Singular value description of a digital radiographic detector: Theory and measurements

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The $\mathcal{H}$ operator represents the deterministic performance of any imaging system. For a linear, digital imaging system, this system operator can be written in terms of a matrix, $\mathbf{H}$, that describes the deterministic response of the system to a set of point objects. A singular value decomposition of this matrix results in a set of orthogonal functions (singular vectors) that form the system basis. A linear combination of these vectors completely describes the transfer of objects through the linear system, where the respective singular values associated with each singular vector describe the magnitude with which that contribution to the object is transferred through the system. This paper is focused on the measurement, analysis, and interpretation of the $\mathbf{H}$ matrix for digital x-ray detectors. A key ingredient in the measurement of the $\mathbf{H}$ matrix is the detector response to a single x-ray (or infinitesimal x-ray beam). The authors have developed a method to estimate the 2D detector shift-variant, asymmetric ray response function (RRF) from multiple measured line response functions (LRFs) using a modified edge technique. The RRF measurements cover a range of x-ray incident angles from 0° (equivalent location at the detector center) to 30° (equivalent location at the detector edge) for a standard radiographic or cone-beam CT geometric setup. To demonstrate the method, three beam qualities were tested using the inherent, Lu/Er, and Yb beam filtration. The authors show that measures using the LRF, derived from an edge measurement, underestimate the system's performance when compared with the $\mathbf{H}$ matrix derived using the RRF. Furthermore, the authors show that edge measurements must be performed at multiple directions in order to capture rotational asymmetries of the RRF. The authors interpret the results of the $\mathbf{H}$ matrix SVD and provide correlations with the familiar MTF methodology. Discussion is made about the benefits of the $\mathbf{H}$ matrix technique with regards to signal detection theory, and the characterization of shift-variant imaging systems.

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I. INTRODUCTION

The two main components of x-ray detector characterization are the detector's response to a point object and a description of its noise characteristics. The detector response to a single x-ray beam, or ray response function (RRF), is directly linked to the inherent ability of an imaging system to discern objects of various sizes and shapes. Knowledge of the complete set of all RRFs (for all possible x-ray locations, angles, and energies) is key to the complete description of the deterministic properties of an x-ray imaging system.

Many authors have developed methods to estimate the presampled detector response using various test devices, such as slits, edges, or pinholes.^{1–13} Most of the methods developed are one-dimensional approximations of the detector response, with every method having higher-order approximations and fewer assumptions. Building upon those methods, a standard edge measurement technique for the calculation of the detective quantum efficiency was developed in 2003.¹⁴ Fetterly *et al.* have developed a multiple pinhole method for estimating the presampled 2D detector response. Although the method Fetterly developed provides a 2D estimation of the detector response, it makes an assumption that

the detector has a uniform response at all locations. Furthermore, the method cannot be used to estimate the dependence of the detector response on the angle of incidence of the x-ray.

Once the detector response function is determined, it can be used in the study of the system's basis functions and their transfer through the imaging chain. Knowledge of the imaging system basis functions provides insight as to what types of objects are transferred through an imaging system. Decomposition of an object into the basis functions of the system can be used for predicting the object detection probability, as well as for object reconstruction.¹⁵

The deterministic properties of an imaging system have been presented in the past in terms of the modulation transfer function (MTF), by assuming that the imaging system basis functions were exponential wave functions. This approach makes the assumption that the imaging system is shift invariant, which requires the detector to have infinite extent and the system to have no preferred origin. Digital imaging systems have finite detectors and shift-variant imaging properties, in violation of the assumptions required for the application of Fourier theory. Singular value decomposition of the

imaging matrix ($\mathbf{H}$ matrix) is a generalization of the Fourier methodology, being a measure of the transfer of whatever basis functions are revealed by the singular value decomposition (SVD) process. The theory behind the $\mathbf{H}$ matrix is well established and has evolved quite rapidly over the past few years, although practical implementations for its measurement and use for object reconstruction exist mostly in SPECT imaging.^{15,16}

I.A. Paper outline

In this paper, we start with the general formulation of Barrett and Myers¹⁵ and apply it to the $\mathbf{H}$ matrix methodology for the evaluation of x-ray projection detectors that are used in planar radiography, tomosynthesis, and cone-beam CT imaging systems. As an example, we calculate the $\mathbf{H}$ matrix of a digital flat panel detector using estimates of the 2D detector response function from measurements. We then derive the system basis functions through singular value decomposition. Finally, an analysis and interpretation of the results is presented.

More specifically, in Sec. II B we develop an expression for a discrete image based on linear systems theory. In Sec. II C we define the discrete-to-discrete $\mathbf{H}$ matrix and provide its SVD. In Sec. III A, we clarify with mathematical definitions the various functions involved in the estimation of—and propose models for—the detector response function. We propose a methodology for determining the axial asymmetric detector response in Sec. III and propose a practical method for the estimation of the 2D detector response function based on a modified edge technique in Sec. IV. The technique was applied in the estimation of the 2D detector response as a function of the x-ray angle of incidence, and x-ray beam quality, of a flat panel digital detector used for projection radiography and cone-beam CT.¹⁷ We present the measured response functions, the estimated detector ray response functions, as well as the SVD of $\mathbf{H}$ in Sec. V. We demonstrate theoretically that the use of the LRF, compared to the RRF, underestimates the performance of a digital detector. Finally, to demonstrate the power of our method we calculate the *measurable* and *null* parts of a standard cross pattern in the Sec. VI.

