Describing Behavior with Ratios of Count and Time

J. M. Johnston and Clyde W. Hodge
Auburn University

Describing behavior with ratios of count and time is a popular measurement tactic in the field of behavior analysis. The paper examines some count and time ratios in order to determine what about behavior each describes and why one ratio may sometimes be more useful than another. In addition, the paper briefly considers some terminological issues, derived quantities, dimensional analysis, some advantages and disadvantages of ratios, and selection of useful quantities for measurement.

Key words: methodology, rate, frequency, duration, interresponse time, dimensional quantities, units of measurement, ratios, dimensional analysis

DIMENSIONAL MEASUREMENT

Describing behavior with ratios of count and time has long been an extremely popular practice among behavior analysts. Popular traditions and unexamined practices warrant careful consideration of how such ratios are constructed and used, however. It may be that some of these practices actually lead us to miss features of our data that might be important. The ubiquity of count and time ratios has also led to some confusions about how we use certain terms. These problems may constrain our ability to interpret data as thoroughly as might be possible.

Constructing ratios involving count and time is a component of dimensional measurement, which is the way that all natural sciences observe and record phenomena. This feature of scientific method requires specifying the facets of a physical event or variable that are the focus of interest and how these features will be quantified. Properties, dimensional quantities, and units of measurement constitute the foundation for dimensional measurement. Properties are fundamental qualities of objects or events that can be represented in distinct dimensions, whereas dimensional quan-

ties, also referred to simply as dimensions or quantities, are quantifiable aspects of properties (Johnston & Pennypacker, 1980). The distinction between properties and dimensional quantities is often not clearly drawn in practice, however, and experimental interests are usually expressed in terms of quantities (see Ellis, 1966, chapter 2, for a discussion of this issue, as well as McDowell, 1988). Units of measurement are determinate amounts of dimensional quantities, usually defined as a fixed and standard amount of each. When scientists attach a number to a unit so as to describe the amount of a quantity that is present, the result is called a measure of that event.

For example, an object may have property of physical extent, which in turn can be described by quantities such as length and weight. These quantities are measured in terms of basic units such as meters and grams. The list of such quantities and units routinely used in science and technology is extensive (e.g., see Chertov, 1964; Cook & Rabinowicz, 1963; Zebrowski, 1979), and there is a sizeable literature concerning this critical aspect of measurement (e.g., Ellis, 1966; Ford & Cullmann, 1959; Ipsen, 1960). This literature addresses the origin and definition of quantities and units, as well as the tactics guiding their use.

As a physical event, behavior can be described in dimensional terms (Johnston & Pennypacker, 1980, chapter 7). Although there are a great many established quantities that may be used to de-

We would like to thank Marc Branch, M. Christopher Newland, and H. S. Pennypacker for their comments on an earlier draft of this paper. Reprints may be obtained from the senior author at Psychology Department, Auburn University, Auburn, AL 36849.

scribe different dimensions of behavior, attention has traditionally focused on a relatively short list of possibilities. These include countability, latency, and duration (all of which characterize the single response), as well as rate and interresponse time (which apply only to multiple instances of behavior, such as a response class). Of these, rate is one of the most popular quantities used in the analysis of behavior.

RATIOS OF COUNT AND TIME

Interest in Count and Time

One of the reasons for the popularity of rate in the analysis of behavior is probably Skinner's seminal use of it as a measure of response strength (Skinner, 1938). In contrast to traditional preparations of the era, Skinner measured behavior under conditions in which the organism was "free" to respond at almost any time. Rate of responding captured the dynamic character of behavior better than the latency or duration measures that usually accompanied discrete-trials procedures (Skinner, 1966). The extensive reliance on rate and its usefulness throughout the field's more than 50 years of experimental literature assures this measure a continued primacy among dimensional quantities.

