of all beamformers at the design cyst radius of 0.4 mm. This plot corresponds to the graph described on the left of Fig. 2. From Fig. 5 we see that there exists a tradeoff between improved cystic resolution performance and SNR performance for the multitap FIR beamformer. Although the 5-tap and 7-tap FIR beamformers improve the contrast resolution by more than 20 dB compared with the rectangular DAS beamformer, their performance rapidly degrades as a function of SNR. On the other hand, the SMF beamformer offers a slight increase in cystic resolution and its performance is more robust than the rectangular DAS beamformer in the presence of noise. This figure highlights the utility of the metric when comparing different beamforming strategies and suggests that given reasonable channel SNR (~ 30 dB) the 3-tap FIR beamformer offers significant improvements in CR compared with the rectangular DAS and SMF beamformers. The metric shows that the longer tap (>3 taps) FIR beamformers should only be used in high SNR environments because their performance rapidly degrades in the presence of noise. In poor SNR environments (<0 dB), the SMF beamformer should be used because it has better sensitivity than the DAS and FIR beamformer.

IV. EXPERIMENTAL RESULTS

The novel performance metric presented in this paper was used to compare different receive beamformer architectures using the spatial response characteristics of an Ultrasonix Sonix RP ultrasound scanner (Ultrasonix Medical Corporation, Burnaby, BC, Canada). We have previously acquired an experimental data set that allows us to characterize the L14–5/38 linear probe ISR using a 64-element transmit aperture focused at 4.0 cm ($f/2$) and a 64-element receive

aperture focused at 2.0 cm ($f/1$). The specifics of the acquisition of this data set are well described in [14]. The experiment consisted of translating a point target in the azimuth-range plane of the transducer and collecting the pulse-echo response at every point target location.

The acquired 4-D (space-space-time-receive channel) RF data set characterizing the 64-element L14-5/38 probe ISR was filtered in MATLAB using a 101st-order bandpass filter with cutoff frequencies at 4 and 8 MHz. An experimental ISR was made by interpolating the space-space-time PSF for each receive element at the time instant associated with a particular receive focus. The temporal pulse-echo pulse was found to extend 71 samples at 40 MHz. Therefore, to produce an experimental SMF beamformer ISR, we formed 71 ISRs around a 2.0 cm receive focus and separated in time by 25 ns. The SMF ISR was made using the same procedure as that used in the simulations. Experimental FIR beamformer ISRs were made by acquiring multiple ISRs with receive foci separated by $18.6 \mu\text{m}$ (equivalent to a 40 MHz filter sampling rate assuming a sound speed in water of 1490 m/s). The FIR-QCLS filters were designed for a 0.35 mm cyst radius. Although we averaged multiple times at each spatial location to reduce electronic noise in the experimental ISRs, the responses are not entirely “noiseless.” For the purposes of presenting experimental results using the metric however, we assume that the responses are noise free.

The experimental ISRs for the different beamformers are shown in Fig. 6. Each ISR is measured on a 2 mm axial by 1.95 cm lateral plane. The ISRs are envelope detected and log compressed to 60 dB in each image. The SMF beamformer reduces the energy in the sidelobe regions of the ISR compared with the rectangular DAS beamformer. The FIR beamformers reduce sidelobe energy and maintain a narrow mainlobe.

Fig. 7 shows the experimental GCR surfaces for the rectangular DAS beamformer and the 3-tap FIR beamformer. The cystic resolution curves at infinite input channel SNR are also shown (assuming that the acquired responses were noise free). The channel SNR axis is relative to the middle element in the receive aperture of the DAS beamformer. These experimental results show similar trends to the simulation results in Fig. 4. The improvements in CR performance for the DAS beamformer between the simulation and experiment are attributed to the differences in system parameters. The limited spatial window acquired will also affect the results of the experimental ISRs. The 3-tap FIR beamformer’s performance begins to degrade when the channel SNR approaches 30 dB. The rectangular DAS beamformer on the other hand maintains an almost constant cystic resolution value until 20 dB channel SNR. Fig. 8 shows 2 slices through the GCR surfaces for all the investigated beamformers. The plot on the left shows the slice at infinite channel SNR and the plot on the right shows a slice at a constant cyst radius of 0.35 mm (the design cyst for the FIR filters). In the cystic resolution versus cyst radius plot, the 7-tap FIR beamformer improves contrast by more than 8 dB compared with the rectangular DAS beamformer over a large range of cyst sizes. The SMF beamformer improves contrast by 2 to 3 dB compared with the DAS beamformer but has worse contrast performance than all the FIR beamformers for cysts larger than 0.35 mm in radius. The cystic resolution versus receive channel SNR plot on the right in Fig. 8 highlights the loss in sensitivity when using the optimal contrast resolution FIR beamformer. Increasing tap length continually degrades the performance of the FIR beamformer at a given SNR value. Note that the SMF beamformer is more robust than the DAS and FIR beamformer in the presence of noise. These results are to be expected given the different design goals of the SMF and FIR beamformers.

