Source of level dependent minima in rabbit distortion product otoacoustic emissions

P. F. Fahey^{a)}

Department of Physics/EE, University of Scranton, Scranton, Pennsylvania 18510

B. B. Stagner

VA Loma Linda Healthcare System, Loma Linda University Medical Center, Loma Linda, California 92357

G. K. Martin

VA Loma Linda Healthcare System, Loma Linda University Medical Center, Loma Linda, California 92357
and Department of Otolaryngology, Loma Linda University Medical Center, Loma Linda, California 92357

(Received 30 May 2008; revised 19 September 2008; accepted 27 September 2008)

Sharp level dependent minima (commonly called nulls or notches) in the distortion product otoacoustic emissions (DPOAEs) have been postulated to be due to two different mechanisms. It is shown here that the level dependent nulls in rabbit $2f_1-f_2$ DPOAEs carry the signature of the mixing of a third order nonlinear term with a fifth order nonlinear term. This suggests that the minima are not due to the mixing of signals from two different physical sites of origin, but rather are due to the nature of the nonlinearity itself. Model simulations show that null production is indifferent to several properties of nonlinear input/output functions.

© 2008 Acoustical Society of America. [DOI: 10.1121/1.3003078]

PACS number(s): 43.64.Bt, 43.64.Jb [WPS]

Pages: 3694–3707

I. INTRODUCTION

Direct and indirect measurements of the mechanical response of the inner ear show evidence of a compressive nonlinearity. This mechanical compressive nonlinearity is one source of the large dynamic range of hearing. The mechanical nonlinearity is manifested by the generation of harmonics and intermodulation distortion products (DPs) when the inner ear is driven by pure tones. The compressiveness is manifest by the saturation of the DPs and that the phase differs by π between adjacent odd order DPs. Because the DPs generated by the mechanical response of the cochlea propagate out of the inner ear via the middle and outer ears, the properties of the inner ear mechanical nonlinearity can be noninvasively studied. Signals generated within and then propagating out of the inner ear have been named otoacoustic emissions (OAEs), and when they are DPs they are labeled DPOAEs. The most easily observable, hence the most studied DPOAE, is the one whose frequency is $f_{DP}=2f_1-f_2$.

It has been observed that $2f_1-f_2$ DPOAE levels can have abrupt decreases and then equally abrupt increases when either (a) the input frequencies are scanned or, alternatively, (b) when the levels of the input tones are scanned. Commonly associated with these abrupt changes in level (or “nulls”) is an equally abrupt change in phase of close to π radians. Hence, these sharp nulls in DPOAE level resemble wave interference patterns due to two or more sources.

When the primary frequencies are swept (by varying either f_2 alone or f_1 alone or varying both at constant f_2/f_1) the DPOAEs often show patterns that resemble classical wave interferometry. Experiments have shown that there is evidence of two sources of some DPOAEs. For example, in humans, there are many measurements of “fine structure.” The fine structure consists of nulls closely spaced in frequency that modulate the overall level versus frequency pattern. This fine structure has been extensively studied and its source is thought to be understood. For example, [Stover et al. \(1996\)](#), [Brown et al. \(1996\)](#), [Knight and Kemp \(2000\)](#), and [Kalluri and Shera \(2001\)](#), among many others, made convincing arguments for two separate wave sources. [Kalluri and Shera \(2001\)](#) demonstrated the concordance of results using different techniques.

In experimental animals nulls similar to the human fine structure have also been observed. In experimental animals the signal time delays are much shorter than in humans ([Whitehead et al., 1996](#)); hence, the null frequency spacing is much larger and the appellation, “fine structure,” is not as appropriate. Using the technique of [Stover et al. \(1996\)](#), [Fahey and Allen \(1997\)](#) made the case for two separate wave sources in cats. [Bergevin \(2007\)](#) made the argument that chicken DPOAEs also show evidence of two sources. In all of the aforementioned swept frequency studies the data are consistent with the first source being the site of generation at the f_2 place and the second source being re-emission from the DP place. The rabbit preparation used in this study does not show swept frequency nulls like the animal preparations mentioned above.

In experimental animals (such as cats, gerbils, guinea pigs, and rabbits) the observation of sharp nulls in the

^{a)}Author to whom correspondence should be addressed. Electronic mail: faheyp1@scranton.edu

DPOAE level when varying the input levels of the primaries also looks like two wave mixing. One possible mechanism would be that there are two separate sources (from a generation site and also from a physically distinct re-emission site) and that the one source grows relative to the other source as level is changed. A second possibility would be within the generation mechanics of nonlinearity itself. Lukashkin and Russell (1999) modeled how a compressive nonlinearity itself can produce sharp nulls with a changing level. Lukashkin *et al.* (2002) observed this π phase change associated with a level dependent null in guinea pigs, and Whitehead *et al.* (1992a) earlier observed a π phase change associated with a level dependent nulls in rabbits.

Since both the two-site explanation, where the DP signal from the generation site grows relative to the DP signal from the re-emission site, and the single-site explanation (only a generation site) of the nulling would exhibit a π phase change at the same parameters for which the null occurs, we look here for other elements in the signature of the single-site nonlinearity induced null. Using simulations akin to Frank and Kossel (1996), Lukashkin and Russell (1999), and Fahey *et al.* (2000), it is found that one signature of the single-site mechanism, when modeled, is that the level of the $3f_1 - 2f_2$ DP has a local maximum at parameters where the $2f_1 - f_2$ null occurs and that this maximum is comparable to the level of $2f_1 - f_2$ just before or just after the null. In the rabbit data shown below this signature is observed. Another signature in simulations is that there is a line of nulling for a particular constant value of either L_1 or L_2 (where L_1 and L_2 are the levels of the input tones at f_1 and f_2) accompanied by an abrupt phase shift. In the rabbit data presented here this signature is observed. Therefore, it seems likely that rabbits present level dependent sharp nulls that are due to properties of the basilar membrane nonlinearity alone and not due to any re-emission site. It will be shown below how to toggle the properties of nonlinearities to create nulls and, thereby, it will be shown which properties of the nonlinearity are indifferent to null production.