II. IMAGING SYSTEM MODELING

II.A. Geometric description of a radiographic imaging system

We define the imaging system geometry for estimating the detector performance in Fig. 1. An infinitesimal x-ray beam intersects the scintillator plane at the location described by the 2D vector $\mathbf{r}_{in}$ with a direction defined by the 2D vector $\hat{\mathbf{s}}$. $\hat{\mathbf{s}}$ is described by the polar angle ϕ (about the $\mathbf{z}$ -axis counter-clockwise where $\phi \in [0, 2\pi]$) and the azimuthal angle θ (the angle from the $\mathbf{z}$ -axis to the vector $\hat{\mathbf{s}}$ with $\theta \in [0, \pi/2]$).

The scintillator has a thickness dc . Positions on the scintillator exit surface are described by the vector $\mathbf{r}_{out}$. The detector digitization circuitry, i.e., the flat panel thin film transistor (TFT) array, lies on the plane immediately adjacent to

FIG. 1. System setup and geometry definitions for modeling a general radiographic imaging system. For this paper we consider objects that lie on the detector, as well as point focal spots.

the scintillator exit plane. We assume that the digitization circuitry has $M_a \times M_b$ pixels of size $da \times db$ along the $\mathbf{x}$ and $\mathbf{y}$ -axis.

II.B. Object phase-space, images, and response functions

Connecting with the notation in Barrett and Myers,¹⁵ we define $\mathbf{f}$ as the distribution of scattering and absorption coefficients in the 3D object, where $\mathbf{r}$ is a 3D vector in object space. The typeface conventions used in this paper are defined in Appendix A. Square-integrable functions can be regarded as vectors in Hilbert space, $\mathbf{f} \in \mathcal{U}$.

A general imaging system maps the object space into image space

$$\mathbf{g} = \mathcal{H}\mathbf{f}, \quad \mathcal{H}: \mathcal{U} \mapsto \mathcal{V}. \quad (1)$$

$\mathcal{H}$ describes the deterministic properties of any imaging system in the absence of noise, and it includes the response of the system due to the source, as well as due to the detector.

Images can be functions of continuous variables, however digital images are discrete sets of numbers (pixels) in the finite-dimensional Hilbert space $\mathcal{V}$. For digital x-ray imaging systems we can write the image formation process as the following continuous-to-discrete (CD) mapping:

$$\mathbf{g} = \mathcal{D}\mathcal{B}\mathbf{f}, \quad (2)$$

where $\mathcal{B}$ is the Boltzmann transport operator that maps the object to a phase-space distribution at the face of the detector for a particular source, $\mathcal{D}$ is the detector response operator, $\mathcal{D}$ is the CD pixel discretization operator, and $\mathbf{g}$ is a discrete vector containing the image values.¹⁵

For projection imaging it is convenient to rewrite Eq. (2) as

$$\mathbf{g} = \mathcal{D}\mathcal{D}\mathbf{w}, \quad (3)$$

where $\mathbf{w}$ is the output of the operator $\mathcal{B}$, i.e., $\mathbf{w} \equiv \mathcal{B}\mathbf{f}$, also known as the object phase-space at the detector entrance, and is dependent on the x-ray/electron location, direction, and energy. Given an object $\mathbf{f}$, and a spatial-spectral focal spot radiance $\mathcal{L}$ which is dependent on the location of the source, the source energy spectrum, and beam direction, we define

the continuous phase-space at the detector entrance plane as $w(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathbf{f}, \mathcal{L})$. E_{in} is the energy carried by the infinitesimal beam which accounts for scatter and beam hardening due to transmission through the object.

The noise-free continuous image $g(\mathbf{r}_{\text{out}}; \mathcal{L})$ of a linear system at point $\mathbf{r}_{\text{out}}$ on the scintillator output plane can be defined as

$$g(\mathbf{r}_{\text{out}}; \mathcal{L}) = \int_{\infty} d\mathbf{r}_{\text{in}} d\hat{\mathbf{s}}_{\text{in}} dE_{\text{in}} \times \mathfrak{d}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}) w(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathbf{f}, \mathcal{L}). \quad (4)$$

In the expression above we define $\mathfrak{d}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}})$ as the detector response at any point $\mathbf{r}_{\text{out}}$ on the detector exit surface, given an infinitesimal x-ray beam with direction $\hat{\mathbf{s}}_{\text{in}}$ and energy E_{in} incident at point $\mathbf{r}_{\text{in}}$ on the scintillator entrance plane. $\mathfrak{d}$ is essentially a *black box* operator that takes the state of the x-ray beam entering the detector, *blurs* it appropriately given its incident location, angle, and energy, and presents it to the digitization circuitry to be integrated as $g(\mathbf{r}_{\text{out}}; \mathcal{L})$. This definition of $\mathfrak{d}$ was chosen based on the experimental measurement method presented in Sec. IV A. Here the notation of integrating over infinity ensures that all subintegrals have the appropriate bounds. The detector response function depends on the point $\mathbf{r}_{\text{in}}$ where the x-ray beam intersects the detector entrance surface in order to account for detector nonuniformities, as well as the angle $\hat{\mathbf{s}}$ with which the x-ray approaches the detector surface. (A simple example of a non-uniform planar detector is the CsI scintillator of an image intensifier (II), which is thicker on the detector periphery compared to the detector center.) (The response of the detector will be different depending on whether an x ray intersects the detector perpendicular or at an angle, as we see in Refs. 18 and 19.)

