Matching Theory in Natural Human Environments

J. J McDowell
Emory University

Matching theory is a mathematical account of behavior, many aspects of which have been confirmed in laboratory experiments with nonhuman and human subjects. The theory asserts that behavior is distributed across concurrently available response alternatives in the same proportion that reinforcement is distributed across those alternatives. The theory also asserts that behavior on a single response alternative is a function not only of reinforcement contingent on that behavior, but also of reinforcement contingent on other behaviors and of reinforcement delivered independently of behavior. These assertions constitute important advances in our understanding of the effects of reinforcement on behavior. Evidence from the applied literature suggests that matching theory holds not only in laboratory environments, but also in natural human environments. In addition, the theory has important therapeutic implications. For example, it suggests four new intervention strategies, and it can be used to improve treatment planning and management. Research on matching theory illustrates the progression from laboratory experimentation with nonhuman subjects to therapeutic applications in natural human environments.

Matching theory (Herrnstein, 1970) is a mathematical account of behavior. It has been a part of basic behavior analysis for over a quarter of a century, and it has been a topic of considerable interest for at least the last 18 years. Most basic scientists would agree that matching theory has advanced significantly our understanding of how the environment governs behavior. In spite of its lengthy, active, and important history, however, and in spite of the publication of a number of articles that call attention to its clinical significance (Epling & Pierce, 1983; McDowell, 1981a, 1982; Myerson & Hale, 1984), matching theory remains virtually unknown to applied behavior analysts. The purpose of this article is to describe matching theory, to explain its significance, and to show how it applies to human behavior in natural environments.

Because it deals with research in the basic science, this article also addresses the issue of the relationship between ba-

sic research and clinical application. Many authors have noted that these fields are becoming isolated from each other. Some authors have expressed alarm at this turn of events (e.g., Pierce & Epling, 1980), others have been more sanguine about it (e.g., Poling, Picker, Grossett, Hall-Johnson, & Holbrook, 1981), and still others have applauded it (e.g., Cullen, 1981). Some of the authors in the last category believe that basic research has become too trivial and esoteric to be relevant to socially important human behavior. For example, Cullen (1981) criticizes basic research in these words:

There seems to be a paucity of studies now identifying major and significant functional relationships, but there are many which identify weak and practically useless relationships. . . . There are many radical behaviorists who have little interest in rats and pigeons. It is *social behavior*, about which further study of rats and pigeons is unfortunately likely to add little, which is of importance now (p. 83, emphasis in the original).

A number of writers, including Cullen, have been especially critical of mathematical work in the basic science. For example, Ferster (1978) has implied that the proliferation of mathematical theories indicates that operant conditioners are getting bored with behavior. Catania (1981) has expressed a similar view, and Cullen (1981) complains that the mathematical accounts are irrelevant to applied concerns and, moreover, that they are too hard.

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These criticisms of basic research are not well founded. In fact, as will be shown in this article, contemporary research in the basic science *is* identifying significant functional relationships, the study of rats and pigeons *is* adding to our understanding of social behavior in humans, and contemporary mathematical research, far from reflecting boredom with behavior, bears importantly on human social behavior.

A MATCHING THEORY PRIMER

Matching theory deals with choice behavior. If a person is confronted with a variety of alternatives and selects one alternative to the exclusion of the others, then the person is said to have made a choice. For example, a disruptive child may either attend to her arithmetic assignment or get out of her seat and walk around the classroom. Choice may also be conceptualized as selecting a particular alternative over all other alternatives considered as an aggregate. Thus a variety of alternatives can be compressed into just two, the target alternative and everything else. For example, a husband may pick a fight with his wife rather than do any of the many other things he might have done instead.

Many choice situations occur frequently rather than just once. A delinquent boy frequently chooses between staying home and going out on the street. Choice can also be described as continuous. For example, in the course of an evening the husband may choose continuously between fighting with his wife and doing other things. He may fight for a while, watch television for a while, fight some more, and so on. At any moment he may switch from fighting to watching television or vice versa. To say that choice is continuous means that the opportunity to choose one activity over another is continuously available.

Sometimes there are consequences for choosing one alternative over another. The delinquent boy's choice to go out on the street may result in enjoyable activities or it may serve to escape or avoid an unpleasant circumstance at home, such

as the nagging of a critical parent. Alternatively, if the boy chooses to remain at home he might find that there is an interesting program on television. Of course, not all choices of a particular alternative produce a consequence. Sometimes nothing of interest happens on the street and sometimes nothing of interest happens at home.

The Concurrent Schedule

A situation in which behavior is free to alternate continuously between two or more alternatives, and in which consequences for choosing each alternative occur occasionally, is called a concurrent schedule of reinforcement. Figure 1, which shows the front panel of a pigeon chamber, illustrates how a concurrent schedule is set up in the laboratory. The circles represent keys which a pigeon can peck. In a situation like the one illustrated in the figure, the pigeon is free to peck either key at any time, or to peck neither key, and it may distribute its pecks between the two keys in any way whatever. In a typical 1-hr session, the pigeon chooses continuously between the two alternatives. It may peck on the left key for a while, switch to the right key, go back to the left key, and so on, alternating between the two keys throughout the session. Notice that this laboratory situation incorporates the characteristics of choice behavior mentioned earlier. Choices of one alternative over the other occur frequently (i.e., the pigeon pecks each key many times), and choice is continuous. To make an analogy with the examples used earlier, the pigeon's choice of the left key is like the disruptive child's choice to get out of her seat, while its choice of the right key is like the child's choice to do arithmetic problems at her desk. To take an example where a variety of alternatives is compressed into just two, the pigeon's choice of the left key is like the husband's choice to fight with his wife, while its choice of the right key is like the husband's choice to do something other than fight with his wife.