II. METHODS

The data presented here were measured as per previously published protocols (Fahey *et al.*, 2006). Some lower level resolution data that motivated this study were presented by Stagner *et al.* (2005). The data shown here were acquired with the high level resolution of 1 dB difference between adjacent levels of both L_1 and L_2 . This high level density is useful for bringing out some features of the data shown below.

A. Subjects

Young female albino rabbits ($n=4$) were tested in a sound-treated booth while awake, confined in a standard plastic rabbit holder. The human data are from one young adult and were corroborated in another young adult. The animal studies were approved by the Animal Care and Use Committee and the humans studies by the Human Subjects Institutional Review Board of the VA Loma Linda Health-care System.

B. DPOAE measurement

The f_1 and f_2 primary tones for eliciting DPOAEs were generated by a digital signal processor (DSP) board mounted in a Macintosh personal computer and presented over ER-2 speakers. Ear canal sound pressure was measured with an ER-10 microphone, sampled at 44100 Hz, and synchronously averaged by the DSP board. 2048 point spectra over a frequency range from 0 to ≈ 22 kHz were collected at the input sound pressure levels (SPLs) shown in the figures (each of L_1 and L_2 ranging from 25 to 80 dB SPL in 1 dB steps). For measurements made at $f_2/f_1=1.0125$, a 4096 point spectrum was used. The DPOAE levels and phase were extracted from the spectra. A control run in a 1.5 cc hard-walled cavity produced no consistent artifacts at any of the emission frequencies measured.

III. MODEL RESULTS

The goal of the modeling study is twofold. First, signature features of null creation that are due to the form of nonlinear input/output (I/O) functions will be numerically demonstrated. The following will be shown.

- (1) There are null lines that are parallel to each of the two input level axes. (The intersection of these lines when the DP is plotted as a function of L_1 and L_2 is called a null spot in the text that follows.)
- (2) There are phase discontinuities of π radians at levels that produce the nulls.
- (3) The level of the fifth order DP at $3f_1 - 2f_2$ is comparable to the level of the $2f_1 - f_2$ DP just before the occurrence of a null. At the levels at which a $2f_1 - f_2$ null spot occurs $3f_1 - 2f_2$ is near a local maximum. Likewise a null in the $3f_1 - 2f_2$ DP is associated with a local maximum in the $4f_1 - 3f_2$ DP, etc. Obviously, the same correlations also apply to the upper sidebands ($2f_2 - f_1$, $3f_2 - 2f_1$, etc.).
- (4) There are nonlinear I/O functions that can generate multiple null spots.

Second, the effect of various attributes of I/O functions on null production will be explored. For example, the modeling shows the following.

- (1) Contrary to implications in literature, some purely odd symmetry saturating I/O functions produce nulls.
- (2) Not all saturating nonlinearities with even symmetry produce nulls.
- (3) Abruptness of the onset of clipping is not a determinant of null production.
- (4) Nulls can be produced by I/O functions that are linear for one polarity of the signal and saturating for the other polarity. That is, both polarities of the signal need not be clipped.
- (5) The nulls in the DP of order n do not occur at the same values as the zeros in the n th derivatives of the I/O functions.

A. Saturating nonlinearities

Hyperbolic tangent type functions, $y=A \tanh(f(x-a)) + B$ where a , A , and B are constants and f is a polynomial,

FIG. 1. (Color online) The nonlinearities $y=\tanh(x)$ and $y=\tanh(x+x^3)$ are plotted in the top lefthand side, and the same nonlinearities, except with linear output for negative signal polarities, are shown in the bottom lefthand side. On the righthand side are the nonlinearity $y=\tanh([x-2]/2)+\tanh(1)$ in the top and that same function except that now it has been broken up so that only one polarity of the signal is clipped. For all of these I/O functions both the function and its first derivative are continuous.

constitute a fairly general class of saturating nonlinear I/O functions that have often been used to model inner ear nonlinear responses (Weiss and Leong, 1985; Frank and Kossil, 1996; Lukashkin and Russell, 1999; Fahey *et al.*, 2000). [Kossil and Coro (2006) also used these functions, and these functions blended with an expansive cubic term.] This class of functions [with $f(x-a)=(x-a)/2$] was successfully used by Weiss and Leong (1985) to model hair cell receptor potentials as a function of stereocilia deflection angle. Functions of this type are common in the description of the physics of systems having two equilibrium states that are separated in energy, e.g., the thermodynamic description of

the magnetic moment of paramagnetic materials often studied in introductory thermal physics courses (e.g., Feynman *et al.*, 1964). In the context of the mammalian inner ear, the two states would be assumed to be the open and closed states of stereociliary motion activated outer hair cell ion channels. Therefore there is a natural saturation when all channels are (a) open or (b) closed. The specifics of the transition from outer hair electrical properties to basilar membrane mechanical properties is not known. However, this mechanical nonlinearity would show saturation as long as it is the outer hair cell electrical response that is somehow controlling the nonlinearity. If the energy states associated with the channels are

FIG. 2. (Color online) Graphs of the output of three DPs as a function of the amplitude of the two input tones. The input is $x=A_1 \cos(\omega_1 t)+A_2 \cos(\omega_2 t)$, where the A 's are the tone amplitudes and the ω 's are the angular frequencies. Here $A_1=A_2$. On the lefthand side are DPs for the single-polarity I/O function, $y=\tanh(x)$ if $x \geq 0$ and linear otherwise, and in the middle for the single-polarity I/O function, $y=\tanh(x+x^3)$ if $x \geq 0$ and linear otherwise; and on the righthand side the two-polarity clipper $y=\tanh([x-2]/2)+\tanh(1)$. The middle and the righthand sides show nulls in the three DPs plotted. The null in the $2f_1-f_2$ DP is near the local maximum of the $3f_1-2f_2$ DP and the null in the $3f_1-2f_2$ DP is associated with a maximum in the $4f_1-3f_2$ DP.

mechanically nonlinear, then arguments of the tanh would be nonlinear, containing one or more terms of the form x^n where n is an integer.