Based on Eq. (2) we can write the discrete image $\mathbf{g}$ by integrating $g(\mathbf{r}_{\text{out}}; \mathcal{L})$ over the pixel sampling function $D_{\mathbf{m}}$ as follows:

$$\mathbf{g}_{\mathbf{m}; \mathcal{L}} = \int_{\infty} d\mathbf{r}_{\text{out}} D_{\mathbf{m}}(\mathbf{r}_{\text{out}}) \int_{\infty} d\mathbf{r}_{\text{in}} d\hat{\mathbf{s}}_{\text{in}} dE_{\text{in}} \times \mathfrak{d}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}) w(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathbf{f}, \mathcal{L}), \quad (5)$$

where $\mathbf{m} \in [1, M]$ and $M = M_a \times M_b$, the total number of pixels in the x and y detector directions.

II.C. The H matrix and its singular value decomposition

The imaging equation above, Eq. (5), represents a continuous-to-discrete transformation. However, we often want to make use of discrete-to-discrete approximations to this equation, for example, when the $\mathcal{H}$ operator is to be calculated using a computer. In such cases it may be desirable to discretize the object as well, although care must be taken in sampling the object space finely enough to adequately represent the continuous-to-discrete nature of the real system.¹⁵

Given the phase-space $w_t(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathbf{f}, \mathcal{L})$ of an infinitesimal test object we can define $\mathbf{H}$ as the discrete-to-discrete (DD) approximation of $\mathcal{H}$:

$$\begin{aligned} \mathbf{H}_{\mathbf{mn}; \mathcal{L} w_{\text{in}}} &= [\mathbf{H}]_{\mathbf{mn}; \mathcal{L}} [\mathbf{w}_t]_{\mathbf{n}} \\ &\equiv \int_{\infty} d\mathbf{r}_{\text{out}} D_{\mathbf{m}}(\mathbf{r}_{\text{out}}) \\ &\times \int_{\infty} d\mathbf{r}_{\text{in}} d\hat{\mathbf{s}}_{\text{in}} dE_{\text{in}} w_{\mathbf{n}}(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}) \\ &\times \mathfrak{d}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}) w_t(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathbf{f}, \mathcal{L}). \end{aligned} \quad (6)$$

Here $\mathbf{H}_{\mathbf{mn}; \mathcal{L}}$ is the $[\mathbf{m}, \mathbf{n}]^{\text{th}}$ element of the matrix $\mathbf{H}$ given a specific source radiance, and $\mathbf{w}_{\text{in}}$ is the discrete phase-space (at position $\mathbf{r}_{\text{in}}$ with angle $\hat{\mathbf{s}}_{\text{in}}$ and energy E_{in}) of an infinitesimal test object. $w_{\mathbf{n}}(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}})$ is the continuous-to-discrete operator that discretizes the phase-space of the object over position, angle, and energy at the detector input. For a small focal spot of a monoenergetic source located at infinity, $\hat{\mathbf{s}}_{\text{in}}$ and E_{in} are constants, therefore the dimensionality of $\mathbf{H}$ is $M \times N = M_a M_b \times N_a N_b$, where M is the total number of detector pixels and N is the total number of elements in the discretized object phase-space. In order to correctly approximate $\mathcal{H}$, care must be taken when calculating $\mathbf{H}$ such that the phase-space discretization sampling is much finer than the pixel area.

$\mathbf{H}$ maps a discrete object phase-space $\mathbf{w}$ to the discrete data $\mathbf{g}$:

$$\mathbf{g} = \mathbf{H} \mathbf{w}, \quad (7)$$

where $\mathbf{w} = \int_{\infty} d\mathbf{r}_{\text{in}} d\hat{\mathbf{s}}_{\text{in}} dE_{\text{in}} w_{\mathbf{n}}(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}) w(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathbf{f}, \mathcal{L})$.

The $\mathbf{H}$ matrix is rectangular in general, therefore for its singular value decomposition we can construct a square matrix using the dot product of $\mathbf{H}^{\dagger}$ and $\mathbf{H}$, which is Hermitian. The eigenanalysis of this product is then written as

$$\lambda_{\mathbf{n}} \mathbf{u}_{\mathbf{n}} = \mathbf{H}^{\dagger} \mathbf{H} \mathbf{u}_{\mathbf{n}}. \quad (8)$$

The significance of Eq. (8) is that it generates a set of eigenvectors that form the basis of the imaging system. Specifically, $\lambda_{\mathbf{n}}$ and $\mathbf{u}_{\mathbf{n}}$ are the sorted eigenvalues and eigenvectors of $\mathbf{H}^{\dagger} \mathbf{H}$, and $\lambda_{\mathbf{n}}$ is the magnitude of each eigenvector transferred through the system. Note that the singular values of $\mathbf{H}$ are equal to $\sqrt{\lambda_{\mathbf{n}}}$.

Since the $\mathbf{u}_{\mathbf{n}}$ are orthonormal, they form a complete basis for vectors of length N ; consequently, any object phase-space can be written as an expansion in terms of these same eigenvectors:

$$\mathbf{w} = \sum_{\mathbf{n}=1}^N \alpha_{\mathbf{n}} \mathbf{u}_{\mathbf{n}}, \quad (9)$$

where the expansion coefficients $\alpha_{\mathbf{n}}$ can be calculated from $\alpha_{\mathbf{n}} = \mathbf{u}_{\mathbf{n}}^{\dagger} \mathbf{w}$.

A special case of Eq. (8) is when the eigenvectors of the system are exponential wavefunctions. In such a case the normalized Fourier transformation of the detector response function returns the system eigenvalues, or the modulation transfer function (MTF) of the system.