The authors point out that the experimental GCR curves on the right in Fig. 8 are likely shifted because the experimental ISRs contain some level of electronic noise. As a result, we would expect the cystic resolution versus SNR curves to shift slightly to the left for all beamformers. The true shape of the surfaces might be different as well because noise affects cystic resolution. Although the experimental results shown do not use “noiseless” responses, we still expect the

performance relationships between the multitap FIR, DAS, and SMF beamformer to be similar to those shown in the figures.

V. DISCUSSION

The pulse-echo imaging performance metric presented in this paper unifies 2 important system characteristics: beamformer signal-to-noise ratio and contrast resolution. We improved on the metrics outlined in [11], [13] by showing how cystic resolution and beamformer SNR are related. Both are functions of the 4-D, spatio-temporal PSF energy, the electronic noise, and the receive beamformer weights. Results from the performance metric are easily interpreted by plotting GCR surfaces. As shown in this manuscript, the metric allows for straightforward comparison between different system architectures. One feature of the new metric not shown in the manuscript is that any configurable system parameter such as apodization, $f/\#$, or bandwidth can be optimized. This optimization procedure simplifies to picking the set of system parameters that achieves the best cystic resolution performance at a given cyst size and channel SNR. The metric, when used as a cost function, provides one quantitative goal for improving image quality.

Historically, beamformer comparison was accomplished by measuring or simulating the system's beamplot or using a contrast detail phantom [2]. The beamplot is useful when imaging point targets but fails to predict performance accurately when the target is weakly reflecting and in a continuous scattering background. A contrast detail phantom requires a real system and a large time investment to investigate different system parameters. Our metric is different than the ideal observer methodology because it does not use the entire RF image data set to quantify lesion detectability. And rather than measuring the discrimination efficiency between different visualization tasks, the GCR metric is specially suited to compare different beamforming strategies or the effects of changing system operating conditions. The metric, however, is adaptable to match a target of interest. For example, one can change the target-masking function to represent a hypoechoic or hyperechoic lesion and not an anechoic cyst [11]. The expression derived in this paper is only valid for anechoic cysts.

The GCR metric predicts that the SMF beamformer significantly improves beamformer sensitivity while offering a small gain in contrast resolution compared with the rectangular DAS beamformer. The metric also shows that the multitap optimal contrast resolution FIR beamformer offers large gains in cystic resolution compared with the rectangular DAS and SMF beamformer, but its performance degrades rapidly in the presence of noise. Given reasonable channel SNR however (~ 30 dB), the 3-tap FIR beamformer outperforms all other investigated architectures. Our analysis of the different beamformers suggests that the 5-tap and 7-tap FIR beamformers should only be used in situations with high echo SNR.

The new performance metric does not include second-order speckle statistics or speckle size. The metric does not measure target detectability or beamformer architecture system complexity. The analysis presented in this paper uses a simplistic model for electronic noise. Adapting the metric to deal with a more sophisticated noise model will certainly improve the metric's robustness. Such studies are left for future research. Further analysis must be performed to determine whether theoretical gains in contrast resolution are realized in real systems and lead to improved detection. Testing the FIR beamformer using real data should also clarify the degree of performance degradation with noise. This paper shows that the GCR metric is better suited to characterize and compare systems than the beamplot. By plotting cystic resolution as a function of receive channel SNR, a system's clinical imaging performance can be assessed. Lastly, the metric quantifies fundamental performance limits and condenses contrast resolution, spatial resolution, and SNR into one, unified framework.

V. CONCLUSION

This paper proposes the use of a new performance metric for broadband, pulse-echo imaging systems. It is especially suited to ultrasound systems because it combines cystic resolution and echo signal-to-noise ratio. The metric is easily adaptable to characterize many different imaging scenarios and beamformer architectures. System optimization is straightforward because tradeoffs between SNR and contrast resolution are easily compared. In this paper, the metric was used to compare 2 beamforming strategies to the conventional DAS beamformer. Results indicate that one beamformer, which uses channel unique FIR filters on each receive channel, offers significant gains in cystic resolution (>15 dB) for a modest increase in system complexity. Further results show that another beamformer, which spatially matched filters the returning echoes, increases beamformer sensitivity and is more robust in the presence of noise than the conventional delay-and-sum beamformer.

ACKNOWLEDGMENT

The authors thank the anonymous reviewers for their insightful and constructive comments. The authors would like to thank Kevin Owen for his assistance in developing the experimental control software. We thank Kris Dickie and Ultrasonix Medical Corporation for technical support. The authors thank IBM for their Shared University Research Program and access to the high performance computing cluster.

This work was supported by U.S. Army Congressionally Directed Research Program Grant W81XWH-04-1-0590 and NIH Grant R01-EB005433.