Rate may have been found useful because it conveniently integrates two different quantities of interest in the form of a ratio that provides more information than either quantity considered separately. If a sample of responding is described by either the number of responses emitted or the total duration, but not both, in each case we are likely to find ourselves wanting the other quantity in order to make more sense of the one we have. For instance, knowing that a student correctly completed five math problems is of little value unless we know how long it took to do so. Similarly, knowing that a child tantrumed for 45 minutes during the day would be interpreted differently if this aggregate duration had resulted from one tantrum versus nine.

A preference for both count and time information may also reflect our appreciation that how often behavior occurs is one of the most general and useful features to know about it (but cf., for example, Nevin, 1974, 1979, regarding resistance to change). An integration of the quantities of count and time is implicit in the connotation of "oftenness." However, depending on what temporal features are described and how the ratios are constructed, these two quantities allow a variety of possible ratios, each of which describes a somewhat different aspect of behavior.

This paper examines some derived quantities of count and time, some quite common in the behavioral literature and others still relatively untried. It focuses on what each ratio tells us, why we may wish to use one instead of another, and what terminological issues these quantities raise. In addition, the paper will briefly consider related issues, such as derived quantities, dimensional analysis, advantages and disadvantages of ratios, and how to select useful quantities for measurement. Throughout, count and time ratios are considered in the context of the goal of describing behavior, rather than variability in behavior (see Johnston & Pennypacker, 1980, chapter 17). The use of ratios for describing variability in sets of observations raises other issues that will not be treated here.

Count and Total Time

Count over total time. Certainly the most common ratio of count and time is obtained by dividing the total number of occurrences of the target behavior by the total time that observation occurred. This quotient is called either *frequency* or *rate*, each of which has a long history of varying scientific and colloquial usages. Before Skinner emphasized rate (which he sometimes termed frequency), the use of the terms rate and frequency had already become thoroughly confused.

The Oxford English Dictionary acknowledges that the two words have gen-

erally been used interchangeably in scientific contexts. Although references to frequency as a ratio of the number of times an event occurs in relation to time (e.g., cycles per second in measuring the frequency of a tone) can be found more than a century ago, statistical references to frequency as the number of times an event occurs in a given sample were also used (Boole, 1854). The extensive use of statistics in psychology and the social sciences may make frequency more familiar as countability than this usage is in the natural sciences. On the other hand, in scientific contexts rate more consistently refers to some measure of count over time, although rate probably has more varied colloquial usages than frequency.

Given that behavior analysis has its origin in psychology and yet looks to the natural sciences as its model, it is understandable that behavior analysts may often be confused about the meaning of frequency. In the interest of improved communication, we recommend that behavior analysis reserve frequency for reference to countability and use rate to refer to ratios of count over time (in contrast to the recommendation of Johnston & Pennypacker, 1980, chapter 7), which will be the practice in this paper.

Terminology aside, there is a more important problem with this type of count/time ratio. Using the total time of observation as the time measure inextricably mixes two very different "kinds" of time. The total time during a period of observation is fully composed of the duration of each response, as well as the interresponse time (a type of latency) between the end of each response and the beginning of the next.¹ These two temporal quantities have very different be-

havioral implications. Duration is time during which the response class is occurring. Assuming that only one response in the class can occur at a time, no other instance is possible when one response is already occurring. In contrast, interresponse time is time during which responding can presumably occur, given other prerequisites for responding.

Because duration and interresponse time represent importantly different aspects of behavior, combining them in a total time measure can only risk obscuring relations of possible interest when rate is calculated by dividing count by total session time. If a series of rate measures decreases across sessions, for instance, it will probably be important to know if the decrement occurred because the duration of responses increased or the interresponse times increased. In other words, if the rate of an employee's performance on a task decreases, it would probably be important for someone interested in understanding this decrement to know whether it results from an increase in the duration of work responses or an increase in the time between responses (during which incompatible and inappropriate behavior may occur).

When either or both quantities are free to vary and such variations might be interesting, then using this ratio may obscure relevant information. Much human behavior seems to vary significantly in duration and interresponse time, and such variations may be independent.² When data describing rate of responding

¹ Technically, the session is started during the interresponse time between the first response and the last one that occurred before timing began. Similarly, the session is ended during the interresponse time between the last response and the one that follows after the session. These "pseudo-latencies" contribute a small degree of distortion into rate measures, although the errors tend to cancel each other (see Johnston & Pennypacker, 1980, chapter 7, for further discussion of this point).