The rate at which each alternative is chosen in a concurrent schedule is an im-

portant dependent variable. Just as one can count the number of times per session that a pigeon pecks a key, one can count the number of times per week or per month that the husband fights with his wife. Even though these two behaviors differ topographically, they can be studied in the same way, namely, in terms of their time rate of occurrence. If we designate the left key in Figure 1 as key-1 and the right key as key-2, then we can represent the rate of pecking key-1 by R_1 and the rate of pecking key-2 by R_2 . The upper case R s stand for response rate and the numerical subscripts on the R s tell which key the response rate refers to. Response rates are usually expressed in units of responses/minute (rsp/min).

On concurrent schedules like the one illustrated in Figure 1, consequences for choosing each alternative are arranged by permitting brief periods of access to a food hopper filled with grain. When the grain is made available (i.e., during reinforcement), the pigeon can insert its head in the rectangular opening shown in the figure and eat. When the grain is made unavailable (i.e., between reinforcements), the pigeon can still insert its head in the opening, but it cannot reach the grain. The consequences for pecking each of the two keys are made occasional by arranging food presentations on each key according to a variable-interval (VI) schedule. On a VI schedule, reinforcement is "set up" at irregular intervals of time. After reinforcement is set up, the schedule waits for the pigeon to peck the key. When the key is pecked, the grain is made available for a brief period of time (e.g., 4 s), after which it becomes unavailable and the timing of the next interval begins. The average of the intervals used to arrange reinforcement defines the VI schedule. For example, on a VI 1-min schedule, the average time between the end of one reinforcement and the setup of the next is 1 min.

If a VI schedule is in effect on one key of a concurrent schedule and another VI schedule is in effect on the other key, then a concurrent VI VI schedule is in effect. Each of the individual VI schedules is referred to as a component of the con-

Figure 1. The front panel of a two-key pigeon chamber. The quantities, R_1 and R_2 , represent the rates of responding on each of the two keys (rsp/min = responses per minute). The quantities, r_1 and r_2 , represent the rates of reinforcement obtained for responding on each of the two keys (rft/hr = reinforcements per hour). The rectangular opening below the response keys permits access to a food hopper.

current schedule. In the typical concurrent VI VI schedule, both component VI schedules operate (i.e., the timing of their intervals continues) regardless of what the pigeon is doing. For example, even if the pigeon is pecking only on key-1, both VI schedules continue timing their intervals. If reinforcement is set up on key-2, the VI schedule on that key will stop and wait for the pigeon to switch over to key-2 and peck it. The individual VI schedules also continue operating when the pigeon is pecking neither key, as for example, when it is preening.

Just as pecking on each key of a concurrent schedule can be expressed as a rate, the reinforcement obtained from pecking each key can be expressed as a rate. The rate of reinforcement obtained from pecking key-1 can be represented by r_1 , and the rate of reinforcement obtained from pecking key-2 can be represented by r_2 . These rates are usually expressed in units of reinforcements/hour (rft/hr). Because the actual reinforce-

ments are spaced irregularly in time, the reinforcement rate is an average. For example, a rate of 5 rft/hr amounts to one brief opportunity to eat every 12 minutes on the average. A rate of 200 rft/hr amounts to one brief opportunity to eat every 18 seconds on the average. Clearly, the latter schedule is much richer than the former.

To summarize, the concurrent schedule is a laboratory environment that incorporates the significant properties of human choice behavior discussed earlier. Choices of one alternative over the other occur frequently, choice is continuous, and occasional consequences occur for choosing each alternative. The pigeon is free to distribute its pecks across the two alternatives in any way whatsoever, or it may choose to peck neither key.

It is important to recognize that organisms respond on concurrent schedules far more often than necessary to obtain all the scheduled reinforcers. For example, pigeons may peck thousands of times to obtain a hundred or fewer reinforcers. The interesting question is how the organism distributes its behavior across the two alternatives. For example, with 5 rft/hr on key-1 and 200 rft/hr on key-2, it might seem reasonable for the pigeon to peck key-2 exclusively because it provides 40 times more reinforcers per unit time than key-1. On the other hand, it could be argued that the very rich schedule on key-2 should be pecked infrequently—just enough to pick up the reinforcers as they are set up. Actually, the pigeon has a very wide latitude for choice. Between exclusive pecking on key-1 and exclusive pecking on key-2 lies a large number of distributions of pecks on the two keys, many of which would produce high overall rates of reinforcement. Notice that the question of interest in the laboratory is the same as the question of interest in the natural human environment, namely, what governs the distribution of behavior across concurrently available response alternatives?

The Matching Equation

In the laboratory, choice behavior is typically studied by arranging a series of

about 5 concurrent VI VI schedules. Each concurrent schedule remains in effect until the response allocation across the two keys remains stable from one day to the next. Four pieces of data are obtained from each concurrent schedule: the two reinforcement rates that the organism obtains for responding (r_1 and r_2) and the two response rates (R_1 and R_2) that it produces.

The outcome of this type of experiment is very simple and, in spite of the wide latitude for choice that concurrent schedules allow, very orderly. The typical finding is that the proportion of responses the organism allocates to each alternative is determined by the proportion of reinforcements obtained from that alternative. Specifically, the response proportion is usually found to equal the reinforcement proportion. For example, if 1/5 of the reinforcers are obtained from key-1 and 4/5 are obtained from key-2, then 1/5 of a pigeon's total number of pecks will occur on key-1 and 4/5 of its total pecks will occur on key-2. This outcome, where the proportion of responses on one alternative equals, or matches, the proportion of reinforcements obtained from that alternative, can be represented by the following simple algebraic expression:

$$\frac{R_1}{R_1 + R_2} = \frac{r_1}{r_1 + r_2} \quad (1)$$

Recall that R_1 and R_2 represent the response rates on the two keys and that r_1 and r_2 represent the reinforcement rates obtained from pecking on the two keys. Evidently, the left side of this equation represents the proportion of responses allocated to key-1 and the right side represents the proportion of reinforcements obtained from pecking key-1. Of course, Equation 1 can be written for the second response alternative as well as for the first:

$$\frac{R_2}{R_1 + R_2} = \frac{r_2}{r_1 + r_2}$$

Clearly, if Equation 1 is true, then the expression above is also true, because the two response proportions and the two reinforcement proportions sum to unity.

