Effect of Multivariate Process Instability on Principal Component Analysis: A Case Study

Submitted: February 26, 2004; Accepted: October 11, 2004; Published: April 29, 2005.

Donald S. Holmes¹ and A. Erhan Mergen²

¹Stochos Inc, 14 North College Street, Schenectady, NY 12305

²Rochester Institute of Technology, Rochester, NY 14623

ABSTRACT

With the rising use of principal component analysis/partial least squares (PCA/PLS) in the process analytical technology (PAT) initiative of the pharmaceutical industry, it seems appropriate to view that approach from a statistical process control (SPC) perspective. The purpose of this study was to demonstrate the effect of process instability (ie, state of statistical out-of-control) on use of PCA/PLS. The demonstrated differences in results should encourage PCA/PLS users to incorporate SPC as an active part of their process analytical control (PAC) toolkit to check for stability prior to drawing conclusions based on PCA/PLS analysis.

Keywords: principal component analysis, partial least squares, covariance matrix, mean square successive differences (MSSD)

INTRODUCTION

A principal components analysis (PCA) is concerned with explaining the variance-covariance structure through a few linear combinations of the original variables, which depend solely on the variance/covariance (ie, dispersion) matrix.¹ However, the elements of the variance/covariance matrix are greatly influenced by the state of statistical control of the data upon which the matrix is based. Thus, failure to check the stability of the data prior to estimating the variance/covariance matrix may lead to distortion in the results of the PCA analysis. Lack of stability from batch-to-batch, lot-to-lot, or even time-to-time in continuous operations of the process will produce spurious values for the weightings of and the number of "significant" components.

Holmes and Mergen² compared the regular estimate of variance/covariance matrix to the mean square successive differences (MSSD) estimate to test the hypothesis that a multivariate process is stable. This method is a multivariate extension of testing the equality of 2 variance estimators. The MSSD estimator of variance (q^2), which is an unbiased variance estimator, is given by Hald³ as follows:

Corresponding Author: Donald S. Holmes, Stochos Inc., 14 North College Street, Schenectady, NY 12305. Tel: (518) 372-5426; Fax: (518) 372-4789; E-mail: dsholmes@stochos.com

$$q^2 = \frac{\text{MSSD}}{2}, \quad (1)$$

where MSSD is defined as

$$\text{MSSD} = \frac{1}{n-1} \sum_{i=2}^n (X_i - X_{i-1})^2, \quad (2)$$

where n is the number of data points, X_i 's are the data points and $i=1,2,\dots,n$.

The multivariate equivalents to the average and variance are, respectively, a vector of variable means and the covariance matrix.

Assume, as given, a set of n multivariate vectors from a multivariate normal distribution with p variables of the form shown below.

$$\mathbf{X}_i = \begin{bmatrix} X_{1,i} \\ X_{2,i} \\ \cdot \\ \cdot \\ \cdot \\ X_{p,i} \end{bmatrix} \quad (3)$$

where p is the number of variables.

The mean vector and the covariance matrix are estimated as

$$\bar{\mathbf{X}} = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{X}_p \end{bmatrix}, \text{ where } \bar{X}_j = \frac{\sum_{i=1}^n X_{ji}}{n}, \text{ for } j = 1, \dots, p \quad (4)$$

$$\text{Cov} = \frac{\sum_{i=1}^n (\mathbf{X}_i - \bar{\mathbf{X}})(\mathbf{X}_i - \bar{\mathbf{X}})^T}{n-1}, \quad (5)$$

where X_i is a multivariate vector and is the mean vector as defined in Equations 3 and 4, respectively, and T indicates transposition.

The multivariate equivalent to the MSSD variance is the mean square successive difference covariance matrix, which is discussed in Holmes and Mergen.⁴ The elements of the MSSD covariance matrix are

$$\text{Cov}_{\text{MSSD}} = \frac{\sum_{i=2}^n (X_i - X_{i-1})(X_i - X_{i-1})^T}{2(n-1)} \quad (6)$$

Again X_i is a multivariate vector as defined in Equation 3.