Weiss and Leong (1985) showed that functions of this class can exhibit level dependent nulls in harmonics, and Lukashkin and Russell (1999) and Fahey *et al.* (2000) showed level dependent nulls in intermodulation DPs. Since the A above is just a scale factor (twice the difference between the asymptotic limits as $x \rightarrow \infty$ and $x \rightarrow -\infty$), it is no loss of generality to set $A=1$. In order to guarantee that the output is zero at zero input $B=\tanh(a/2)$. Adjusting the a parameter about zero gives the nonlinearity some even symmetry. It will be shown here that I/O functions with purely odd symmetry can also exhibit level dependent nulling. However, these purely odd symmetry functions producing nulling are hyperbolic tangents with nonlinear arguments.

The forms of some model nonlinear I/O functions are shown in Fig. 1. On the top lefthand side are the odd symmetry nonlinearities $y=\tanh(x)$ and $y=\tanh(x+x^3)$. These I/O functions clip both the positive and the negative polarity of an input signal. On the bottom lefthand side are two single-polarity nonlinearities $y=\tanh(x)$ if $x \geq 0$ but $y=x$ otherwise and $y=\tanh(x+x^3)$ if $x \geq 0$ but $y=x$ otherwise. The nonlinearities on the bottom clip only the positive polarity of the

signal. On the top righthand side is the nonlinearity $y=\tanh([x-a]/2)+b$ where $a=2$ and $b=\tanh(a/2)$. This nonlinearity [in the form $y=(b+1)-2/(1+e^{a-x})$] was used in the study of Fahey *et al.* (2000). On the bottom righthand side this same function has been broken up into two functions, each of which clips only one polarity of the signal.

Some of these nonlinearities exhibit DP nulls and some do not. Both the two-polarity clipper $y=\tanh(x)$ and its one-polarity version [$y=\tanh(x)$ if $x \geq 0$ and $y=x$ otherwise] do not exhibit nulls. Neither does the one-polarity nonlinearity, $y=\tanh([x-2]/2)+\tanh(1)$ if $x \leq 0$ and $y=x$ otherwise. All of the other I/O functions do show nulling in the $2f_1-f_2$ DP and higher order DPs. [Note that $y=\tanh([x-2]/2)+\tanh(1)$ if $x \leq 0$ and $y=x$ otherwise clips at lower levels than $y=\tanh([x-2]/2)+\tanh(1)$ if $x > 0$ and $y=x$ otherwise. Furthermore, the infinite peak clipper, $y=\text{sgn}(x)$ ($+1$ if $x \geq 1$ and -1 otherwise) does not produce nulls in the DPs. Hence, null production cannot be used as a measure of the level and severity of peak clipping.]

Figure 2 shows the $2f_1-f_2$, $3f_1-2f_2$, and $4f_1-3f_2$ DPs at the output of three different example nonlinearities. And it is evident that the null occurs in the third order DP ($2f_1-f_2$) when the fifth order DP ($3f_1-2f_2$) has grown to roughly

FIG. 3. Contour graphs of the $2f_1 - f_2$ level output for the $y = \tanh(x + x^3)$ nonlinearity as a function of the two levels L_1 and L_2 . The lines of nulling parallel to each of the input level axes are evident.

the same size as the third order DP and the null in $3f_1 - 2f_2$ occurs near the maximum of the seventh order $4f_1 - 3f_2$ DP. In the bottom panels of the figure, it is seen that there is a discontinuity of π radians in the phase of the DP at the levels producing the null. Furthermore, it is evident that clipping on only one polarity of the signal is sufficient for null production. Were the nonlinearities of the form $y = \tanh(x)$ and $y = \tanh(x + x^3)$, with symmetric clipping on both polarities of the signal, the figure would look the same except for a doubling in DP amplitudes. Simulations with I/O function $y = \tanh([x - 2]/2) + \tanh(1)$; $x \geq 0$ and linear otherwise show nulling. But $y = \tanh([x - 2]/2) + \tanh(1)$; $x \leq 0$ and linear otherwise do not show nulling.

One of the signatures of nulling in the $2f_1 - f_2$ DP generated by the structure of the nonlinearity alone is that the fifth order DP ($3f_1 - 2f_2$ DP) is at roughly the same magnitude as the $2f_1 - f_2$ DP just before the null. A second signature of null production is a π phase discontinuity. A third signature of nulling is null lines and π phase discontinuities parallel to each of the input level axes as is shown in Fig. 3. Here the amplitude and phase of the third and the fifth order DPs are plotted as both input tone levels are varied. (The amplitude graph shows the same nulling features as Fig. 5 of

Lukashkin and Russell, 2002.) The input levels, L_1 and L_2 , are in decibels and the output DP levels are also in decibels.

The I/O function $y = \tanh(\alpha x + \beta x^5)$ shows a fourth signature of nulling—two sets of intersecting null lines meeting in two null spots. The placing of the spots relative to each other can be manipulated by changing the constants α and β . For example, Fig. 4 shows the pair of null spots for $2f_1 - f_2$ and $3f_1 - 2f_2$ with their associated π phase discontinuities (when $\alpha = 1 = \beta$). The same patterns shown in Figs. 3 and 4 exist for the upper sideband DPs ($2f_2 - f_1$ and $3f_2 - 2f_1$) as long as one just switches the labels on the axes. Figure 8(f) in the study by Kossl and Coro (2006) shows a different example of a two null spot I/O function.