Equation 1 is frequently referred to as the matching law. It constitutes a precise, deterministic, and mathematical understanding of choice behavior. According to the equation, organisms distribute their behavior across available response alternatives in the same proportion that reinforcements are distributed across those alternatives.

The Single-Alternative Hyperbola

Equation 1 applies to behavior in multi-alternative environments. It is not clear how it applies to behavior in single-alternative environments where there is only one key to peck or one lever to press. Notice, however, that organisms can and do exhibit behaviors in single-alternative environments other than the instrumental response. For example, a bird may occasionally preen or flap its wings rather than peck the key. Similarly, a rat may rear, explore the chamber, or wash its snout in between presses on the lever. Just like a concurrent schedule, then, a single-alternative schedule of reinforcement entails more than just one response alternative. Notice also that reinforcers may be produced by the "extraneous" behaviors. Exploring, for example, may turn up an interesting odor. Preening might remove a source of skin irritation, which is escape and is supported by negative reinforcement.

Considerations like these led Herrnstein (1970) to suggest that single-alternative schedules could be conceptualized as concurrent schedules and that, consequently, all behavior could be conceptualized as choice behavior. According to this view, responding on single-alternative schedules should be governed by the matching equation, where the two alternatives are instrumental responding (e.g., key pecking) and extraneous responding (i.e., all behavior other than key pecking) considered as an aggregate. The instrumental responding is supported by schedule-delivered reinforcement and the extraneous responding is supported by extraneous reinforcement. Letting R_e represent the aggregate rate of extraneous responding and r_e represent the aggregate rate of extraneous reinforcement, the

matching equation for a single-alternative environment is written

$$\frac{R}{R + R_e} = \frac{r}{r + r_e}, \quad (2)$$

where R and r (without subscripts) represent the rates of instrumental responding and schedule-delivered reinforcement. Equation 2 is the same as Equation 1, except that the two response alternatives are R and R_e instead of R_1 and R_2 . According to Equation 2, responding on single-alternative schedules entails choosing between instrumental responding and extraneous responding and must therefore be governed by the matching law.

One of the most important features of Equation 2 is that it can be written in a form that expresses the absolute rate of responding on the instrumental alternative as a function of the absolute rate of reinforcement obtained for that responding. Remember that the original matching equation relates only response and reinforcement proportions. If we let the quantity, $R + R_e$, which is the denominator on the left side of Equation 2, equal a constant, k , then Equation 2 can be written

$$\frac{R}{k} = \frac{r}{r + r_e}.$$

Solving this expression for R produces

$$R = \frac{kr}{r + r_e}. \quad (3)$$

Equation 3 expresses the response rate, R , on the instrumental alternative as a function of the reinforcement rate, r , obtained for that responding. The equation is hyperbolic in form. A graph of Equation 3, where response rate, R , is plotted against reinforcement rate, r , appears as a curve that begins at the origin, rises, and then levels off. According to Equation 3, the absolute rate of responding on a single response alternative is a hyperbolic function of the absolute rate of reinforcement obtained for that responding. Some authors (e.g., de Villiers, 1977) refer to this equation as a quantitative

statement of the law of effect. The significance of Equation 3 will be discussed in detail later.

Time-Allocation

The rate at which behavior occurs is the principal dependent variable in basic behavior analysis. The matching equation and the single-alternative hyperbola are both written in terms of response rates. But not all behavior can be represented informatively by its time rate of occurrence. Consider, for example, the out-of-seat behavior of a disruptive child. A child may get out of his seat once during a class period, remain out of his seat briefly, and then sit down. Another child may get out of his seat once during a class period and remain out of his seat for the rest of the period. The two children's rates of out-of-seat behavior are the same, namely, one episode per class period, but the second child's disruptive behavior is obviously worse than the first child's. In cases like these it is more informative to time the behavior than to count it.

The issue of timing versus counting behavior has been addressed by basic researchers interested in matching theory. For example, Baum and Rachlin (1969) arranged a concurrent schedule for pigeons where one alternative was standing on one side of an experimental chamber and the other alternative was standing on the other side. This is just like the example of the disruptive child. Standing on one side of the chamber is analogous to the child staying in his seat, and standing on the other side is analogous to the child getting out of his seat. In Baum and Rachlin's experiment reinforcement was arranged for responding on each alternative. They measured the total amount of time the pigeons spent standing on each side of the chamber and found that the proportion of time spent on each alternative equalled the proportion of reinforcements obtained from that alternative. In other words, they found that matching held when behavior was measured in terms of its total duration. If T_1 represents the total amount of time spent on the first alternative and T_2 represents

the total amount of time spent on the second alternative, then the matching equation can be written in terms of time spent on each alternative:

$$\frac{T_1}{T_1 + T_2} = \frac{r_1}{r_1 + r_2} \quad (4)$$

This equation is often said to represent time-allocation matching. Clearly, it increases the generality of matching theory because it extends the theory to behavior that cannot be represented informatively by counting.

Just as a single-alternative equation was derived from Equation 1, a single-alternative equation can be derived from Equation 4. The derivation is exactly analogous to the derivation of Equation 3. The result is

$$T = \frac{kr}{r + r_e} \quad (5)$$

In this equation T represents the total amount of time spent engaged in the instrumental alternative and k represents the total session time.

Equations 1, 3, 4, and 5 constitute matching theory. The theory asserts that all behavior is choice behavior governed by the matching principle. Equations 1 and 4, which are mathematical statements of the matching principle, govern behavior in multi-alternative environments. Equations 3 and 5, which are derived from the mathematical statements of the matching principle, govern behavior in single-alternative environments.