The benefit of obtaining the complete basis of an imaging system is that we can predict which parts of an object phase-space can be transferred through the system based on the true system basis. Furthermore, if the rank R of $\mathbf{H}$ is less than N , the eigenvalues λ_n are real and positive for $n < R$ and zero otherwise, meaning that they are not transferred by the imaging system. We refer to such functions as null functions of the imaging system. We can therefore separate an object phase-space $\mathbf{w}$ into the measurable part $\mathbf{w}_{\text{meas}}$ and the null part $\mathbf{w}_{\text{null}}$,

$$\mathbf{w} = \mathbf{w}_{\text{meas}} + \mathbf{w}_{\text{null}}, \quad (10)$$

where

$$\mathbf{w}_{\text{meas}} = \sum_{n=1}^R \alpha_n \mathbf{u}_n \quad (11)$$

and

$$\mathbf{w}_{\text{null}} = \sum_{n=R+1}^N \alpha_n \mathbf{u}_n \quad \text{with } \mathbf{H}\mathbf{w}_{\text{null}} = 0. \quad (12)$$

We can use the measurable $\mathbf{w}_{\text{meas}}$ and the null object phase-space $\mathbf{w}_{\text{null}}$ as means of detector evaluation.

III. RESPONSE FUNCTIONS AND THEIR ESTIMATES

In this section we will discuss a method for measuring $\mathbf{H}$ in a radiographic imaging system. By definition $\mathbf{H}$ in Eq. (6) can be measured by moving a small test object in object space, while recording the system response.

Experimental difficulties in radiography prevent the literal adoption of the $\mathbf{H}$ definition. Instead, in practice, physical test objects f_i are used. Some convenient test objects that can be used for this task include a pinhole, a slit, or an edge. The detector response to the specific object can be recorded and the system response $\mathbf{H}$ to an infinitesimal beam can be inferred from the measurements.

In Sec. III A we will define the response functions we expect when the test objects are a ray, as line, and an edge, and in Sec. III B we will provide a method to estimate the detector response given the line response.

III.A. Response function definitions

In this section we provide the definitions of the point, ray, line, and edge response functions as they will be used in this paper.

When an x-ray photon interacts inside the scintillator it deposits its energy by generating secondaries and, depending on the detector technology, optical photons, or electron-hole pairs.

The detector responds differently depending on the interaction location (especially depth) inside the scintillator. This

detector response is known as the point response function (PRF). Such a response is very difficult to measure experimentally because of the difficulty in generating single events inside a scintillator,²⁰ however, it can be readily simulated using Monte Carlo methods.²¹

The photons of an infinitesimal x-ray beam will interact inside the detector along the line track of the beam. For the characterization of clinical systems, a ray response function (RRF) can be used for description of the detector's response $\mathfrak{d}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}, E)$ since it integrates the cumulative responses of all points along the x-ray track.

Definition 1: Ray response function (RRF) at $\mathbf{r}_{\text{out}}$ on the scintillator exit plane is defined as the cumulative response to all the point processes inside the scintillator generated by an infinitesimal x-ray beam that enters the scintillator at position $\mathbf{r}_{\text{in}}$, with angle $\hat{\mathbf{s}}$, and energy E .

$$\text{RRF}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}, E_{\text{in}}) \equiv \mathfrak{d}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}, E_{\text{in}}). \quad (13)$$

The $\hat{\mathbf{s}}$ dependence comes from the fact that a ray can intersect the detector surface at an angle other than normal, which is a critical contribution to an asymmetric response.

The phase-space

$$w_l(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}, E_{\text{in}}; \mathcal{L}) = \delta(\mathbf{x}_{\text{in}}) \lim_{y_0 \rightarrow \infty} \text{rect}(y_{\text{in}}/y_0) w(\hat{\mathbf{s}}, E_{\text{in}}; \mathcal{L})$$

of a line object in the form of a narrow, infinitely long slit can also be used to characterize the response of an imaging system. $w_{\mathcal{L}}(\hat{\mathbf{s}}, E_{\text{in}}; \mathcal{L})$ is the projection of the focal spot radiance at the detector input.

Definition 2: The line response function (LRF) is the cumulative contribution of RRFs at the scintillator output generated from an infinite line placed on the scintillator input:

$$\begin{aligned} \text{LRF}(\mathbf{r}_{\text{out}}; \mathcal{L}) &= \int_{\infty} d\mathbf{r}_{\text{in}} d\hat{\mathbf{s}} dE_{\text{in}} \text{RRF}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}, E_{\text{in}}) \\ &\times w_l(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}, E_{\text{in}}; \mathcal{L}). \end{aligned} \quad (14)$$

This measurement is usually performed with a well collimated beam (with a small solid angle) on a small detector area. Therefore, if we assume that the incident radiance is locally uniform in the region where we perform the measurement, the LRF reduces to a 1D function:

$$\begin{aligned} \text{LRF}(\mathbf{r}_{\text{out}}; \mathcal{L}) &= \int_{\infty} d\hat{\mathbf{s}}_{\text{in}} dE_{\text{in}} dy_{\text{in}} \text{RRF}(\mathbf{r}_{\text{out}} \\ &- \mathbf{r}_{\text{in}}; \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}) w_{\mathcal{L}}(\hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathcal{L}), \end{aligned}$$

i.e., a convolution with a line becomes a projection, or an integral over one dimension for each $\mathbf{x}_{\text{out}}$. The consequence is that the $\text{LRF}(\mathbf{x}_{\text{out}})$ becomes wider than $\text{RRF}(\mathbf{r}_{\text{out}})$ because of the additive contributions from the y -dimension.²²