Always correlated with nulling patterns in the amplitude are discontinuities in the phase of π radians (or half a cycle). Also, there is a slight asymmetry in L_1 - L_2 space. These patterns wherein the null follows along either a constant L_1 or constant L_2 lines would constitute another signature of single-site nonlinearity induced nulling. Where the $L_1 = \text{constant}$ null line intersects the $L_2 = \text{constant}$ null line is designated as the null spot. In the data, null spots can be more evident than null lines because the DPOAEs can come from an array of primary generation sites.

FIG. 4. Contour graphs of the $2f_1-f_2$ output for the $y=\tanh(x+x^5)$ nonlinearity as a function of the two levels L_1 and L_2 . The lines of nulling parallel to each of the input level axes are evident.

From the modeling examples above, it is evident that nulls in the DP amplitude can be produced by the structure of the nonlinear I/O function alone (Frank and Kossel, 1996; Lukashkin and Russell, 1999; Fahey *et al.*, 2000). Furthermore, the signature of a null in the $2f_1-f_2$ DP produced by a single-site process contains (a) null lines that are parallel to the two level axes (Lukashkin and Russell, 2002), (b) phase discontinuities of π radians at levels that produce the nulling (Lukashkin *et al.*, 2002), (c) the presence of the fifth order DP at $3f_1-2f_2$ at levels comparable to the $2f_1-f_2$ levels just before or just after the nulling (Fahey *et al.*, 2000, in Figs. 4, 7, and 8), and (d) the possibilities of multiple sets of null lines and null spots.

B. Controlling null production

1. Usefulness of Taylor series

It is possible to gain some insight regarding nulling by examining the Taylor series (expanded about zero) of the functions used in the plotted simulations.

$$\tanh(x + \alpha x^3) \approx x + \left(-\frac{1}{3} + \alpha\right)x^3 + \left(\frac{2}{15} - \alpha\right)x^5 + \left(\frac{2}{3}\alpha - \frac{17}{315} - \alpha^2\right)x^7 + O(x^8). \quad (1)$$

So, if $\alpha=0$,

$$\tanh(x) \approx x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + O(x^8), \quad (2)$$

and if $\alpha=1$,

$$\tanh(x + x^3) \approx x + \frac{2}{3}x^3 - \frac{13}{15}x^5 - \frac{122}{315}x^7 + O(x^8). \quad (3)$$

The asymmetric hyperbolic tangent's Taylor representation up to the fifth order term is

$$\tanh([x-2]/2) + \tanh(1) \approx \frac{5}{24}x + \frac{3}{38}x^2 + \frac{1}{76}x^3 - \frac{1}{636}x^4 - \frac{1}{709}x^5 + O(x^6). \quad (4)$$

The $2f_1-f_2$ DP at the lowest levels is produced by the cubic term in the Taylor series. Comparing the cubic terms for $y=\tanh(x+x^3)$, $y=\tanh(x)$, and $y=\tanh([x-2]/2) + \tanh(1)$ it is evident that for values of $|x| \ll 1$ the relative sizes of the amplitudes of $2f_1-f_2$ as shown in Fig. 2 are predicted by the relative sizes of the coefficients of x^3 . Also,

the phases for $|x| \ll 1$ are predicted by the sign of the x^3 term. If the sign is positive, then the phase is zero; if negative, the phase is $\pm \pi$ radians. Such is also the case for the correlation of the $3f_1 - 2f_2$ DP and the x^5 term, etc.

Consider the two nonlinear functions $y = \tanh(x + \alpha x^3)$ and $y = \tanh([x - \beta]/2) + \tanh(\beta/2)$. In the first case the parameter α changes the shape of the I/O function and in the second case the parameter β shifts the function along the y axis while preserving the shape and guaranteeing that $y=0$ at $x=0$. In both cases, the change of either α or β can change the function from a function that produces nulls in the DPs to one that does not. [Figure 5 in Meenderink and van Dijk, 2005 shows a similar result for a function of the class $y = \tanh([x - \beta]/2) + \tanh(\beta/2)$.] In the simulations above, when $\alpha=0$ and $\beta=0$, there were no nulls and when $\alpha=1$ and $\beta=2$ nulls were produced.

Varying α , it is found that there is a critical value, α_c , such that for values of $\alpha > \alpha_c$ the cubic term in the Taylor series of $y = \tanh(x + \alpha x^3)$ has a positive coefficient, for values of $\alpha < \alpha_c$ the coefficient of the cubic term is negative, and for $\alpha = \alpha_c = 1/3$ the coefficient is zero. The same situation applies for $\beta = \ln(2 + \sqrt{3}) = \beta_c$. When $\beta > \beta_c$, the coefficient of the cubic term in the Taylor series of $y = \tanh([x - \beta]/2) + \tanh(\beta/2)$ is positive, for $\beta < \beta_c$ the coefficient is negative, and it is zero at $\beta = \beta_c$. So, at $\alpha = \alpha_c$ and $\beta = \beta_c$, the third order DP, $2f_1 - f_2$, is produced by higher order terms—primarily the fifth order term of the Taylor series. For nulling of the $2f_1 - f_2$ DP to occur for these functions $\alpha > \alpha_c$ and $\beta > \beta_c$. (Does this mean that nulling occurs for every function where the third order term in the Taylor series is positive? No, there are many counterexamples where the third order term is negative and nulling also occurs.) Because physiological changes to the inner ear that affect either shape of the I/O nonlinearity or that shift the zero (change the offset) of the nonlinearity would be able to change a preparation from one that exhibits nulling to one that does not. A good example of this was shown by Whitehead *et al.* (1992b) where the application of diuretics reversibly changed the level dependent nulling.