THE SIGNIFICANCE OF MATCHING THEORY

As noted earlier, matching theory constitutes a precise, deterministic, and mathematical understanding of choice. It states that organisms distribute their behavior across concurrently available response alternatives in the same proportion that reinforcements are distributed across them. The single-alternative hyperbola identifies the form of the function relating absolute response and re-

inforcement rates, and it expresses the functional relationship between these two variables quantitatively. To understand the most revolutionary consequence of matching theory, it is necessary to consider Equation 3 or Equation 5 more carefully. The arguments based on the two equations are analogous, so only Equation 3 will be considered here.

Although it was derived from Equation 1, Equation 3 is often regarded as the more fundamental statement of matching theory because the dependent and independent variables in this equation are absolute response and reinforcement rates rather than proportions of responses and reinforcements. Moreover, it is easy to show that Equation 1, the matching equation, can be obtained from Equation 3 when the latter is written to describe each alternative in a multi-alternative environment (e.g., de Villiers, 1977).

Two plots of Equation 3, the single-alternative hyperbola, are shown in Figure 2. Each hyperbola passes through the origin (i.e., when $r = 0$, $R = 0$), increases with negative acceleration, and approaches an asymptote as reinforcement rate (r) becomes large. The asymptote, which is the maximum possible rate of responding, is represented by the parameter, k , in Equation 3. The horizontal line, $R = k$, in the figure represents the hyperbola's asymptote. Both hyperbolas in Figure 2 approach the same asymptote but they have different values of r_e . As shown in the figure, when r_e is small the hyperbola approaches its asymptote rapidly; that is, responding is nearly asymptotic even at fairly low reinforcement rates. When r_e is large the hyperbola approaches its asymptote more slowly; that is, responding remains well below its asymptotic value even at fairly high reinforcement rates. Remember that r_e represents the aggregate rate of reinforcement obtained from all response alternatives other than the instrumental alternative. A large value of r_e corresponds to a rich environment and a small value corresponds to a lean environment.

Behavior is usually considered to be a function of its consequences. But Equation 3 indicates that there is more to the

Figure 2. Two plots of Equation 3, the single-alternative hyperbola. Both hyperbolas approach the same asymptote, which is represented by the line, $R = k$. For one hyperbola, r_e is small (i.e., the environment is lean); for the other hyperbola, r_e is large (i.e., the environment is rich). The dashed lines illustrate how a given reinforcement rate, r , may support a relatively low response rate, R_a , in a rich environment but a relatively high response rate, R_b , in a lean environment.

regulation of a specific behavior than just the reinforcement that is contingent on it. According to the equation, the rate of reinforcement supplied to the organism from other concurrent sources (r_e) also contributes to the regulation of the behavior. As shown in Figure 2, a particular rate of contingent reinforcement, r , may produce a low response rate, R_a , or a high response rate, R_b , depending on how much additional reinforcement the environment supplies to the organism. Equation 3 represents an advance over the traditional view of reinforcement because it takes into account the environmental context in which responding and reinforcement occur. As might be expected, this more naturalistic understanding of the effects of reinforcement is especially important for applied work.

LABORATORY RESEARCH SUPPORTING MATCHING THEORY

During the 27 years since the matching equation was originally proposed (Herrn-

stein, 1961), a large body of data from laboratory experiments has accumulated in support of matching theory. Pigeons, rats, crows, cows, squirrel monkeys, and humans are some of the species that have served as subjects in these experiments. Behaviors studied include key pecking, lever pressing, treadle pressing, chain pulling, standing, talking, and macrosaccadic eye movements. Reinforcers used include food, water, intracranial brain stimulation, shock avoidance, money, opportunities to listen to comedy records (de Villiers, 1977), and opportunities to view sexually interesting slides (Cliffe & Parry, 1980). Extensive reviews of the literature leave no doubt that matching theory provides an excellent description of behavior on concurrent schedules (Baum, 1974, 1979; de Villiers, 1977; Wearden & Burgess, 1982). For example, Baum (1979) examined 103 data sets from 23 laboratory studies of behavior on concurrent schedules and found that forms of Equations 1 and 4 accounted for an average of 90% of the variance of the individual data sets.

Equation 3, the single-alternative hyperbola, has also been studied extensively in laboratory experiments. However, its time-allocation counterpart (Equation 5) has received little experimental attention. The hyperbolic form of Equation 3 has been confirmed in a large number of experiments using nonhuman (de Villiers & Herrnstein, 1976) and human (Bradshaw, Szabadi, & Bevan, 1976, 1977, 1978; McDowell & Wood, 1984, 1985) subjects. For example, McDowell and Wood (1984) studied human subjects' lever pressing on various VI schedules of monetary reinforcement. They collected 34 sets of response rate versus reinforcement rate data from 8 human subjects and found that for the 27 data sets showing substantial response-rate variability, Equation 3 accounted for a median of 95% of the data variance.

The interpretation of r_e as the rate of extraneous reinforcement has also been tested in laboratory experiments. Consider, for example, a situation in which the reinforcement rate, r , for an instrumental response is held constant. Equa-

tion 3 states that in this situation an increase in r_e will result in a decrease in the rate of the instrumental response, R , even though the rate of reinforcement contingent on the response has not changed. This is because as r_e increases, the denominator on the right side of Equation 3 increases and so the ratio, $r/(r + r_e)$, decreases. Similarly, if r_e is decreased while r is held constant, Equation 3 requires the rate of instrumental responding to increase, even though the rate of contingent reinforcement has not changed. The magnitude of r_e can be increased by adding reinforcement to the environment. For example, reinforcement can be arranged for a behavior other than the instrumental response. Research using nonhuman (de Villiers, 1977; Rachlin & Baum, 1972) and human (Bradshaw, Szabadi, & Bevan, 1976) subjects has confirmed this interpretation of r_e . That is, instrumental response rates have been found to vary inversely with the rate of experimentally arranged extraneous reinforcement in the manner required by Equation 3. It has also been found that this effect holds even if the additional reinforcement is delivered independently of behavior (e.g., Rachlin & Baum, 1972). In other words, r_e comprises all additional reinforcement provided by the environment, including free or noncontingent reinforcement.