² It might appear that these quantities are necessarily dependent within a session because total duration plus total interresponse time equals total session time. This is true for any one session length, however, only if total duration and interresponse time are being considered. If individual durations and interresponse times are of interest, then these values can vary between sessions of the same length. Furthermore, if session lengths vary, even total duration and interresponse time can vary independently. Finally, these relations are true by definition when the behavior can only occur during sessions. However, when the behavior can also occur independently of experimental sessions, which are then an arbitrary imposition of measurement, these quantities may vary independently.

have been calculated using total session time, it is impossible to know to what extent variations in rate measures are due to variation in duration, interresponse time, or both.

Although mixing duration and interresponse time has this inherent disadvantage, it is not always serious. For instance, whenever one of the temporal quantities is known to be relatively constant or extremely brief, this commingling may be unimportant. This might be true if durations or interresponse times were knowingly constrained by some aspect of the situation. This may be the case when the nature of the response class assures that the quantity is relatively trivial (i.e., as with the duration of a pigeon's key peck), or when some aspect of the environment produces either consistent or trivially brief values (e.g., very brief presses on a word processor keyboard aided by both the return spring under each key and the aversive effects of holding a key down).

Having examined some interpretative consequences of mixing duration and interresponse time in total session time calculations of rate measures, we can now revisit the original terminological problem from a slightly different perspective. Although it may be argued that our label for a quantity is less important than clearly understanding what it does and does not tell us, calling the ratio of count over total session time *rate* seems to involve some inconsistency between colloquial connotations and technical language. If rate is generally intended to refer to "oftenness," then using this label for a quantity that incorporates duration is misleading. Oftenness implies a quality of "being able to occur" or "oftenness, given possible occurrence." Our ability to detect this aspect of behaving is limited by the extent of the total duration of a sample of responding, and the ratio of count over total time can be misleading. Practically speaking, however, rate seems unalterably attached to the count/total time ratio. However, *total rate* or *overall rate* will distinguish this construction from other forms of rate.

Total time over count. If total count over total session time expresses the

number of responses emitted per unit of session time, then the inverse ratio describes the average amount of session time per individual response. As with its sibling ratio, this quantity is potentially useful within the limits described above. However, we may be unaccustomed to looking at count and time information in this form, so such measures may seem confusing or, at best, only vaguely interesting. Perhaps the popularity of count over total time has discouraged examining the utility of total time over count, or perhaps we are simply more interested in count than time.

Another possibility is that this ratio makes the confusion caused by mixing duration and interresponse time too obvious to accept. With this ratio, a description of "seven minutes per response" plainly raises the question of "seven minutes of what?" (i.e., duration or interresponse time). The answer must confront the awkward duration-interresponse time dilemma. The analogous description of "seven responses per minute" associated with count over total time seems not to offer a comparable problem, though, only because we customarily omit reference to the proper unit of measurement for countability—the cycle. The technically proper expression is, "seven *cycles* per minute," and then asking for the referent of cycles may be answered by identifying the response class (see Johnston & Pennypacker, 1980, chapter 7).

Whatever the reason for the apparent lack of interest, this ratio would seem to have descriptive potential equal to that of count/total time. There is no *general* reason why countability should be more important than temporal quantities, although it may be a challenge to identify situations in which mixing duration and interresponse time is important or even desirable. If this is the case, then one of the "cleaner" ratios discussed below will always be preferable.

Finally, the ratio of total time over count has no accepted name in psychology. "Average session time per response" is as close as we tend to come, and this is more description than label. In the natural sciences, it is referred to as *period*

(as in "the period swing of a pendulum"), and this may be an appropriate label.