The multivariate process stability test is derived from the work described in Kramer and Jensen,⁵ which is based on Wilks' idea⁶ that the determinant of the covariance matrix is the multivariate analog of the variance. The steps are as follows:

1. Calculate 2 covariance matrices S_1 and S_2 (let S_1 be the regular covariance matrix and S_2 be the MSSD covariance matrix as defined in Equations 2 and 6, respectively). S_1 and S_2 are matrices of order $p \times p$, where p is the number of variables.

2. Calculate the weighted average (S) of S_1 and S_2 using Equation 7.

$$S = \frac{(n_1 - 1)S_1 + (n_2 - 1)S_2}{n_1 + n_2 - 2} \quad (7)$$

where n_1 and n_2 are the observations in samples used to determine the 2 covariance matrices, respectively. If $n_1 = n_2 = n$, Equation 4 can be written as

$$S = \frac{S_1 + S_2}{n} \quad (8)$$

3. Calculate M , a factor used in test statistics G shown in Equation 13, for using S , S_1 , and S_2 in Equation 9:

$$M = (n_1 + n_2 - 2) \log |S| - (n_1 - 1) \log |S_1| - (n_2 - 1) \log |S_2| \quad (9)$$

where the logarithms are to the base 10. Again for the case $n_1 = n_2$, Equation 9 will be as follows:

$$= \log \left[\frac{|S|^{2(n-1)}}{|S_1|^{n-1} |S_2|^{n-1}} \right] \quad (10)$$

4. Calculate m , a factor used in test statistics G shown in Equation 13, using Equation 11.

$$m = 1 - \left[\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} - \frac{1}{n_1 + n_2 - 2} \right] \left[\frac{2p^2 + 3p - 1}{6(p+1)} \right], \quad (11)$$

where p is the number of variables. For the case $n_1 = n_2$, Equation 11 can be expressed as

$$= 1 - \left[\frac{2}{n-1} - \frac{1}{2(n-1)} \right] \left[\frac{2p^2 + 3p - 1}{6(p+1)} \right] \quad (12)$$

5. Calculate G using Equation 13.

$$G = 2.3026(m)(M), \quad (13)$$

which approximately follows a chi-square distribution for large samples with $p(p+1)/2$ degrees of freedom. The value of G will then be compared with the critical value of chi-square with $p(p+1)/2$ degrees of freedom at a selected significance level, and a decision will be made on the equality of the 2 matrices. If the test result is significant, this implies that 2 matrices are not equal, thus the process is not stable.

The analysis below is based on the data set that is included as Appendix 1. The data represent the records of the composition of the powder of 100 batches of a tablet mix. It is the simulated production data describing the concentration of percentage by mass of 9 components (labeled V1 through V9 in Appendix 1). Figures 1, 2, 3, 4, 5, 6, 7, 8, and 9 display the plots of the 9 components.

THE TECHNIQUE

Test for Stability of the Variance/Covariance Matrix

First, the statistical stability of the system is determined using a multivariate method, proposed by Holmes and Mergen² and described above, to test for process stability. This method tests the equality of 2 variance/covariance matrices. One matrix is the variance/covariance matrix as usually defined. The elements of this one are independent of the order in which the data were collected. The other matrix is one based on the concept of the MSSD. The elements of this matrix are dependent upon the order of the data. If the 2 matrices are not similar, the process from which the data were collected is statistically unstable. The 2 variance/covariance matrices are shown in Tables 1 and 2.

The value of chi-square for the test is 348.7 with 45 degrees of freedom. This is a very strong signal that the process is unstable with respect to the data set being used as an example.

It should be noted that a T^2 control chart, an essential PAC tool, would yield the same result: the process is not stable in the statistical sense. However, we needed the 2 dispersion matrices for use in the PCA that follows.

Figure 1. Plot of component no 1.

Figure 2. Plot of component no 2.

Figure 3. Plot of component no 3.

Figure 4. Plot of component no 4.

Figure 5. Plot of component no 5.

Figure 6. Plot of component no 6.

Figure 7. Plot of component no 7.

Figure 8. Plot of component no 8.

Figure 9. Plot of component no 9.