Evidence supporting the interpretation of r_e as the rate of extraneous reinforcement is especially important because this evidence also supports the expanded understanding of reinforcement that Equation 3 entails. This evidence indicates that the rate at which a behavior occurs depends not only on the rate of reinforcement contingent on the behavior, but also on the rate of additional reinforcement provided by the environment.

THE VALIDITY OF MATCHING THEORY IN NATURAL HUMAN ENVIRONMENTS

A rather direct transition from the laboratory to the natural human environment is possible because the properties of behavior that are studied in laboratory

research also characterize behavior in natural human environments. It is reasonable to suppose that lawful relationships between response rate and reinforcement rate, or between time allocation and reinforcement rate, that are discovered in laboratory experiments also hold in natural human environments. On the other hand, because there are differences between laboratory environments and natural human environments, it is also reasonable to demand evidence that functional relationships discovered in the laboratory in fact hold in natural human environments. Although this is not a widely pursued area of research, some data have been reported that bear on the validity of matching theory in natural environments.

Conger and Killeen (1974) were the first to study human behavior on concurrent schedules in a social environment. These investigators arranged concurrent schedules of social reinforcement for the verbal behavior of five subjects. Each subject participated in a discussion of drug abuse with three experimenters. One response alternative of the concurrent schedule consisted of talking to one of the experimenters, the other response alternative consisted of talking to the second experimenter. The third experimenter facilitated the discussion. A variable-interval schedule was used to arrange social reinforcement, which consisted of praising statements such as "that's a good point," on the two alternatives. Each of the first two experimenters delivered a social reinforcer when responding occurred following the illumination of a cue light, which signaled that reinforcement was set up on that alternative. Conger and Killeen recorded the total amount of time subjects spent talking to each of the first two experimenters. They found that Equation 4, the time-allocation matching equation, provided an excellent description of their subjects' behavior. That is, the proportion of time the subjects spent talking to each experimenter was governed by the proportion of social reinforcements obtained from that experimenter, as required by Equation 4.

McDowell (1981a) was the first to re-

port the governance of naturally occurring human behavior by Equation 3, the single-alternative hyperbola. He analyzed assessment data from a case study of self-injurious scratching in an otherwise normal 10-year-old boy (Carr & McDowell, 1980). At the start of the study the boy had approximately 30 lesions on his scalp, face, back, arms, and legs. Some lesions were open and as large as 1 cm in diameter; others were smaller and covered by scabs. The boy's skin also exhibited scar tissue from healed lesions and areas of skin discoloration where new lesions might be produced. Carr and McDowell (1980) showed that the boy's self-injurious scratching was under stimulus control inasmuch as it occurred principally while he and his family were watching television in the living room of their home. In addition, data from an experimental assessment procedure using a reversal design showed that the behavior was reinforced by verbal reprimands from family members. McDowell (1981a) examined the pretreatment data, which consisted of naturally occurring rates of self-injurious scratching and naturally occurring rates of verbal reprimanding. The data were obtained during four 20–30 min observation sessions conducted in the boy's living room while he and the rest of the family were watching television. McDowell found that Equation 3 provided an excellent description of the pretreatment data. The data are plotted in Figure 3, along with a least-squares fit of Equation 3 (McDowell, 1981b). As shown in the figure, the single-alternative hyperbola accounted for nearly all the variance in the data. This is especially noteworthy because the behavior occurred in an uncontrolled environment where other factors that might have influenced the behavior had ample opportunity to do so. It may be worth emphasizing that the rates of reprimanding represented in Figure 3 occurred naturally; that is, they were not experimentally arranged as were the rates of social reinforcement in Conger and Killeen's (1974) study of behavior on concurrent schedules. Thus the data in Figure 3 demonstrate the relevance of matching theory

Figure 3. Self-injurious behavior governed by Equation 3, the single-alternative hyperbola. The filled circles represent naturally occurring rates of self-injurious scratching and naturally occurring rates of verbal reprimanding. The smooth curve is a plot of Equation 3. Listed in the lower right corner are the percentage of variance accounted for by the equation, and estimates of k and r_e . The units on r_e are reprimand-equivalents per hour. (From "On the validity and utility of Herrnstein's hyperbola in applied behavior analysis" by J. J McDowell, 1981, in C. M. Bradshaw, E. Szabadi, and C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour*, pp. 311-324, Amsterdam: Elsevier/North-Holland. Copyright 1981 by Elsevier/North-Holland Biomedical Press. Reprinted by permission.)

to the natural ecology of human behavior (cf. Smith, 1983).

Evidence supporting the interpretation of r_e as the rate of extraneous reinforcement is widespread in the applied literature. Numerous investigators have reported increases or decreases in the frequency of a target behavior (R) when the reinforcement frequency for some other behavior (r_e) changed in the opposite direction. For example, Ayllon, Layman, and Kandel (1975), Ayllon and Roberts (1974), Kirby and Shields (1972), Marholin and Steinman (1977), and Winnett and Roach (1973) found that the rate of disruptive behavior (R) decreased when reinforcement was arranged for academic behavior (r_e), just as Equation 3 requires. Similarly, Brawley, Harris, Allen, Fleming, and Peterson (1969) and Lovaas, Litrownik, and Mann (1971) found that the frequency of self-stimulatory behavior decreased when appropriate behavior was reinforced. In addition, Lovaas, Freitag, Gold, and Kassarila