Ratios of Count and Duration

Count over duration. This ratio describes the average number of responses per unit of total duration. Although it may seem similar to the traditional concept of rate (count over time), it is fundamentally different. Whereas rate connotes the average number of times that a response occurs per measurement interval during which responding is possible, count over duration describes average count during periods when responding is occurring. Although such a ratio may well describe features of responding that are useful in some experiments, it seems to make less behavioral sense than count over interresponse time.

Many investigators are not accustomed to this quantity, and most probably find it difficult to conceptualize. This may be because of the previously cited fact that an operant cannot occur when it is already occurring. It seems meaningful to look at total count averaged over the time available for it to occur (i.e., rate), but averaging count over time when responding cannot occur is difficult to interpret. For instance, if we divide the total number of math problems correctly answered by their total duration, the same measure that seems clear in the case of rate (i.e., responses per minute) is now confusing.

However, when the contingencies for initiating a response are different from those for ending a response, this ratio may be more clearly interpretable. (The variables controlling initiation of watching a television that is already on might be auditory, whereas the variables controlling cessation of watching might be visual.) Such a ratio may represent response initiations or terminations per unit of duration, which might provide useful information under some conditions. Nevertheless, the ratio of count over duration has no established name. Perhaps *response continuity* is closest to what this ratio describes.

Duration over Count. The inverse of total count over total duration, on the

other hand, expresses the familiar and usually more behaviorally meaningful quantity of average duration per response, which is usually called *average duration*. Examples are common in the behavioral literature (e.g., average time spent playing, studying, etc.).

Ratios of Count and Interresponse Time

Count over interresponse time. This ratio might be said to be the quantity that we ought to call rate. It avoids the difficulty described above in interpreting a quantity that places count over duration plus interresponse time. Total count over total interresponse time simply describes the average number of responses that occur during periods when responding is possible. For example, a ratio of the number of inhalations of cigarette smoke over the total number of minutes not actually inhaling is a form of rate corrected for periods during which responding cannot occur (i.e., duration). Because the term rate is often assigned to the ratio involving total session time, this quantity has no unique identifier. If count divided by total session time is qualified as *total* or *overall rate*, it would permit count over interresponse time to be called, simply, *rate*, although some distinctive modifier may be necessary.

Interresponse time over count. Because this quantity uses a measure of the time during which responding can occur, it makes more behavioral sense than duration over count. Total interresponse time over total count describes the average time between responses and is appropriately labeled *average interresponse time*.

Other count and latency ratios. Of course, interresponse time is only one type of latency. The time between a stimulus event and the subsequent response is another quantity that may be informative with some experimental preparations (e.g., the time between an alarm going off and a corrective action being taken). Accordingly, the two quantities placing count and this type of latency in ratios may sometimes describe useful aspects of responding.

USING COUNT AND TIME RATIOS

Other Ratios Involving Count and Time

The quantities of duration and latency are in the category of quantities that are called *substantial variables* (Ipsen, 1960) or *fundamental quantities* (Chertov, 1964; Ellis, 1966; Ford & Cullmann, 1959) because they may be measured directly and their units have been defined independently in terms of some artificial standard. (The nature of the quantity of countability does not require an independently defined unit because it merely requires enumeration of instances, although it is still usually considered a fundamental quantity.) The count and time ratios discussed above are termed *simple natural variables* (Ipsen, 1960) or *derived quantities* (Chertov, 1964; Ellis, 1966) because they are derived from fundamental quantities on the basis of physical relationships.

Any of these ratios can themselves participate in other ratios as either numerator or denominator. The familiar quantity of acceleration is a good example of this practice. It may be defined as a ratio of rate (count over time) over time, or count divided by time squared. In the case of behavior, acceleration and deceleration (increasingly referred to in behavior analysis as *celeration*, as proposed by Pennypacker, Koenig, & Lindsley, 1972) describe the change in the rate of responding over time. Many ratios of count and time are possible, but there is no a priori guarantee that any particular ratio will be meaningful in the study of behavior.