As long as the Taylor series is convergent, the origin of nulls is easily understood by the cancellation of the terms in the series. Equation (5) of Engbretson and Eldredge (1968) shows the first three terms of the Taylor series expansion for the amplitude of the $2f_1 - f_2$ DP in terms of the third, fifth, and seventh derivatives evaluated at zero and the input amplitudes, A_1 and A_2 . Let A_{DP} be the amplitude of the DP. Then

$$A_{DP} \approx A_1^2 A_2 \left[\frac{3}{4} a_3 + \frac{5}{8} a_5 (2A_1^2 + 3A_2^2) + \frac{105}{64} a_7 (A_1^4 + 4A_1 A_2^2 + 2A_2^4) + \dots \right], \quad (5)$$

where a_n is the n th derivative of the I/O function evaluated at zero and divided by $n!$. For small values of $A_1 = A_2 = A \leq 0.5$ the cancellation between the first two terms in the Taylor series expansion above occurs very near the values shown in the simulations for $y = \tanh(x + x^n)$ where $n=3, 5$ and $x=A$. This is the same lesson as Fig. 2 where the nulling occurs due to the growth of the next highest order odd term. If we add the ninth order term to the series above,

$\frac{315}{320} a_9 [2A_1^6 + 15A_1^4 A_2^2 + 20A_1^2 A_2^4 + 5A_2^6]$, the resulting series yields the value of the null almost exactly for $y = \tanh(x + x^3)$ and $y = \tanh(x + x^5)$.

2. Zeros of I/O functions' derivatives

The experiments of Bian (2004) show minima in the second and third order DPOAEs that correlate with the zeros in the respective derivatives of a model I/O function. In these experiments the DPOAEs are measured at various phases of a very low frequency bias tone. The possibility that the nulls in the n th order DPOAEs in this study are correlated with the zeros in the respective n th derivatives of the I/O functions was examined. They do not. For example, $y = \tanh(x)$ has no nulls in the intermodulation DPs but all of the odd order derivatives have zeros. For the I/O functions exhibiting nulls in the odd order DPs, the nulls occur at different input values ($A_1 = A_2 = x$) than the zeros of the respective derivatives. But,

FIG. 5. DPOAE data for the $2f_1 - f_2$ level and phase (given by the direction of the arrows) in the top panel and the phase alone in the bottom panel as a function of the two input levels L_1 and L_2 . The lines of nulling parallel to the L_1 and L_2 axes are evident in the level as is an abrupt π phase discontinuity at about $L_1 \approx 67$ dB SPL and also $L_2 \approx 62$ dB SPL. Then, there is a second less abrupt π phase change in the mid-70 dB SPL region. Note that the phase in the figures is expressed in units of cycles. In the bottom panel, the phase has been discretized into four contour levels. The data density in this figure and the other data figures is 1 dB between adjacent levels. Here $f_2/f_1 = 1.05$.

FIG. 6. $2f_1 - f_2$ DPOAE level and phase as functions of the two input levels L_1 and L_2 . The lines of nulling and phase discontinuities parallel to the level axes are evident or hinted across a broad frequency range ($2 \text{ kHz} < f_2 < 11 \text{ kHz}$). Here $f_2/f_1 = 1.25$. The phase has been discretized into four contour levels in order to emphasize the phase transition patterns.

interestingly, for the model functions used in this study, the first nulls in the third order DPs are near the values of the zeros in the second derivatives. In the [Bian \(2004\)](#) experiments the offset of the I/O function is being controlled by the bias tone. That is, the I/O function is dynamically changed. In the experiments reported here the data are consistently a stationary I/O function.

IV. THE DATA

What features in the model results can one find in the data? The model shows the intermodulation DP output from naked nonlinearities (just the I/O functions—no filtering on

the input or on the output) but the DPOAE data are filtered and summed signals of intermodulation DPs are generated by a continuous section of a nonlinear basilar membrane. The input to the nonlinearity is filtered by the basilar membrane and the intermodulation DPs created by the nonlinearity are filtered first by the basilar membrane and then by the middle ear. These signals are summed in the ear canal. However, some of the signatures shown in the model should (and do) survive the processing of the DPs from the naked nonlinearities. In [Fig. 5](#) below, it is evident that there are null lines in level and correlated phase discontinuities in the $2f_1 - f_2$ DPOAE that are parallel to the L_1 and L_2 axes. This

FIG. 7. Shown here are the $3f_1-2f_2$ DPOAE level and phase (direction of arrows) measured at the same time as the $2f_1-f_2$ DPOAE data in Fig. 6.

amplitude nulling and phase discontinuity pattern in the DPOAE look very much like Fig. 3. Here $f_2/f_1=1.05$, so the two primaries are close in frequency and at the sites of basilar membrane nonlinearity would have almost the same amplitudes. When f_2/f_1 is small, both of the amplitude null lines are often prominent. Unfortunately for f_2/f_1 small, the DPOAE amplitude is also small. The phase data suggest that there is a second set of intersecting null lines and there is a hint of this in the amplitude data. In Fig. 5 (especially visible in the bottom panel) there are abrupt π phase discontinuities at $L_1 \approx 67$ dB SPL and $L_2 \approx 65$ dB SPL and then there are gradual π phase changes in the mid-70 dB SPL range. Given that the higher levels of input result in detuning of the basilar membrane and would be expected to increase emission from multiple sources, one would expect to smear out the abrupt phase change expected from the I/O function simulations.

Most often $2f_1-f_2$ DPOAEs are studied when $f_2/f_1 \approx 1.2-1.25$ because in this ratio range the amplitudes are maximum. Just such data are seen in Fig. 6. These amplitude data for rabbits are similar to the amplitude data for guinea pigs shown in Fig. 6 of [Lukashkin and Russell, 2002](#). Both the level nulling and the phase discontinuities do not exactly

match the model pattern in Fig. 3. However, the similarities—in particular, phase transitions parallel to the L_1 axis—with the model figure are evident. This figure also shows that the level dependent nulling with associated phase discontinuity is found over a wide frequency range in the rabbit ($2 \text{ kHz} < f_2 < 11 \text{ kHz}$). It should be noted that the phase contour plots have been generated with four contour levels in order to emphasize the larger phase changes, which emphasize the patterns. Necessarily, between contour levels, there are small phase changes that are not shown. Given that the DPOAE signal is generated over a region of the basilar membrane where the effective input levels would be varying with respect to location, it is not surprising that the null lines would wash out a bit. Referring to Fig. 3, in the vicinity of a null line, the phases have only two values (0 or π), while the levels have many values. If the dominant basilar membrane sources are mostly on one side or on the other of the null line, the phase pattern will look like the model while the null line would become obscured. Notice that the $f_2=2, 4$, and 8 kHz data suggest that there are two sets of intersecting null lines with two null spots.