The phase-space

$$\begin{aligned} w_e(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}, E_{\text{in}}; \mathcal{L}) &= \vartheta(x_{\text{in}}) \lim_{y_0 \rightarrow \infty} \text{rect}(y_{\text{in}}/y_0) \\ &\times w_{\mathcal{L}}(\hat{\mathbf{s}}, E_{\text{in}}; \mathcal{L}) \end{aligned}$$

of an edge object, defined in terms of the Heaviside function $\vartheta(x)$ (zero for $x \leq 0$ and 1 otherwise) is an attractive object

for measuring the response of a detector, because of the experimental simplicity associated with the measurement:

Definition 3: Edge response function (ERF) is the cumulative contribution of LRFs at the scintillator output that are generated from an edge that covers the semi-infinite input plane of an isotropic, or a locally isotropic scintillator.

Starting from the general definition,

$$\text{ERF}(\mathbf{r}_{\text{out}}; \mathcal{L}) = \int_{\infty} d\mathbf{r}_{\text{in}} d\hat{\mathbf{s}}_{\text{in}} dE_{\text{in}} \text{RRF}(\mathbf{r}_{\text{out}}; \mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}) \times w_e(\mathbf{r}_{\text{in}}, \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathcal{L}), \quad (15a)$$

for a locally isotropic detector the $\text{ERF}(\mathbf{r}_{\text{out}}; \mathcal{L})$ reduces to

$$\text{ERF}(\mathbf{r}_{\text{out}}; \mathcal{L}) = \int_{\infty} dx_{\text{in}} \vartheta(x_{\text{in}}) \int d\hat{\mathbf{s}}_{\text{in}} dE_{\text{in}} dy_{\text{in}} \text{RRF}(\mathbf{r}_{\text{out}} - \mathbf{r}_{\text{in}}; \hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}) w_{\mathcal{L}}(\hat{\mathbf{s}}_{\text{in}}, E_{\text{in}}; \mathcal{L}), \quad (15b)$$

or simply to

$$\text{ERF}(x_{\text{out}}; \mathcal{L}) = \int_{\infty} dx_{\text{in}} \vartheta(x_{\text{in}}) \text{LRF}(x_{\text{out}} - x_{\text{in}}; \mathcal{L}). \quad (15c)$$

III.B. Estimating the detector point response from measurements

Based on the Monte Carlo simulations we performed on indirect mammographic detectors,¹⁹ we assume that an x ray with oblique angle of incidence will result in an asymmetric detector response. Other examples where detectors might exhibit asymmetric RRFs might be line transfer CCD configurations, slit scanning detectors, or detectors coupled with variable thickness scintillators, such as image intensifiers (II).

Even though a conventional analysis of a 1D LRF measurement yields a symmetric RRF function, we can estimate the 2D asymmetric RRF($\mathbf{r}_{\text{out}}$) by approximating its profile normal to lines at various orientations using the central-slice theorem. This can be done by using a series of independent 1D LRF measurements obtained by measuring the detector response at different orientations. (An example could be measurements in the positive x direction, the negative x direction, the positive y directions and the negative y directions.) The LRF measurements can be obtained either directly with a slit, or estimated from the derivatives of ERF measurements.⁵

We assume that along the path through a scintillator, an x ray will have interactions at multiple depths, and each interaction produces a Gaussian response at the scintillator exit. The RRF and in extension the LRF at the exit surface of the scintillator will therefore be the weighted sum of those Gaussians:

$$\text{LRF}_{\text{meas}}(x_{\text{out}}; \mathcal{L}) = \sum_{i=1}^Q \alpha_i e^{-\beta_i^2 x_{\text{out}}^2}, \quad (16a)$$

where Q is the number of the mixture terms. We assume that the constants α_i and β_i are functions of $\mathcal{L}$ and can be ob-

tained with a nonlinear fit to the measured data. Digital detectors are made of discrete pixels, therefore in order to obtain an approximation to the continuous RRF the presampled $\text{LRF}_{\text{meas}}(x_{\text{out}}; \mathcal{L})$ must be measured. As we will see in Sec. IV A we can sample the LRF at much finer steps than the pixel size. However, from Eq. (5) we see that any measurement on a digital detector includes the effects of the pixel sampling function $D_{\mathbf{m}}(\mathbf{r}_{\text{out}})$. If the measurement is performed on a small area of the detector with the detector at a large distance from the focal spot, using a monochromatic beam then $w_{\mathcal{L}}$ is constant and the incoming rays can be considered parallel to each other. In relation to Eqs. (5) and (14) the LRF_{meas} takes the form

$$\text{LRF}_{\text{meas}}(x_{\text{out}}; \mathcal{L}) = \int_{\infty} d\mathbf{r}_{\text{out}} D_{\mathbf{m}}(\mathbf{r}_{\text{out}}) \int_{\infty} dy_{\text{in}} \text{RRF}(\mathbf{r}_{\text{out}} - \mathbf{r}_{\text{in}}). \quad (16b)$$

Each LRF can be analyzed to yield a symmetric RRF. Furthermore, by using the central slice theorem we can rewrite Eq. (16a) as

$$\text{LRF}_{\text{meas}}(x_{\text{out}}; \mathcal{L}) = \int_{\infty} dy_{\text{in}} \left\{ \gamma \sum_{i=1}^Q \alpha_i \beta_i e^{-\beta_i^2 (r_{\text{out}} - r_{\text{in}})^2} \right\}, \quad (16c)$$

with γ the normalization constant.