Table 1. Regular Variance/Covariance Matrix*

	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	0.1951	-0.0741	-0.0017	-0.0058	-0.1512	0.0160	0.0454	-0.0051	-0.0091
V2	-0.0741	0.0621	0.0012	0.0032	0.0769	-0.0486	-0.0334	0.0026	0.0078
V3	-0.0017	0.0012	0.0001	0.0001	0.0016	-0.0006	-0.0011	0.0000	0.0001
V4	-0.0058	0.0032	0.0001	0.0004	0.0052	-0.0016	-0.0027	0.0002	0.0003
V5	-0.1512	0.0769	0.0016	0.0052	0.1583	-0.0620	-0.0468	0.0051	0.0100
V6	0.0160	-0.0486	-0.0006	-0.0016	-0.0620	0.1037	0.0053	-0.0032	-0.0073
V7	0.0454	-0.0334	-0.0011	-0.0027	-0.0468	0.0053	0.0458	-0.0002	-0.0029
V8	-0.0051	0.0026	0.0000	0.0002	0.0051	-0.0032	-0.0002	0.0012	0.0008
V9	-0.0091	0.0078	0.0001	0.0003	0.0100	-0.0073	-0.0029	0.0008	0.0017

*Regular indicates data order independent.

Table 2. MSSD Variance/Covariance Matrix*

	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	0.0386	-0.0009	-0.0001	-0.0004	-0.0272	-0.0137	0.0028	-0.0009	-0.0001
V2	-0.0009	0.0142	0.0003	0.0003	0.0061	-0.0198	-0.0042	0.0003	0.0013
V3	-0.0001	0.0003	0.0000	0.0000	0.0003	-0.0004	-0.0002	0.0000	0.0000
V4	-0.0004	0.0003	0.0000	0.0001	0.0004	-0.0001	-0.0003	0.0000	0.0001
V5	-0.0272	0.0061	0.0003	0.0004	0.0475	-0.0218	-0.0044	0.0014	0.0008
V6	-0.0137	-0.0198	-0.0004	-0.0001	-0.0218	0.0633	0.0005	-0.0008	-0.0021
V7	0.0028	-0.0042	-0.0002	-0.0003	-0.0044	0.0005	0.0074	-0.0001	-0.0005
V8	-0.0009	0.0003	0.0000	0.0000	0.0014	-0.0008	-0.0001	0.0002	0.0002
V9	-0.0001	0.0013	0.0000	0.0001	0.0008	-0.0021	-0.0005	0.0002	0.0003

*MSSD indicates data order dependent.

Table 3. A Comparison of the Loadings Associated With the First 2 Principal Components*

PC No. 1 Loadings	V1	V2	V3	V4	V5	V6	V7	V8	V9
Regular	0.1629	-0.1865	-0.1386	-0.1622	-0.1824	0.1052	0.1387	-0.0890	-0.1565
MSSD	0.0977	-0.1642	-0.0674	0.1632	-0.2550	0.1249	-0.2019	0.1796	0.2223
PC No. 2 Loadings	V1	V2	V3	V4	V5	V6	V7	V8	V9
Regular	-0.1093	-0.0449	0.2642	0.2075	-0.0157	0.3878	-0.3626	-0.4231	-0.3107
MSSD	-0.1629	-0.3344	-0.3944	-0.2630	-0.0277	0.2976	0.1592	0.0102	-0.1646

*PC indicates principal component; and MSSD, mean square successive differences. Differences of both the magnitude and the sign reversal of the loadings are clearly indicated.

Demonstrate the Effects of Instability

To see the effect of the instability on the PCA, 2 PCA runs will be made. The effects will then be evaluated by comparing the results of 2 PCA analyses: one with the usual variance/covariance matrix; the other with the MSSD variance/covariance matrix. Since the 2 matrices are significantly different, the 2 PCA analyses should also lead to different conclusions.

A comparison of the loadings associated with the first 2 principal components (PCs) is shown in Table 3.

A scan of the differences in the 2 correlation matrices (Appendix 2) should also help the PAT implementation team understand the effect of process instability on PCA analysis.