FIG. 8. Shown here are the $2f_2-f_1$ DPOAE level data measured at the same time as the $2f_1-f_2$ DPOAE data in Fig. 6.

At the same time that the $2f_1-f_2$ DPOAE was measured the $3f_1-2f_2$ DPOAE and the $2f_2-f_1$ DPOAE were also measured. The level data from these measurements are shown in Figs. 7 and 8, respectively. Examining Figs. 6 and 7 together shows that the amplitude null spots (where the null line parallel to L_1 intersects the null line parallel to L_2) occur in the vicinity where the amplitudes of the $3f_1-2f_2$ are near their maxima. This is reminiscent of Figs. 2 and 3. The level of the $3f_1-2f_2$ DPOAE will be decreased relative to what would be expected from the model simulations because of the basilar membrane filtering. At the site of $3f_1-2f_2$ DP creation this signal is a lower frequency tone than the $2f_1-f_2$ tone, so that it does not couple as well into the basilar membrane as its sibling the $2f_1-f_2$ DP. In order to lessen the differential effects of basilar membrane filtering on the primary tones (and also at a higher input level on the DPs), one may use smaller values of f_2/f_1 , such as $f_2/f_1=1.0125$. The correlation between null spots and a local maximum in the next highest order DP can be seen in Fig. 9 where there are four orders of odd upper sideband DPOAEs that show this correlation. For the $2f_2-f_1$ emission the null occurs at approximately L_1

$=L_2=70$ dB SPL. In the $3f_2-2f_2$ panel $L_1=70=L_2$ is on the peak. In this panel the null is at $L_1=70$, $L_2=72$ dB SPL. In the $4f_2-3f_1$ panel $L_1=70$, $L_2=72$ is on the peak and so on for the $5f_2-4f_2$ panel. Note that the phase at the lower level local maxima alternates in sign from one order DPOAE to the next just as is shown in the computer simulations. As would be expected from the modeling there would be very similar nulling patterns in the upper sideband DPOAEs as in the lower sideband DPOAEs, and this is apparent in Figs. 8 and 9. At some values of f_2 the correlation of nulls and local maxima across orders was much more evident in the phase data than in the level data.

The correlations seen in the modeling results are generally well replicated in the experimental results. The modeling shows DP creation from nonlinearities defined by I/O functions. But DPOAEs result from a summation of DPs that have been filtered upon creation by the basilar membrane and whose input primaries were filtered by the basilar membrane before mixing in the basilar membrane nonlinearity. Also, there is the potential of some filtering due to the middle and outer ear reverse transfer functions that could be

FIG. 9. The amplitudes of $2f_2 - f_1$, $3f_2 - 2f_1$, $4f_2 - 3f_1$, and $5f_2 - 4f_1$ DPOAEs as functions of L_1 and L_2 show a general correlation between a null spot and a relative maximum of the next highest order. The amplitude of the maximum of the higher order is approximately at the level of the lower order just before the null spot. $f_2/f_1 = 1.0125$. The phase at the maxima flips by π between adjacent orders. Notice that the range of input levels varies from one panel to the next.

important when comparing DPOAEs of different orders. Given the processing of the DPs to make DPOAEs, the correspondence of experimental and model results seems sound. Any putative filtering effects of the basilar membrane and middle ear can be minimized in the limit $f_2/f_1 \rightarrow 1$.

We conclude that the correlation of features in the data with the features in the model presents a compelling case that the level dependent nulls in $2f_1 - f_2$ are due to the structure of the basilar membrane nonlinear I/O function.

V. DISCUSSION

A. Similar results in other species

Rabbits are not alone in showing some evidence of level dependent nulling due to the nature of their nonlinear mechanical response. In cats, Fig. 7 of Fahey and Allen, (1997) shows a level dependent null with an abrupt π phase change—the nulling and associated phase change were not due to mixing from a second source. As mentioned previously, nulls in amplitude with associated π radian discontinuities in phase have been observed in pigmented guinea pigs (Lukashin *et al.*, 2002). The studies in gerbil by Mills and Rubel (1994) and Mills *et al.* (1993) show evidence of level dependent nulling with some examples of null lines parallel to either the L_1 or the L_2 axis. Gerbils have also been

shown to exhibit nonlinearity generated nulling in either the manipulated cochlea (Frank and Kossel, 1996) or the very low frequency biased cochlea (Frank and Kossel, 1996; Bian, 2004). However, the inner ear pressure measurements of Dong and Olson (2005) in gerbils over the range 60–80 dB SPL show no evidence of nulling. Their Figs. 10 and 11 generally show that the phase response of each of the primary tones near the probable site of DP creation is about 1/2 cycle out of phase with the two third order DPs at frequencies $2f_1 - f_2$ and $2f_2 - f_1$ and is in phase with the fifth order DPs at frequencies $3f_1 - 2f_2$ and $3f_2 - 2f_1$. This would be consistent with an I/O function that does not produce nulls, such as $y = \tanh(ax)$. Since for both $y = \tanh([x-2]/2) + \tanh(1)$ and $y = \tanh(x+x^3)$ the cubic term in the Taylor series is positive as is the linear term, the functions would only be consistent with the Dong and Olson (2005) data if the levels used in their study (60, 70, and 80 dB SPLs) were above the nulling levels for both $2f_1 - f_2$ and $3f_1 - 2f_2$. In chinchillas, Rhode (2007) showed that nulls in both the $2f_1 - f_2$ DPOAE and the DP measured on the basilar membrane occur at the same values of the input levels. In frogs, DPOAE amplitude patterns similar to some of the data shown here have been observed (Meenderink and van Dijk, 2005). However, the sharp π phase discontinuity is missing,

FIG. 10. Shown here are the $2f_1-f_2$ DPOAE level data measured in a human with the same apparatus as the rabbit measurements. The DPOAE phases are depicted by the directions of the arrows. $f_2/f_1=1.2$.

although there is sometimes seen a gradual change in phase of π radians. Meenderink and van Dijk (2005) attributed their data to two different nonlinear regimes rather than to one physical nonlinearity. Level dependent nulling has been well characterized in moths by Kossl and Coro (2006). Therefore, this phenomenon would appear to be a fairly general feature of auditory nonlinearities over a range of animal preparations.