By comparing Eqs. (16b) and (16c) we see that to obtain the RRF we must deconvolve the pixel sampling function from the integrand of Eq. (16c). The standard assumption about the pixel function is that it is a perfect square. However, the deconvolution of a square function cannot be solved analytically because it involves dividing by the pixel sinc function in Fourier space, which has zeros. Instead of a sinc, we approximate the pixel function with a Gaussian that has the same width as the square pixel: $D_{\mathbf{m}}(\mathbf{r}_{\text{out}}) = \exp(-d^2(\mathbf{r}_{\text{out}} - \mathbf{r}_{\mathbf{m}})^2)$ with $d = 2/da$, where $da = db$ is the pixel size, and $\mathbf{r}_{\mathbf{m}}$ is the center of the $\mathbf{m}^{\text{th}}$ pixel. This is a good approximation since the difference between the areas of the pixel sinc function and the Gaussian, when integrated up to 31.4 mm^{-1} where the sinc function has its first zero, is less than 7%. Given this approximation, and setting $r_{\text{in}} = r_{\mathbf{m}}$, the pixel deconvolution can be calculated analytically. The RRF obtained from Eq. (16c) is therefore

$$\text{RRF}(r_{\text{out}} - r_{\text{in}}) = \gamma \sum_{i=1}^Q \frac{\alpha_i \beta_i}{d^2 - \beta_i^2} \exp\left(-\frac{d^2 \beta_i^2}{d^2 - \beta_i^2} (r_{\text{out}} - r_{\text{in}})^2\right), \quad (17)$$

where

$$\gamma = \left[\sum_{i=1}^Q \frac{\alpha_i \beta_i}{d^2 - \beta_i^2} \right]^{-1}.$$

Equation (17) depends only on the radial distance $r_{\text{out}} - r_{\text{in}}$, i.e., it is rotationally symmetric. Nevertheless, the x and y asymmetric profiles of the 2D RRF can be estimated by the corresponding halves of the 1D RRFs obtained by four edge-

test-object measurements. As we will see in Sec. IV A, such measurement can be performed in a single step by using a square test object, which can provide the horizontal and vertical RRFs simultaneously. The assumption this technique makes is that the response of the detector is the same at least in the region covered by the square.

A note of caution: in the definitions of Sec. III A we used infinite edges or lines. The experimental consequence is that there exists an optimal balance in the size of the square test object: not too small (so that we sample the LRF finely enough) and not too large (so that the detector response remains the same within the test region). Since the 2D RRF is expected to have a finite extent, then the length of an edge of the test square should be a multiple of the RRF FWHM (full width half max). For example, we found that for our experimental setup (see Sec. IV), a $5 \times 5 \text{ cm}^2$ plate provided a good balance. Care must be taken to avoid the regions near the tips of the square test device. Data from those regions do not obey the assumptions of Sec. III A, since they receive interference from both perpendicular directions.²³

IV. EXPERIMENTAL SETUP

We measured the detector response function of a custom-made laboratory x-ray cone-beam CT imaging system. The detector used in our experimental setup is a Varian 4030CB (Varian Corp. Salt Lake City, UT) with $n_a \times n_b = 2048 \times 1596$, $d_a = d_b = 195 \text{ }\mu\text{m}$ pixels, and $600 \text{ }\mu\text{m}$ thick columnar CsI(Tl) scintillator. The x-ray tube is a Varian B180 (Varian Corp., Salt Lake City, UT) with 0.3 and 0.6 mm nominal focal spots. Three beam qualities were used at a fixed 100 kVp tube voltage. The three beam filter combinations used to adjust the beam spectrum were the inherent tube filtration of 1 mm Al, the inherent 1 mm Al plus 0.25 mm Yb, and the inherent 1 mm Al with a combination of 0.127 mm Lu and 0.127 mm Er. The latter two filter combinations give sharp energy peaks with mean energies around 59 keV, an energy close to the peak quantum absorption of the CsI(Tl) scintillator. With this filter combination we take advantage of the K-edge emissions of Tungsten (W) to create quasi monochromatic beams. Custom electronics and software control the image acquisition process, event flow control, and synchronization. The detector's linearity was checked and is linear in the range 100 mR to 2 R. In all our measurements the detector is fixed in the 14 bit mode with no binning, acquiring images at 1 frame per second.

IV.A. Measurement of the detector response function

The presampled detector response function was measured using a square dysprosium (Dy) test-object, with an edge response function technique we have developed in the past.¹³ The technique permits the simultaneous measurement of the asymmetric detector response function profiles at the vertical and horizontal directions. The choice of dysprosium was made based on the beam filtration we used. Dy has a K-absorption edge at 53.8 keV, while Yb has an edge at 61.3 keV. As a result the combination of Yb–Dy provides strong attenuation for the 100 kVp Tungsten (Wu) beam with

a relatively small Dy thickness. A relatively thin edge (0.25 mm) is advantageous when measuring response functions at oblique x-ray incidence angles because it reduces beam inhomogeneity near the tip of the edge.