CONCLUSION

The example above demonstrates that it is essential, in implementing the PAT initiative, that the statistical stability of the

variance/covariance matrix be checked prior to drawing conclusions from a PCA analysis. Results based on a variance/covariance matrix of an unstable process may lead to erroneous conclusions (results). This, of course, then carries over to the use of PCA concepts in PLS.

REFERENCES

1. Johnson R, Wichern D. Principal Components. In: *Applied Multivariate Statistical Analysis*. 3rd ed. Upper Saddle River, NJ: Prentice Hall; 1992:356-357.
2. Holmes D, Mergen E. A multivariate test for randomness. *Qual Eng*. 1998;10:505-508.
3. Hald A. Statistical Control. In: *Statistical Theory with Engineering Applications*. New York, NY: John Wiley & Sons; 1952:357-360.
4. Holmes D, Mergen E. Improving the performance of T-square control chart. *Qual Eng*. 1993;5:619-625.
5. Kramer C, Jensen D. Fundamentals of multivariate analysis. II. Inference about two treatments. *J Qual Technol*. 1969;1:189-204.
6. Wilks S. Certain generalizations in the analysis of variance. *Biometrika*. 1932;24:471-494.

Appendix 1. Production Data Describing the Concentration of Percentage by Mass of 9 Components (labeled V1 through V9)

V1	V2	V3	V4	V5	V6	V7	V8	V9
20.94	5.29	0.22	0.23	3.32	63.17	4.33	0.20	1.02
20.75	5.26	0.22	0.20	3.50	63.34	4.43	0.18	1.00
20.76	5.25	0.22	0.18	3.46	63.16	4.51	0.20	1.00
20.80	5.21	0.22	0.18	3.37	63.23	4.42	0.23	1.03
20.73	5.18	0.21	0.18	3.31	63.44	4.47	0.26	1.04
21.00	5.15	0.21	0.18	3.25	63.22	4.49	0.25	1.05
20.97	5.32	0.21	0.17	3.46	62.87	4.73	0.24	1.04
20.98	5.18	0.20	0.17	3.37	63.02	4.74	0.24	1.04
20.84	5.27	0.21	0.18	3.41	63.07	4.60	0.25	1.05
20.94	5.18	0.20	0.18	3.26	63.23	4.56	0.28	1.07
21.10	5.24	0.20	0.17	3.30	63.13	4.49	0.26	1.05
21.10	5.11	0.21	0.18	3.26	63.32	4.44	0.27	1.07
21.10	5.27	0.21	0.19	3.33	63.23	4.29	0.25	1.05
21.21	5.18	0.21	0.19	3.56	63.10	4.29	0.24	1.04
21.17	5.20	0.21	0.18	3.36	63.32	4.44	0.25	1.04
21.35	5.15	0.21	0.20	3.23	63.22	4.41	0.25	1.04
21.29	5.42	0.21	0.19	3.26	63.07	4.20	0.21	1.07
20.93	5.53	0.21	0.19	3.69	62.88	4.15	0.20	1.08
20.62	5.36	0.21	0.17	3.66	63.39	4.27	0.19	1.07
20.87	5.11	0.20	0.17	3.08	63.81	4.55	0.18	1.06
20.57	5.20	0.20	0.15	3.29	63.72	4.71	0.18	1.04
20.83	5.37	0.21	0.15	3.43	63.01	4.74	0.20	1.08
20.63	5.39	0.21	0.17	3.63	62.96	4.78	0.20	1.06
20.79	5.36	0.21	0.16	3.62	63.05	4.59	0.21	1.04
20.93	5.32	0.21	0.19	3.43	63.22	4.48	0.21	1.03
20.80	5.45	0.21	0.18	3.63	62.92	4.46	0.24	1.04
20.81	5.15	0.20	0.17	3.33	63.49	4.64	0.23	1.01
21.08	5.13	0.21	0.16	3.18	63.46	4.54	0.27	1.06
21.06	5.27	0.21	0.19	3.38	63.15	4.44	0.29	1.09
21.09	5.51	0.21	0.18	3.40	62.90	4.46	0.30	1.10
20.84	5.43	0.20	0.19	3.47	63.01	4.51	0.27	1.07
20.94	5.59	0.21	0.19	3.77	62.32	4.47	0.27	1.08
20.51	5.59	0.21	0.19	3.95	62.75	4.33	0.28	1.08
20.66	5.36	0.20	0.17	3.56	63.24	4.36	0.24	1.04
21.15	5.18	0.20	0.17	3.16	63.41	4.34	0.18	1.03
21.25	5.28	0.20	0.18	3.36	62.97	4.32	0.15	1.01
20.88	5.45	0.20	0.18	3.59	63.01	4.30	0.17	1.04
20.80	5.35	0.21	0.20	3.66	63.16	4.39	0.15	1.01
20.90	5.56	0.21	0.21	3.73	62.79	4.22	0.17	1.03
20.74	5.43	0.21	0.19	3.73	63.00	4.32	0.19	1.01
21.14	5.33	0.20	0.19	3.40	62.96	4.40	0.20	1.00
21.37	5.17	0.20	0.18	3.38	62.91	4.58	0.22	1.01
21.54	5.06	0.20	0.17	3.04	63.12	4.73	0.24	1.03
21.61	5.26	0.20	0.17	3.14	62.75	4.59	0.24	1.03
21.31	4.97	0.20	0.15	3.02	63.23	4.87	0.22	0.99
21.35	4.89	0.19	0.17	2.62	63.63	4.92	0.22	1.00
21.64	4.94	0.20	0.14	2.65	63.31	4.89	0.23	1.02
21.84	4.95	0.19	0.14	2.75	63.17	4.85	0.22	1.01
21.57	5.07	0.20	0.14	2.98	63.14	4.77	0.20	0.98
21.29	4.94	0.20	0.16	2.96	63.79	4.60	0.21	0.99