One method of obtaining clues about meaningful functional relationships is through dimensional analysis (Bridgeman, 1922; Barenblatt, 1987). The process of creating derived quantities will result in some that lack units of measurement and are thus called *dimensionless variables*, quantities, or numbers (e.g., the Mach number, which is the ratio of the speed of an object to the speed of sound in the medium through which it travels). Dimensional analysis is a tech-

nique for deriving dimensionless numbers that may lead to promising or interesting experimental findings through the delineation of all viable functional relations. For example, dimensional analysis can show that the period swing of a pendulum is independent of its mass and depends on length, gravitational acceleration, and swing amplitude (Ellis, 1966; Ford & Cullmann, 1959). Thus, dimensional analysis may be used to aid the search for physical relationships.

Dimensional analysis cannot preempt empirical analysis, however, because empirical evidence is always required to confirm any supposed functional relationship. For example, it is conceivable that the period swing of a pendulum may depend on mass or length in a way different from that specified by the dimensional formula. That is, the relationship suggested by dimensional analysis may not exist under all conditions (such as varying atmospheric conditions), or it may be modified by a dimensionless constant. Furthermore, meaningful functional relations may be implied through the use of dimensional analysis, but its use will not lead to the discovery of "basic" laws such as gravity. Fundamental physical relations are not always derivable from other relations, and they are often not unitless (Ellis, 1966). Therefore, dimensional analysis may be used to propose experimental directions that must be tested empirically or as a check on the meaningfulness of calculations.

The field of behavior analysis has not often attempted dimensional analysis (cf. Nevin, Mandell, & Atak, 1983). Such analyses would involve examining various combinations of fundamental quantities in an attempt to learn something new about a particular facet of behavior. The quantities that were suggested by the literature to be relevant would be combined in different ways and candidate dimensionless numbers obtained. Each would then be applied to the analysis of actual data in a search for orderly relations. However, order per se may not always be meaningful (i.e., useful). The features of behavior described by a dimensionless quantity must also be

Figure 1. Fictional session values representing two different quantities ($Q1$ and $Q2$, bottom) and the quotients resulting from dividing one quantity by the other ($Q1/Q2$, top).

shown by experimental research to be important to our understanding of behavior.

Disadvantages of Ratios

When two or more fundamental quantities are combined in a ratio, the viewer's behavior can no longer be controlled by the original quantities (unless, of course, those quantities are retained separately in some form). Although this may well be an intentional and valuable analytical tactic, it nonetheless involves a "loss of information" that can hinder effective interpretation. For instance, when the total number of self-injurious responses per observation period is divided by the total interresponse time to provide an experiment's sole data, viewers cannot know either total count or total interresponse time. Whether this omission is important will depend on the particulars of each such case, but the risk certainly exists that the use of these fundamental quantities only in the form of such a ratio will limit interpretation.

A related limitation of ratios is that the

viewer has no information about how each of the underlying quantities contributes to changes in the ratio. This point is illustrated by Figure 1, which displays a series of session values representing two different quantities ($Q1$ and $Q2$, bottom) and the quotients resulting from dividing one quantity by the other ($Q1/Q2$, top). It is easily seen that a description of the sequential change in $Q1/Q2$ would improperly represent change in either of the quantities that were combined to form the ratio. For example, most would describe the data points in the top panel approximately between sessions 10 and 20 as relatively stable. However, this stability is produced by sharply increasing and decreasing transitory states in $Q1$ and $Q2$, with $Q2$ having much higher values. This limitation (which applies to all ratios) can be serious, and should at least be acknowledged in interpretations. When describing ratio data, the customary phrase, "Responding showed such and such a pattern . . ." should always be qualified by reference to only the particular derived quantity under analysis (i.e.,

Rate of responding showed . . .). Of course, either of the contributing quantities can be independently analyzed, thereby avoiding this problem.³

Selecting Useful Ratios

Which ratios involving count and time will be important in the analysis of behavior? In spite of the popularity of rate (as traditionally calculated), there can be no general answer to this question. The quantity that reveals variability that helps to answer the experimental question may not even involve count or time (e.g., force). Although it may be possible to exclude certain quantities before the fact (as when some aspect of procedure makes the variability in a certain quantity meaningless or trivial), it is more difficult to be sure which will show variability that turns out to be important.