(1965) reported decreases in the frequency of self-injurious behavior when other behaviors were reinforced. And Patterson and Reid (1970) reported a reduction in aggressive boys' problem behavior when mutual positive reinforcement in the family was increased. The opposite effect, namely, an increase in the frequency of a target behavior, R , when r_e is decreased, has also been reported. For example, Sajwaj, Twardosz, and Burke (1972) found that the social behavior of a retarded boy increased in frequency when his conversation with the teacher was put on extinction. The foregoing examples entail changing r_e by adding or removing contingent reinforcement. But r_e can also be changed by changing the amount of free or noncontingent reinforcement in the environment. Examples of the effect of this type of change in r_e can also be found in the applied literature. For example, Brigham, Finfrock, Breunig, and Bushell (1972) found that the frequency of correct handwriting in groups of kindergarten children decreased when the children received noncontingent token reinforcement, just as Equation 3 requires. Similarly, Glynn (1970) found that accurate test performance in history and geography declined in a class of ninth-grade girls when tokens were delivered randomly. Of course, the findings supporting the interpretation of r_e as the rate of extraneous reinforcement were not discussed in terms of matching theory by the investigators who reported them. Rather, they were usually referred to as unexplained side effects of reinforcement or extinction procedures. But Equation 3 asserts that findings like these, far from being side effects, are direct consequences of changing the environmental context of reinforcement.

THERAPEUTIC APPLICATIONS OF MATCHING THEORY

Although the data reviewed in the previous section are not voluminous and do not address every aspect of the theory, they nevertheless suggest that matching theory describes human behavior in natural environments. This is a significant

achievement for the theory. McDowell (1981a, 1982) has shown that the theory also has important therapeutic implications. He noted that the theory suggests four new treatment interventions and that it can be used to improve overall treatment planning and management. McDowell (1981a) has also described how the theory can be used to regulate token economies.

Specific Treatment Interventions

Traditional methods of decreasing the frequency of a problem behavior include punishment, extinction, and differential reinforcement of not responding (DRO). Equation 3 suggests two additional interventions, namely, increasing the rate of reinforcement for a concurrently available response alternative, and increasing the rate of free or noncontingent reinforcement. Both interventions increase r_e which, according to Equation 3, should result in a decrease in the frequency of the target behavior. Notice that the effect of these two interventions cannot be understood in terms of traditional behavior analytic principles. Indeed the only basis for proposing the interventions is Equation 3. It is also important to recognize that the first intervention is different from arranging a DRO schedule. On a DRO schedule, a contingency is applied to the target behavior. If the target behavior does not occur for a specified period of time, then a reinforcer is delivered. But when reinforcement is arranged for a concurrently available response alternative, a contingency is applied to a nontarget behavior. If the nontarget behavior occurs, then a reinforcer is delivered regardless of the occurrence or non-occurrence of the target behavior. According to matching theory, this intervention affects the target behavior because it changes the environmental context of reinforcement in which that behavior occurs.

McDowell (1981a, 1982) has reported a case in which matching theory was used as described above to design a treatment regimen. The client was a mildly retarded 22-year-old male who had been dis-

missed from a sheltered workshop for threatening a staff member with a pair of scissors. The client exhibited other types of oppositional behavior as well, including noncompliance, argumentativeness, and temper tantrums, the last of which frequently escalated in intensity and sometimes culminated in assault. In addition to the incident at the workshop, the young man had a history of assaulting his mother, sometimes with a lighted cigarette. Assessment information indicated that the client's oppositional behavior at home was supported by social reinforcement from his parents. Traditional treatment procedures could not be applied in this case. Extinction was contraindicated because assault during the extinction run was an unacceptable risk. Similarly, punishment of the behavior was ruled out because no one was willing to enforce a time-out contingency with this assaultive client. The treatment procedure decided upon was based on Equation 3. Token reinforcement was arranged for behaviors unrelated to the oppositional behavior. The client earned points, which could be exchanged for money at the end of each week, for personal-maintenance behaviors like shaving, job tasks like helping with dinner, and educational tasks like reading. According to Equation 3 this intervention increases the total amount of reinforcement in the environment and so should decrease the frequency of the oppositional behavior (R), even though the contingencies directly affecting the oppositional behavior do not change. The outcome of the treatment procedure is shown in Figure 4. Before treatment the client exhibited almost daily episodes of escalating oppositional behavior. Token reinforcement of unrelated behavior reduced the frequency of the oppositional behavior by about 80%. During the ninth week of treatment a response cost (point deduction) contingency was added but, as shown in the figure, this failed to reduce the behavior much further. At the end of treatment the mother reported that the young man's oppositional behavior had decreased in intensity as well as in frequency.

Figure 4. Results of a treatment procedure based on Equation 3, the single-alternative hyperbola. The oppositional behavior of a mildly retarded adult male was treated by arranging token reinforcement for personal-hygiene, job, and educational tasks. By the eighth week of treatment the frequency of oppositional behavior had declined by about 80%. (From "On the validity and utility of Herrnstein's hyperbola in applied behavior analysis" by J. J McDowell, 1981, in C. M. Bradshaw, E. Szabadi, and C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour*, pp. 311-324, Amsterdam: Elsevier/North-Holland. Copyright 1981 by Elsevier/North-Holland Biomedical Press. Reprinted by permission.)

Two additional intervention strategies suggested by Equation 3 deal with the problem of increasing the frequency of a desired behavior. Traditionally, this has been accomplished by increasing the rate of reinforcement contingent on the behavior. But Equation 3 suggests two new methods, namely, decreasing the rate of reinforcement for a concurrently available response alternative, and decreasing the rate of free or noncontingent reinforcement. Both interventions reduce r_e which, according to Equation 3, should result in an increase in the frequency of the target behavior. The first of these intervention strategies is illustrated in the study by Sajwaj et al. (1972), which was discussed earlier. Sajwaj et al. increased the frequency of a boy's conversation with his peers by putting conversation with his teacher on extinction. Of course, Sajwaj et al. interpreted this outcome as a side effect of the extinction procedure.