Appendix 1. Production Data Describing the Concentration of Percentage by Mass of 9 Components (labeled V1 through V9)
(Continued)

V1	V2	V3	V4	V5	V6	V7	V8	V9
21.29	5.11	0.21	0.16	3.07	63.41	4.60	0.23	1.04
21.28	5.39	0.20	0.16	3.64	62.06	4.86	0.24	1.12
21.01	5.28	0.20	0.16	3.51	62.67	4.76	0.23	1.11
20.95	5.52	0.20	0.18	3.53	62.63	4.53	0.21	1.10
20.84	5.50	0.21	0.16	3.74	62.73	4.38	0.21	1.07
20.74	5.43	0.22	0.18	3.67	63.12	4.32	0.24	1.10
20.70	5.47	0.21	0.18	3.70	63.05	4.43	0.25	1.09
20.43	5.55	0.22	0.19	3.72	63.26	4.39	0.25	1.13
20.41	5.67	0.22	0.20	4.05	62.78	4.49	0.23	1.10
21.15	5.45	0.21	0.16	3.88	62.39	4.58	0.23	1.09
21.56	4.86	0.19	0.15	2.86	63.53	4.80	0.22	1.03
22.13	4.78	0.19	0.15	2.60	63.19	4.98	0.22	1.01
22.03	4.97	0.19	0.14	2.88	62.92	4.87	0.23	1.03
21.96	4.79	0.19	0.13	2.62	63.25	4.99	0.20	1.00
21.84	5.02	0.20	0.15	2.92	62.65	4.99	0.24	1.07
21.67	4.97	0.19	0.15	2.98	62.97	4.96	0.24	1.04
21.50	4.99	0.20	0.15	2.92	63.27	4.73	0.23	1.05
21.36	5.19	0.21	0.16	3.19	63.25	4.56	0.21	1.05
21.53	5.20	0.22	0.16	3.23	63.27	4.37	0.18	1.06
21.82	5.15	0.21	0.16	3.15	63.11	4.39	0.16	1.05
22.41	5.83	0.22	0.16	1.89	62.79	4.31	0.16	1.09
21.23	5.55	0.21	0.17	3.57	62.72	4.25	0.26	1.08
20.90	5.30	0.21	0.17	3.66	63.29	4.22	0.25	1.07
21.05	5.40	0.21	0.17	3.51	63.12	4.27	0.27	1.09
20.97	5.42	0.21	0.17	3.68	62.85	4.32	0.25	1.08
21.08	5.33	0.20	0.16	3.45	62.90	4.41	0.23	1.05
21.22	5.30	0.20	0.15	3.40	63.02	4.32	0.24	1.06
21.16	5.16	0.20	0.15	3.24	63.54	4.30	0.21	1.01
21.45	5.19	0.20	0.15	3.15	63.32	4.30	0.20	1.01
21.11	5.35	0.20	0.16	3.32	63.17	4.41	0.20	1.03
21.00	5.24	0.21	0.17	3.49	62.97	4.52	0.21	1.03
20.98	5.33	0.22	0.18	3.50	62.96	4.49	0.20	1.02
21.17	5.30	0.22	0.19	3.45	62.92	4.35	0.20	1.03
21.10	5.24	0.22	0.19	3.52	63.04	4.30	0.19	1.04
21.40	5.11	0.21	0.16	3.22	63.21	4.27	0.17	1.01
21.55	5.02	0.21	0.19	2.99	63.30	4.32	0.17	1.00
21.46	5.10	0.21	0.16	3.18	63.16	4.45	0.18	0.98
21.39	4.88	0.20	0.15	2.81	63.62	4.67	0.17	0.97
21.64	4.70	0.20	0.14	2.54	63.57	4.85	0.18	0.97
21.85	4.62	0.20	0.14	2.41	63.62	4.86	0.17	0.96
22.37	4.67	0.20	0.13	2.37	63.18	4.81	0.16	0.96
22.38	4.53	0.20	0.14	2.29	63.57	4.69	0.17	0.95
22.27	4.63	0.20	0.13	2.43	63.42	4.66	0.16	0.94
21.72	4.59	0.19	0.13	2.36	64.9	4.70	0.16	0.92
21.74	4.96	0.21	0.14	2.82	63.11	4.73	0.22	1.00
21.65	4.89	0.20	0.13	2.89	63.05	4.88	0.22	0.98
21.90	4.90	0.20	0.14	2.90	63.04	4.70	0.21	1.00
21.00	5.20	0.22	0.18	3.30	63.43	4.35	0.26	1.07
21.08	5.29	0.21	0.18	3.47	62.88	4.37	0.28	1.08