B. Example of a preparation where there is no evidence of level dependent nulling

Figure 10 shows human $2f_1-f_2$ DPOAEs measured with the same apparatus as used in the the rabbit DPOAE measurements. Here are level and phase data presented at four values of f_2 with $f_2/f_1=1.2$ and there is no evidence or nulling of associated π phase discontinuities. The data obtained when $f_2/f_1=1.05$ also show no level dependent nulling at the levels shown. Meinke *et al.* (2005) presented data showing a hint of nonlinearity induced nulling at $L_2 > 75$ dB SPL for $2f_1-f_2$ DPOAEs measured during binaural stimulation in humans. No similar pattern is apparent in Fig. 10. There is frequency dependent nulling in our human subject that has been sufficiently explained by Kalluri and Shera (2001) and

confirmed by others. This nulling in humans is explained by assuming two sources of the DPOAE—one at the f_2 place and the other at the f_{DP} place.

C. Utility of null lines and null spots

1. Site of generation of $2f_2-f_1$

Comparing the amplitude null lines for $2f_1-f_2$ in Fig. 6 with the $2f_2-f_1$ amplitude null lines in Fig. 8 it is evident that the null lines for $2f_2-f_1$ occur at higher levels than those of $2f_1-f_2$. The physics of the cochlea would suggest that there are two possible routes for $2f_2-f_1$ generation. The signal could be generated at the f_2 place and then excite the basilar membrane as an evanescent signal. This signal would drive the $2f_2-f_1$ place and then become a propagating wave and DPOAE. Alternatively, the two primaries could directly excite the $2f_2-f_1$ place—of course they would be attenuated relative to their excitation at f_2 . In this route $2f_2-f_1$ is made directly at its own place. For the first route the null lines should be at the same values of L_1 and L_2 for both $2f_1-f_2$ and $2f_2-f_1$. For the second route the null lines would be at larger values of L_1 and L_2 for $2f_2-f_1$ than for $2f_1-f_2$. The

data, while not extensive, favor the direct generation of $2f_2 - f_1$ at its own characteristic place, supporting the conclusion in [Martin et al., 1987](#).

2. Internal cochlear calibration

Because the null lines and the null spots are determined by the levels of the input tones at the site of the nonlinearity, once this level is known it can serve as a calibration marker for the level of the tone at the site of the nonlinearity. For example, if $L_1 = L_2$ were chosen to produce nulling in the $2f_1 - f_2$ DPOAE and that was done as the frequencies of the primaries were changed at $f_2/f_1 = \text{constant}$ sweep, then, assuming a scaling symmetric cochlea, one would have a relative measure of how much energy as a function of frequency traverses the middle ear into the cochlea.

D. Can I/O induced nulling produce the appearance of frequency dependent nulling?

Yes. As the modeling (confirmed by the data) show, I/O induced nulling is sensitive to small changes in the levels of the input tones at the site of the nonlinearity. If one or both of the input levels are near its nulling threshold and if that level is also sensitive to a change in frequency, then a frequency change will induce a null. Hence, nonlinearity induced nulling can masquerade as frequency induced nulling. This has been observed in rabbits at values of $f_2/f_1 < 1.1$ when the levels of the primary tones are around 60 dB SPL. The “frequency” dependent nulling goes away with slight changes of level. Using the technique of [Stover et al. \(1996\)](#), it can be shown that there are not two sources with different time delays.

VI. CONCLUSION

This study is basically two experimental studies—one experimental system is the rabbit and the other is the computer. The computer experiments were simulations wherein several properties of saturating I/O functions were tested in order to see if they were either necessary or sufficient for null production. The hope was that the simulations would reveal the way to a general theory of null production in saturating I/O functions. This did not happen.

However, given the correlation between several features in the model simulations and in the rabbit DPOAE data, it is concluded that the nature of the nonlinear I/O function is the likely explanation of the level dependent DPOAE nulling in rabbits. Furthermore, the modeling shows that the presence of nulling in the data is not a determinant of the I/O function shape, the presence or absence of even symmetry, and the asymmetry or severity of clipping. When nulling is due to the nature of the nonlinearity, this nulling in a given odd order DPOAE correlates with a local maximum in the next highest odd order DPOAE.

ACKNOWLEDGMENTS

The authors thank C. Bergevin, C. Shera, and the reviewers for suggestions on improving both the content and expression of this study. This research was supported in part

by funds from the Public Health Service (DC000613 and DC003114) and Department of Veterans Affairs (VA/RR&D C4494R).