Overall Treatment Planning and Management

Matching theory provides a framework for understanding, planning for, and exploiting the "side effects" of reinforcement and extinction procedures. As not-

ed earlier, matching theory asserts that these so-called side effects are in fact direct consequences of changing the environmental context of reinforcement. Obviously, any intervention that adds or removes reinforcement changes the total amount of reinforcement in the environment. According to matching theory, this change will affect not just the target behavior, but every behavior that is concurrently exhibited in that environment. There are times when this effect will cause problems. For example, suppose that two undesirable behaviors are occurring in a given environment, one at a low frequency and the other at a high frequency. Suppose also that social reinforcement is supporting both behaviors. If the objective of treatment is to suppress the high-frequency behavior, then the therapist might elect to put this behavior on extinction. But matching theory (Equation 3) suggests that this intervention may adversely affect the low-frequency behavior. The removal of reinforcement entailed by the extinction regimen reduces r_e for the low-frequency behavior, which means that its frequency will increase. A dynamically oriented clinician may call this undesirable outcome symptom substitution. But a knowledgeable behavior

analyst can prevent such an outcome by, for example, arranging additional reinforcement for a third behavior such that the removal of reinforcement entailed by the extinction regimen is made up for by the addition of reinforcement for the third behavior.

The foregoing example shows that matching theory requires the applied behavior analyst to consider the effect of a treatment regimen on all behaviors concurrently exhibited in the treatment environment. But the theory also requires the applied behavior analyst to consider how the total amount of reinforcement in the environment affects the treatment regimen. According to matching theory, a particular intervention may be more or less effective, depending on the amount of reinforcement that is already present in the environment. As illustrated in Figure 2, a given reinforcement procedure will produce a larger change in behavior if the environment is lean (small r_e) than if it is rich (large r_e). Put another way, to produce a given increment in response rate, a greater increment in reinforcement rate is required in a rich environment than in a lean one. McDowell (1981a, 1982) has given other examples of how matching theory can help the applied behavior analyst make intelligent and effective treatment decisions.

CONCLUSION

The research on matching theory discussed in this article illustrates the developmental progression from laboratory experimentation with nonhuman subjects to therapeutic applications in natural human environments. Functional relationships that are discovered in laboratory research with nonhuman subjects are verified in laboratory experiments with human subjects, then they are verified in natural human environments, and finally their therapeutic implications are drawn and tested. In the case of matching theory the first two steps have been completed successfully. The third step, namely, verifying matching theory in natural human environments, requires additional research. The evidence cited

in this article is encouraging, but it is not extensive. It may be worth noting that only Conger and Killeen (1974) actually designed an experiment to test matching theory in a natural human environment. The rest of the evidence is incidental; that is, the data were collected for other reasons. The fourth step, namely, testing the therapeutic implications of matching theory, also requires additional research.

While applied (i.e., nonlaboratory) work on matching theory languishes for lack of interested researchers, basic research on the theory is proceeding apace. For example, modifications of Equation 1, the matching equation, have been proposed and are now widely accepted (Baum, 1974, 1979, 1983; Mullins, Agunwamba, & Donohoe, 1982; Wearden & Burgess, 1982). Equation 3 has also undergone testing and development in recent years (McDowell, 1986; McDowell & Wood, 1984, 1985). Moreover, there are indications in the literature that matching theory may be subsumed by a more comprehensive mathematical account of behavior. Several candidates for a more general account are currently being developed and tested (e.g., Killeen, 1982; McDowell & Wixted, 1988). There is no doubt that applied behavior analysis lags far behind the basic science. The applied significance of matching theory indicates that it might be worthwhile to catch up.

REFERENCES

- Ayllon, T., Layman, D., & Kandel, H. (1975). A behavioral-educational alternative to drug control of hyperactive children. *Journal of Applied Behavior Analysis*, 8, 137-146.
- Ayllon, T., & Roberts, M. (1974). Eliminating discipline problems by strengthening academic performance. *Journal of Applied Behavior Analysis*, 7, 71-76.
- Baum, W. M. (1974). On two types of deviation from the matching law: Bias and undermatching. *Journal of the Experimental Analysis of Behavior*, 22, 231-242.
- Baum, W. M. (1979). Matching, undermatching, and overmatching in studies of choice. *Journal of the Experimental Analysis of Behavior*, 32, 269-281.
- Baum, W. M. (1983). Matching, statistics, and common sense. *Journal of the Experimental Analysis of Behavior*, 39, 499-501.
- Baum, W. M., & Rachlin, H. C. (1969). Choice