Appendix 2. Regular Variance/Covariance Matrix Approach

Correlation Matrix

	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	1.0000	-0.6738	-0.4628	-0.6592	-0.8604	0.1122	0.4806	-0.3290	-0.5033
V2	-0.6738	1.0000	0.5715	0.6569	0.7753	-0.6063	-0.6266	0.2952	0.7616
V3	-0.4628	0.5715	1.0000	0.6025	0.4927	-0.2148	-0.6054	0.0832	0.3908
V4	-0.6592	0.6569	0.6025	1.0000	0.6651	-0.2531	-0.6437	0.2354	0.4225
V5	-0.8604	0.7753	0.4927	0.6651	1.0000	-0.4835	-0.5492	0.3659	0.6161
V6	0.1122	-0.6063	-0.2148	-0.2531	-0.4835	1.0000	0.0767	-0.2836	-0.5576
V7	0.4806	-0.6266	-0.6054	-0.6437	-0.5492	0.0767	1.0000	-0.0251	-0.3276
V8	-0.3290	0.2952	0.0832	0.2354	0.3659	-0.2836	-0.0251	1.0000	0.5854
V9	-0.5033	0.7616	0.3908	0.4225	0.6161	-0.5576	-0.3276	0.5854	1.0000

Loadings using regular variance/covariance matrix approach are shown below where factors are columns and variables are rows.

Factors

	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	0.1629	-0.1093	-0.4781	-0.5645	-0.2323	-0.4297	-0.0510	-0.6188	-4.1348
V2	-0.1865	-0.0449	-0.2459	0.1347	-0.3900	0.2088	0.4071	2.0881	-2.8615
V3	-0.1386	0.2642	-0.2326	-0.6826	0.8951	0.6059	-0.3401	0.0481	-0.1372
V4	-0.1622	0.2075	0.0804	-0.0494	0.4032	-1.1058	1.0282	-0.3770	-0.3435
V5	-0.1824	-0.0157	0.1200	0.5674	0.1299	0.0499	-0.8375	-1.2742	-3.7622
V6	0.1052	0.3878	0.6628	-0.2273	-0.2770	0.4266	0.4548	0.0076	-2.8710
V7	0.1387	-0.3626	0.1000	0.3106	1.0230	0.2987	0.7169	0.2762	-1.8073
V8	-0.0890	-0.4231	0.5697	-0.6028	0.0514	-0.4134	-0.6104	0.6374	-0.2510
V9	-0.1565	-0.3107	-0.0115	-0.2881	-0.4476	0.6422	0.9614	-1.4117	0.5803