The $5 \times 5 \text{ cm}^2$, 0.25 mm thick Dy square plate was carefully milled and polished so that no edge imperfections larger than $1 \text{ }\mu\text{m}$ could be detected with an inspection under a microscope. The Dy plate was secured on the center of the detector, which was then centered on the x-ray beam, with the detector plane aligned perpendicular to the central x ray. The plate was tilted slightly with respect to the pixel rows, following the configuration that we^{13,24,25} and others⁹ have used in the past, so that on each side the edge passes through at least four pixel corners. The resulting tilt of the plate with respect to the horizontal axis x was about 1° . The reason for this tilt is to achieve a sampling pitch higher than the actual detector pixel size, which results in a super sampled edge response function.^{7,14} Note that in order to perform the pre-sampled ERF measurement we assumed local shift invariance in the small detector region where we performed the ERF measurement. This permits the measurement of the super-sampled LRF by assuming that the response of the neighboring pixel lines is equivalent. This also assumes that the x-ray angle of incidence does not change significantly in the small region of the ERF measurement which can be guaranteed by using a large SID and a small area on the detector. For a flat panel detector this is a good assumption since for a flat field corrected image, the detector response is uniform.

To measure different x-ray incidence angles we rotated the detector about the x - or y -axis, while maintaining the center of the plate on the z -axis, perpendicular to the central x ray. The detector was placed on a pivot that allowed it to be rotated according to the Euler angles: ϕ deg about the z -axis and then θ deg about the y -axis. Measurements were performed for $\phi = 0^\circ$ with the detector rotated about the y -axis at $\theta = 0^\circ$, to 30° at 5° increments, and for $\phi = 90^\circ$ at $\theta = 15^\circ$ for comparison. These angles correspond to x rays intersecting the detector at locations ranging from the center to the outer edges (horizontal and vertical) for typical cone-beam CT configurations.

To increase the signal to noise ratio, 60 images of the test object were averaged for each configuration and were flat-field corrected as described in Refs. 13, 24, and 25. From the mean of the flat-field corrected images we extracted four edge images, corresponding to the four sides of the square. The resulting edge images were about 250×250 pixels; half of their area was dark (i.e., covered by the edge) and half was bright (exposed by the x rays). The derivatives in the direction perpendicular to each of the four edges were then calculated. As an example, the top part of Fig. 2(a) shows a few rows from the derivative of the image of the square-test-object's bottom edge. The edge location is at the top part of the image as marked on Fig. 2(a). The edge tilt can also be seen. The graph below the derivative image shows selected line profiles, as pointed by the arrows, from the top derivative image. The location of the dashed line boundaries shown on the derivative image is chosen by the profile intersections.

FIG. 2. (a) The top image is the derivative of the edge of the square test object x-ray projection. The bottom graph shows selected horizontal line profiles (pointed by the arrows) from the top derivative image. The location of the dashed line boundaries shown on the derivative image is chosen from the profile intersections as shown in the lower graph. The data contained within the top dashed region are used to generate the presampled LRF as described in the text. (b) Compares the LRF (dashed curve) generated from the data contained within the top dashed polygonal region in (a) with the LRF derived from the data not covered by the test object (continuous curve), i.e., the data coming from the bottom polygon. (a) Data selection for the LRF determination. (b) Horizontal LRF for the left edge of the Dy plate.

The data contained within the top dashed polygonal region are used to generate the presampled LRF. To obtain the data contained within the top dashed polygon of Fig. 2(a) we proceeded as follows: We first extracted all the columns within the two vertical dashed lines in Fig. 2(a). This submatrix is then transposed and lexicographically ordered by attaching each of the submatrix rows at the end of their preceding rows, thereby converting the transposed submatrix into a 1D array. The resulting array is the presampled LRF, with a sampling spacing determined as the detector pixel size divided by the number of columns contained between the two vertical dashed lines. The same process was performed a total of three times on the equivalent regions adjacent to the dashed region; their average was used in order to reduce the noise. The first and last sets of columns (i.e., the regions

before and after the first and last minima) were avoided in order to prevent tail contaminations from the perpendicular edges.²³

The data from the last step were used to generate a set of four symmetric LRFs by mirroring, and peak-normalizing only the section of the LRFs covered by the plate. As an example, the dashed curve shown in Fig. 2(b) corresponds to the desired data from the top polygonal dashed region of the derivative image, while the solid curve corresponds to the data below the polygonal region. The model of Eq. (16a) was then fit to each of the four symmetric LRFs, and Eq. (17) was used to derive four symmetric RRFs. By using a χ^2 fit, we found that a three-term Gaussian mixture is sufficient to describe our data. The resulting RRFs were appropriately joined to generate estimates of the x - and y -axis piecewise profiles of the 2D asymmetric RRF.

To obtain a 2D estimate of the detector RRF we generated 2D contours using cubic spline fits between the horizontal and vertical 1D RRF profiles: for a given RRF_0 magnitude we found the four points on the x - and y -axis that satisfied $\text{RRF}(x_{\text{out}}, y_{\text{out}}) = \text{RRF}_0$. Those four points were fit by second-order-continuous cubic splines, as can be seen in Fig. 5. Note here that as with any interpolation, the results should only be trusted at the measured points, which are the two axes; nevertheless, this method provides an analytical piecewise Gaussian-mixture spline estimate of the 2D detector ray response function which can be used to generate the imaging system basis functions. A more accurate estimate of the RRF can be obtained by rotating the test object through a range from 0 to 90° and by reconstructing the RRF using the Fourier central slice theorem.