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Biography

Drake A. Guenther received his B.S.E. degree in biomedical engineering in 2003 from Duke University, Durham, NC. His undergraduate research as a Pratt Fellow explored adaptive beamforming and spatial compounding in ultrasound. In 2008, he received his Ph.D. degree in biomedical engineering from the University of Virginia, Charlottesville, VA. His dissertation explored optimal beamforming strategies for enhancing image quality in state-of-the-art medical ultrasound systems.

His research interests include ultrasound beamforming, signal processing, and angular scatter imaging.

William F. Walker received the B.S.E. and Ph.D. degrees in 1990 and 1995 from Duke University, Durham, NC. His dissertation explored fundamental limits on the accuracy of adaptive ultrasound imaging.

After completing his doctoral work he stayed on at Duke as an assistant research professor in the Department of Biomedical Engineering. At the same time, he served as a senior scientist and president of NovaSon Corporation located in Durham, NC. In 1997, he joined the faculty

of the Department of Biomedical Engineering at the University of Virginia, being promoted to associate professor in 2003. He is an active founder of two ultrasound-based start-up companies, PocketSonics Inc. and HemoSonics LLC. His research interests include aperture domain processing, beamforming, angular scatter imaging, tissue elasticity imaging, low-cost system architectures, blood coagulation, and time delay and motion estimation.

Fig. 1. Different beamformer architectures. The apodization weights and filters in all architectures are spatially and temporally variant. (a) Delay and sum beamformer, (b) dynamic focus FIR beamformer, and (c) spatial matched filter beamformer.

Fig. 2.

Example unified performance metric graph. The signal-to-noise ratio and cystic resolution performance of an arbitrary beamformer can be easily investigated and quantified using the new metric. Note that the x-axis (Rx Channel SNR) is the input signal-to-noise ratio for each channel of the arbitrary beamformer. Receive channel SNR is computed using (21), cystic resolution is computed using (24). Each point on this surface represents the beamformer's cystic resolution performance at a particular cyst radius and receive channel SNR (σ_{load}). Two different slices through the GCR surface are also shown. The slice on the right is a plane of constant SNR, showing the contrast curve that would be produced using Ranganathan's cystic resolution metric. The slice on the left is a plane of constant cyst radius highlighting beamformer cystic resolution performance as a function of input electronic noise.

Fig. 3.

Integrated lateral beamplots (left) and contrast curves (right) for the investigated beamformers. The beamplots show the dramatic reduction in sidelobe levels when using the multitap FIR beamformer. The contrast curves were computed using Ranganathan's metric. Sensitivity of the beamformers is completely ignored in both the beamplots and contrast curves, however, the contrast resolution performance is readily distinguished. The multitap FIR beamformers offer significant gains in contrast resolution compared with the rectangular DAS and SMF beamformer.

Fig. 4. GCR surfaces for the rectangular DAS beamformer and the 3-tap optimal contrast resolution FIR beamformer. The 2 cystic resolution curves at infinite SNR correspond to the rect and 3-tap curves from Fig. 3, respectively. Receive channel SNR is relative to the middle element of the DAS beamformer. Note that although the 3-tap FIR beamformer significantly improves cystic resolution, its performance degrades as a function of input SNR quicker than the rectangular DAS beamformer. This aspect of performance is not apparent with Ranganathan's metric.

Fig. 5.

Cystic resolution vs. receive channel SNR. This data shows a slice through the GCR surfaces for all the investigated beamformers at a cyst radius of 0.4 mm (design cyst size for FIR filters). The curves suggest that the FIR optimal contrast resolution beamformer should only be used in moderate to high SNR scenarios. The SMF beamformer offers a 1 to 2 dB increase in cystic resolution, and its performance is more robust than the rectangular DAS and multitap FIR beamformer in the presence of noise.

Fig. 6. Experimental ISRs for the rectangular DAS, SMF, 1-tap, 3-tap, 5-tap, and 7-tap optimal contrast resolution FIR beamformers. All images are envelope detected and log compressed to 60 dB. The FIR beamformer ISRs have lower sidelobe energy than the rectangular DAS and SMF beamformer ISRs. However, beamformer sensitivity is not readily apparent from these images.

Fig. 7. Experimental GCR surfaces for the rectangular DAS and 3-tap optimal contrast resolution FIR beamformer. The receive channel SNR is relative to the middle element of the DAS beamformer. The 3-tap FIR beamformer's cystic resolution performance begins to degrade when the input channel SNR approaches 30 dB. This figure shows similar trends to the simulation results in Fig. 4.

Fig. 8.

Two perpendicular slices through the GCR surfaces of the experimental results. The plot on the left shows the cystic resolution vs. cyst radius plots at infinite channel SNR, and the plot on the right shows the cystic resolution vs. receive channel SNR curves at a constant cyst radius (0.35 mm, delineated by the dashed vertical line in the figure on the left). Both figures show improvements in contrast resolution when using the multitap FIR beamformer. Note that in the presence of noise, cystic resolution performance steadily degrades as tap length increases. The SMF beamformer offers improvements in cystic resolution and robustness compared with the rectangular DAS beamformer.