The conservative solution to this problem is to measure all of the reasonably relevant fundamental quantities so that all candidates and their possible ratios will be available for analysis. This is not as challenging an option as it might seem. If observation can detect the beginning and end of individual responses and therefore create a record of the number of responses and all of the resulting intervals (i.e., durations and interresponse times), then all quantities based on count and time will be available. If it is not possible to identify individual response cycles (e.g., if it is only possible to know when the behavior is occurring, as with the behavior of watching TV), only duration can be measured.

Having all quantities based on count and time at one's analytical disposal is encouraging, but it does not mean that any possible ratio that shows systematic variability within or across phases is automatically behaviorally meaningful. As discussed above, some ratios may be un-

interpretable for the study of behavior, and others may be inappropriate or trivial in the case at hand. If any set of data describing fundamental count and time quantities is subjected to sufficient manipulation, it is likely to exhibit some kind of order. However, the meaningfulness of the resulting derived quantity must be convincingly shown, and the generality of the relations must then be experimentally established.

In summary, ratios of count and time can bring the viewer's interpretations under control of aspects of responding that might not otherwise be evident. However, casual selection or automatic construction of such ratios can lead to interpretative limitations and errors. Cautious use, guided by the focus of the experimental question and possibly supplemented with other displays of the same raw data, may augment our analytical skills.

REFERENCES

- Barenblatt, G. I. (1987). *Dimensional analysis*. New York: Gordon and Breach.
- Boole, G. (1854). *The laws of thought*. New York: Dover.
- Bridgeman, P. W. (1922). *Dimensional analysis*. New Haven, CT: Yale University Press.
- Chertov, A. G. (1964). *Units of measurement of physical quantities*. New York: Hayden.
- Cook, N. H., & Rabinowicz, E. (1963). *Physical measurement and analysis*. Reading, MA: Addison-Wesley.
- Ellis, B. (1966). *Basic concepts of measurement*. London: Cambridge University Press.
- Ford, R. G., & Cullmann, R. E. (1959). *Dimensions, units and numbers*. New York: Teachers College, Columbia University.
- Ipsen, D. C. (1960). *Units, dimensions, and dimensionless numbers*. New York: McGraw-Hill.
- Johnston, J. M., & Pennypacker, H. S. (1980). *Strategies and tactics of human behavioral research*. Hillsdale, NJ: Lawrence Erlbaum.
- McDowell, J. J. (1988). Behavior analysis: The third branch of Aristotle's physics. *Journal of the Experimental Analysis of Behavior*, 50, 297-304.
- Nevin, J. A. (1974). Response strength in multiple schedules. *Journal of the Experimental Analysis of Behavior*, 21, 389-408.
- Nevin, J. A. (1979). Reinforcement schedules and response strength. In M. D. Zeiler & P. Harzem (Eds.), *Advances in analysis of behaviour: Vol. 1. Reinforcement and the organization of behaviour*. New York: Wiley.
- Nevin, J. A., Mandell, C., & Atak, J. R. (1983). The analysis of behavioral momentum. *Journal*

³ The points in this section fully apply to ratios in which numerator and denominator represent like quantities (percentages). However, because this paper is exclusively concerned with ratios of count and time, we direct the reader to Johnston & Pennypacker, 1980, chapter 7, for a discussion in the context of percentages.

- of the Experimental Analysis of Behavior*, 39, 49–59.
- Pennypacker, H. S., Koenig, C. H., & Lindsley, O. R. (1972). *Handbook of the standard behavior chart*. Kansas City: Precision Media.
- Skinner, B. F. (1938). *The behavior of organisms*. Englewood Cliffs, NJ: Prentice-Hall.
- Skinner, B. F. (1966). Operant behavior. In W. K. Honig (Ed.), *Operant behavior: Areas of research and application*. New York: Appleton-Century-Crofts.
- Zebrowski, E. (1979). *Fundamentals of physical measurement*. Belmont, CA: Wadsworth.