- as time allocation. *Journal of the Experimental Analysis of Behavior*, 12, 861-874.
- Bradshaw, C. M., Szabadi, E., & Bevan, P. (1976). Behavior of humans in variable-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 26, 135-141.
- Bradshaw, C. M., Szabadi, E., & Bevan, P. (1977). Effect of punishment on human variable-interval performance. *Journal of the Experimental Analysis of Behavior*, 27, 275-279.
- Bradshaw, C. M., Szabadi, E., & Bevan, P. (1978). Effect of variable-interval punishment on the behavior of humans in variable-interval schedules of monetary reinforcement. *Journal of the Experimental Analysis of Behavior*, 29, 161-166.
- Brawley, E., Harris, F., Allen, K., Fleming, R., & Peterson, R. (1969). Behavior modification of an autistic child. *Behavioral Science*, 14, 87-97.
- Brigham, T., Finckro, S., Breunig, M., & Bushell, D. (1972). The use of programmed materials in the analysis of academic contingencies. *Journal of Applied Behavior Analysis*, 5, 177-182.
- Carr, E. G., & McDowell, J. J. (1980). Social control of self-injurious behavior of organic etiology. *Behavior Therapy*, 11, 402-409.
- Catania, A. C. (1981). Discussion: The flight from experimental analysis. In C. M. Bradshaw, E. Szabadi, & C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour* (pp. 49-64). Amsterdam: Elsevier/North-Holland.
- Cliffe, M. J., & Parry, S. J. (1980). Matching to reinforcer value: Human concurrent variable-interval performance. *Quarterly Journal of Experimental Psychology*, 32, 557-570.
- Conger, R., & Killeen, P. (1974). Use of concurrent operants in small group research: A demonstration. *Pacific Sociological Review*, 17, 399-416.
- Cullen, C. (1981). The flight to the laboratory. *The Behavior Analyst*, 4, 81-83.
- de Villiers, P. A. (1977). Choice in concurrent schedules and a quantitative formulation of the law of effect. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior* (pp. 233-287). Englewood Cliffs, NJ: Prentice-Hall.
- de Villiers, P. A., & Herrnstein, R. J. (1976). Toward a law of response strength. *Psychological Bulletin*, 83, 1131-1153.
- Epling, W. F., & Pierce, W. D. (1983). Applied behavior analysis: New directions from the laboratory. *The Behavior Analyst*, 6, 27-37.
- Ferster, C. B. (1978). Is operant conditioning getting bored with behavior? *Journal of the Experimental Analysis of Behavior*, 29, 347-349.
- Glynn, E. (1970). Classroom applications of self-determined reinforcement. *Journal of Applied Behavior Analysis*, 3, 123-132.
- Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of frequency of reinforcement. *Journal of the Experimental Analysis of Behavior*, 4, 267-272.
- Herrnstein, R. J. (1970). On the law of effect. *Journal of the Experimental Analysis of Behavior*, 13, 243-266.
- Killeen, P. R. (1982). Incentive theory. In D. J. Bernstein (Ed.), *Nebraska symposium on motivation 1981: Vol. 29. Response structure and organization*. Lincoln: University of Nebraska Press.
- Kirby, F., & Shields, F. (1972). Modification of arithmetic response rate and attending behavior in a seventh-grade student. *Journal of Applied Behavior Analysis*, 5, 79-84.
- Lovaas, O., Freitag, G., Gold, V., & Kassorla, I. (1965). Experimental studies in childhood schizophrenia: Analysis of self-destructive behavior. *Journal of Experimental Child Psychology*, 2, 67-84.
- Lovaas, O., Litrownik, A., & Mann, R. (1971). Response latencies to auditory stimuli in autistic children engaged in self-stimulatory behavior. *Behaviour Research & Therapy*, 9, 39-49.
- Marholin, D., II, & Steinman, W. (1977). Stimulus control in the classroom as a function of the behavior reinforced. *Journal of Applied Behavior Analysis*, 10, 465-478.
- McDowell, J. J. (1981a). On the validity and utility of Herrnstein's hyperbola in applied behavior analysis. In C. M. Bradshaw, E. Szabadi, & C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour* (pp. 311-324). Amsterdam: Elsevier/North-Holland.
- McDowell, J. J. (1981b). Wilkinson's method of estimating the parameters of Herrnstein's hyperbola. *Journal of the Experimental Analysis of Behavior*, 35, 413-414.
- McDowell, J. J. (1982). The importance of Herrnstein's mathematical statement of the law of effect for behavior therapy. *American Psychologist*, 37, 771-779.
- McDowell, J. J. (1986). On the falsifiability of matching theory. *Journal of the Experimental Analysis of Behavior*, 45, 63-74.
- McDowell, J. J., & Wixted, J. T. (1988). The linear system theory's account of behavior maintained by variable-ratio schedules. *Journal of the Experimental Analysis of Behavior*, 49, 143-169.
- McDowell, J. J., & Wood, H. M. (1984). Confirmation of linear system theory prediction: Changes in Herrnstein's k as a function of changes in reinforcer magnitude. *Journal of the Experimental Analysis of Behavior*, 41, 183-192.
- McDowell, J. J., & Wood, H. M. (1985). Confirmation of linear system theory prediction: Rate of change of Herrnstein's k as a function of response-force requirement. *Journal of the Experimental Analysis of Behavior*, 43, 61-73.
- Mullins, E., Agunwamba, C. C., & Donohoe, A. J. (1982). On the analysis of studies of choice. *Journal of the Experimental Analysis of Behavior*, 37, 323-327.
- Myerson, J., & Hale, S. (1984). Practical implications of the matching law. *Journal of Applied Behavior Analysis*, 17, 367-380.
- Patterson, G., & Reid, J. (1970). Reciprocity and coercion: Two facets of social systems. In C. Neuringer & J. Michael (Eds.), *Behavior modification in clinical psychology* (pp. 133-177). New York: Appleton-Century-Crofts.
- Pierce, W. D., & Epling, W. F. (1980). What happened to analysis in applied behavior analysis? *The Behavior Analyst*, 3, 1-9.
- Poling, A., Picker, M., Grossett, D., Hall-Johnson,

- E., & Holbrook, M. (1981). The schism between experimental and applied behavior analysis: Is it real and who cares? *The Behavior Analyst*, 4, 93–102.
- Rachlin, H. C., & Baum, W. M. (1972). Effects of alternative reinforcement: Does the source matter? *Journal of the Experimental Analysis of Behavior*, 18, 231–241.
- Sajwaj, T., Twardosz, S., & Burke, M. (1972). Side effects of extinction procedures in a remedial pre-school. *Journal of Applied Behavior Analysis*, 5, 163–175.
- Smith, T. L. (1983). Skinner's environmentalism: The analogy with natural selection. *Behaviorism*, 11, 133–153.
- Wearden, J. H., & Burgess, I. S. (1982). Matching since Baum (1979). *Journal of the Experimental Analysis of Behavior*, 38, 339–348.
- Winett, R., & Roach, E. (1973). The effects of reinforcing academic performance on social behavior. *Psychological Record*, 23, 391–396.