Number of Eigenvalues: 9

Eigenvalues	% Variance	Cumulative % Variance
4.9308	54.7868	54.7868
1.4609	16.2321	71.0189
0.8787	9.7633	80.7821
0.5809	6.4543	87.2364
0.3979	4.4215	91.6579
0.3738	4.1533	95.8112
0.2463	2.7365	98.5478
0.1113	1.2364	99.7842
0.0194	0.2158	100.0000

Chi-square = 774.819, degrees of freedom = 36.

Appendix 2. Regular Variance/Covariance Matrix Approach (Continued)

Mean Square Successive Difference Variance/Covariance Matrix Approach

Correlation Matrix

	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	1.0000	-0.2315	-0.1252	0.3078	-0.5316	-0.4916	-0.0620	-0.0352	0.3877
V2	-0.2315	1.0000	0.6078	0.0314	0.6822	-0.5985	0.3066	-0.1654	-0.1964
V3	-0.1252	0.6078	1.0000	0.1491	0.3416	-0.3234	-0.3569	-0.2144	-0.0000
V4	0.3078	0.0314	0.1491	1.0000	-0.5066	0.1167	-0.6703	0.1918	0.5010
V5	-0.5316	0.6822	0.3416	-0.5066	1.0000	-0.2977	0.6025	-0.5783	-0.6949
V6	-0.4916	-0.5985	-0.3234	0.1167	-0.2977	1.0000	-0.4865	0.2550	0.1140
V7	-0.0620	0.3066	-0.3569	-0.6703	0.6025	-0.4865	1.0000	-0.3214	-0.5847
V8	-0.0352	-0.1654	-0.2144	0.1918	-0.5783	0.2550	-0.3214	1.0000	0.7656
V9	0.3877	-0.1964	-0.0000	0.5010	-0.6949	0.1140	-0.5847	0.7656	1.0000

Loadings using MSSD variance/covariance matrix approach shown below where factors are columns and variables are rows.

Factors

	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	0.0977	-0.1629	-0.5206	-0.2347	-0.1744	-0.5015	-3.5513	-19.5505	
V2	-0.1642	-0.3344	0.0993	0.3096	0.4749	-0.0322	-2.7758	22.8529	
V3	-0.0674	-0.3944	0.2640	-0.1126	-0.7134	0.3166	-0.8985	3.1266	
V4	0.1632	-0.2630	0.0738	-0.2719	1.0081	0.1525	0.9879	-10.9703	
V5	-0.2550	-0.0277	0.1537	0.0667	0.1054	-1.3402	0.5803	-46.1496	
V6	0.1249	0.2976	0.3926	-0.1464	0.1265	-0.7900	-3.9390	7.4489	
V7	-0.2019	0.1592	-0.2998	0.2871	0.3191	-0.1415	-0.6565	18.3212	
V8	0.1796	0.0102	0.0629	0.6981	-0.0183	0.8496	-1.2429	-32.1177	
V9	0.2223	-0.1646	-0.0391	0.3527	-0.1747	-1.7277	1.7985	19.9517	

Number of Eigenvalues: 8

Eigenvalues	% Variance	Cumulative % Variance
3.6939	41.0437	41.0437
1.9805	22.0060	63.0497
1.5894	17.6597	80.7094
1.0443	11.6034	92.3127
0.5152	5.7242	98.0370
0.1533	1.7034	99.7403
0.0232	0.2574	99.9978
0.0002	0.0022	100.0000
-0.0000	-0.0000	100.0000

Chi-square = 1172.55, degrees of freedom = 36.