- Bergevin, C. (2007). “Comparative approaches to otoacoustic emissions: Towards an understanding of why the ear emits sound,” Ph.D. thesis, Speech and Hearing Bioscience and Technology Harvard-MIT Division of Health Sciences and Technology, Massachusetts Institute of Technology, Cambridge, MA.
- Bian, L. (2004). “Cochlear compression: Effects of low-frequency biasing on quadratic distortion product otoacoustic emission,” *J. Acoust. Soc. Am.* **116**, 3559–3571.
- Brown, A. M., Harris, F. P., and Beveridge, H. A. (1996). “Two sources of acoustic distortion products from the human cochlea,” *J. Acoust. Soc. Am.* **100**, 3260–3267.
- Dong, W., and Olson, E. S. (2005). “Two tone distortion in intracochlear pressure,” *J. Acoust. Soc. Am.* **117**, 2999–3015.
- Engelbreton, A. M., and Eldredge, D. H. (1968). “Model for the nonlinear characteristics of cochlear potentials,” *J. Acoust. Soc. Am.* **44**, 548–554.
- Fahey, P. F., and Allen, J. B. (1997). “Measurement of distortion product phase in the ear canal of the cat,” *J. Acoust. Soc. Am.* **102**, 2880–2891.
- Fahey, P. F., Stagner, B. B., and Martin, G. K. (2000). “Nonlinear interactions that could explain distortion product interference response areas,” *J. Acoust. Soc. Am.* **108**, 1786–1802.
- Fahey, P. F., Stagner, B. B., and Martin, G. K. (2006). “Mechanism for bandpass frequency characteristic in distortion product otoacoustic emission generation,” *J. Acoust. Soc. Am.* **119**, 991–996.
- Feynman, R. P., Leighton, R. B., and Sands, M. (1964). *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA), Vol. II, Chap. 35.
- Frank, G., and Kossil, M. (1996). “The acoustic two-tone distortions $2f_1 - f_2$ and $f_2 - f_1$ and their possible relation to changes in the operating point of the cochlear amplifier,” *Hear. Res.* **98**, 104–115.
- Kalluri, R., and Shera, C. A. (2001). “Distortion-product source unmixing: A test of the two-mechanism model for DPOAE generation,” *J. Acoust. Soc. Am.* **109**, 622–637.
- Knight, R. D., and Kemp, D. T. (2000). “Indications of different distortion product otoacoustic emission mechanisms from a detailed f_1, f_2 area study,” *J. Acoust. Soc. Am.* **107**, 457–473.
- Kossil, M., and Coro, F. (2006). “ L_1, L_2 maps of distortion-product otoacoustic emissions from a moth ear with only two auditory receptor neurons,” *J. Acoust. Soc. Am.* **120**, 3822–3831.
- Lukashkin, A. N., Lukashkina, V. A., and Russell, I. J. (2002). “One source for distortion product otoacoustic emissions generated by low- and high-level primaries,” *J. Acoust. Soc. Am.* **111**, 2740–2748.
- Lukashkin, A. N., and Russell, I. J. (1999). “Analysis of the $f_2 - f_1$ and $2f_1 - f_2$ distortion components generated by the hair cell mechanoelectrical transducer: Dependence on the amplitudes of the primaries and feedback gain,” *J. Acoust. Soc. Am.* **106**, 2662–2668.
- Lukashkin, A. N., and Russell, I. J. (2002). “Modifications of a single saturating non-linearity account for post-onset changes in $2f_1 - f_2$ distortion product otoacoustic emissions,” *J. Acoust. Soc. Am.* **112**, 1561–1568.
- Martin, G. K., Probst, P., Scheinin, S. A., Coats, A. C., and Lonsbury-Martin, B. L. (1987). “Acoustic distortion products in rabbits. II. Site of origin revealed by suppression and pure-tone exposures,” *Hear. Res.* **28**, 191–208.
- Meenderink, S. W. F., and van Dijk, P. (2005). “Characteristics of distortion product otoacoustic emissions in the frog from L_1, L_2 maps,” *J. Acoust. Soc. Am.* **118**, 279–286.
- Meinke, D. K., Stagner, B. B., Martin, G. K., and Lonsbury-Martin, B. L. (2005). “Human efferent adaptation of DPOAEs in the L_1, L_2 space,” *Hear. Res.* **208**, 89–100.
- Mills, D. M., Norton, S. J., and Rubel, E. W. (1993). “Vulnerability and adaptation of distortion product otoacoustic emissions to endocochlear potential variation,” *J. Acoust. Soc. Am.* **94**, 2108–2122.
- Mills, D. M., and Rubel, E. W. (1994). “Variation of distortion product otoacoustic emissions with furosemide injection,” *Hear. Res.* **77**, 18–199.
- Rhode, W. S. (2007). “Distortion product otoacoustic emissions and basilar membrane vibration in the 6–9 kHz region of sensitive chinchilla cochleae,” *J. Acoust. Soc. Am.* **122**, 2725–2737.
- Stagner, B. B., Nemanov, L. S., Martin, G. K., and Lonsbury-Martin, B. L. (2005). “ L_1, L_2 response areas of lower- and upper-sideband DPOAEs measured at three f_2/f_1 ratios in normal and noise-exposed rabbits,” *Assoc. Res. Otolaryngol. Abstr.* **28**, 140.

- Stover, L. J., Neely, S. T., and Gorga, M. P. (1996). "Latency and multiple sources of distortion product otoacoustic emissions," J. Acoust. Soc. Am. **99**, 1016–1024.
- Weiss, T. F., and Leong, R. (1985). "A model for signal transmission in an ear having hair cells with free-standing stereocilia. IV. Mechanoelectric transduction stage," Hear. Res. **20**, 175–195.
- Whitehead, M. L., Lonsbury-Martin, B. L., and Martin, G. K. (1992a). "Evidence for two sources of $2f_1-f_2$ distortion-product otoacoustic emission in rabbit: I. Differential dependence on stimulus parameters," J. Acoust. Soc. Am. **91**, 1587–1607.
- Whitehead, M. L., Lonsbury-Martin, B. L., and Martin, G. K. (1992b). "Evidence for two sources of $2f_1-f_2$ distortion-product otoacoustic emission in rabbit: II. Differential physiological vulnerability," J. Acoust. Soc. Am. **92**, 2662–2682.
- Whitehead, M. L., Stagner, B. B., Martin, G. K., and Lonsbury-Martin, B. L. (1996). "Visualization of the onset of distortion-product otoacoustic emissions, and measurement of their latency," J. Acoust. Soc. Am. **100**, 1663–1679.