IV.B. Estimation, and singular value decomposition of the detector $\mathbf{H}$ matrix

Once we obtained the analytical form of the RRF, we built the $\mathbf{H}$ matrix by scanning a simulated infinitesimal x ray beam across the surface of the detector and simultaneously sampling the RRF using the pixel function. This was achieved by successively sampling the RRF on a $4 \times 4 \text{ mm}^2$ detector subregion, while the RRF maximum was shifted $50 \mu\text{m}$ at a time so as to cover the detector subregion completely. For a given location of the RRF maximum the resulting matrix of the $4 \times 4 \text{ mm}^2$ detector subregion was lexicographically ordered into an array of 400 elements, which correspond to the rows of the $\mathbf{H}$ matrix. The process was repeated for 6400 times until we covered all the detector subregions. The resulting $\mathbf{H}$ had dimensions $\{\mathbf{M} \times \mathbf{N}\} = \{400 \times 6400\}$. $\mathbf{H}^\dagger \mathbf{H}$ generated a matrix of 6400×6400 elements. Singular value decomposition of $\mathbf{H}$ was performed according to Eq. (8) and resulted in 6400 eigenvectors and 400 nonzero eigenvalues. The rank R of $\mathbf{H}$ was therefore 400.

The assumption we made for the calculation of $\mathbf{H}$ is that for a small detector area, covering a small solid angle $d\Omega = d\phi d\theta$, and for fixed angle of incidence, $\text{RRF}(r_{\text{out}})$, can be considered independent of r_{in} .

FIG. 3. (a) and (b) compare the horizontal and vertical detector LRF data for various beam filtrations at $\theta=\phi=0^\circ$ angle of incidence. (c) and (d) compare the horizontal and vertical detector LRF data for various incidence angles for the Yb 0.127 mm filter. (a) Horizontal LRF data for three beam filters. (b) Vertical LRF data for three beam filters. (c) Horizontal LRF data for four beam incidence angles. (d) Vertical LRF data for four beam incidence angles.

V. RESULTS

Figure 3 presents the measured data used for the calculation of the detector RRF. Figures 3(a) and 3(b) compare the presampled LRF data at $\theta=\phi=0^\circ$ angle of x-ray incidence, for the horizontal and vertical directions. Each horizontal (vertical) LRF is comprised by data originating under the right (top) edge of the Dy plate for $x_{\text{out}}(y_{\text{out}}) \leq 0$ and data under the left (bottom) edge of the Dy plate for $x_{\text{out}}(y_{\text{out}}) \geq 0$. The data are presented in this fashion for comparison purposes. Figures 3(c) and 3(d) compare the presampled LRF data at $\phi=0^\circ$ and $\theta=0^\circ$ to 30° angle of x-ray incidence for the horizontal and vertical directions, respectively. In experiments not shown here we found that the LRFs of $\phi=90^\circ$, $\theta=15^\circ$ compared with the $\phi=0^\circ$, $\phi=15^\circ$ overlap, meaning that the detector inherent pixel structure is homogeneous in the two directions.

Figure 4 compares the asymmetric measured data with the fitted piecewise model of Eq. (16a) in the horizontal and the vertical directions, for two angles of incidence: (a) $\theta=0^\circ$ and (b) for $\theta=30^\circ$, both with $\phi=0^\circ$. In Appendix B we provide the constants of the fit. On each figure we also plot the estimated piecewise asymmetric RRF. Notice the asymmetries of the LRF and the RRF.

Figure 5(a) shows the resulting 2D RRF by fitting spline curves on the 1D RRF profiles as described in Sec. IV A. Figure 5(c) is the projection of the contours in (a) displayed in logarithmic scale. Figures 5(b) and 5(c) compare the RRFs for 0° and 30° .

Figure 6(b) shows the first four eigenvectors/eigenvalues of $\mathbf{H}^\dagger \mathbf{H}$ presented in Fig. 6(a). The images in Fig. 6(b) were generated by repartitioning the resulting eigenvectors into 80×80 element (or $2 \times 2 \text{ mm}^2$) matrices.

Figure 7 shows two histograms of eigenvectors with their corresponding eigenvalues from the singular value decomposition of $\mathbf{H}^\dagger \mathbf{H}$ for (a) $\theta=0^\circ$ angle of incidence and (b) for $\theta=30^\circ$ with $\phi=0^\circ$. The eigenvectors were sorted according to a descending order of their eigenvalues and grouped in bins of 0.1 starting from one to zero. Only the first 20 eigenvectors of each column are shown here because of space limitations.

In Fig. 8 we present an example of how a cross pattern is transferred through the detector. For a monochromatic source that emits parallel x rays perpendicular to the detector plane, a cross pattern phase-space has been discretized on a $2 \times 2 \text{ mm}^2$ area in 80×80 bins, each $50 \times 50 \mu\text{m}^2$ large.

FIG. 4. The plots compare the horizontal and vertical LRF and the RRF for two beam incidence angles. (a) Measured LRF data points with LRF Gaussian mixture fit, and RRF for the x and y axes for $\theta=0^\circ$ incidence angle. (b) Measured LRF data points with LRF Gaussian mixture fit, and RRF for the x and y axes for $\theta=30^\circ$ incidence angle.

Figure 8(a) shows the simulated object phase-space, the image obtained by $\mathbf{Hw}_i$, the measurable part of the object phase-space obtained from Eq. (11), and the null part of the object phase-space obtained by Eq. (12). Figure 8(b) shows the measured and null parts of the object phase-space for $\theta=30^\circ$ for comparison with (a). In Fig. 8(b) we see that the vertical line is more blurred than the horizontal, consistent with the asymmetry seen in the RRF in Figs. 5(a) and 5(c).

VI. DISCUSSION

In the attempt to provide a complete description of an imaging system the minimum possible number of assumptions must be made. We started with a general imaging equation of a linear imaging system, Eq. (1), and rewrote it in an appropriate form for a digital flat-panel-based projection radiography imaging system, Eq. (5). From this equation we see that knowledge of the detector response to a single x ray, the RRF, is essential. Therefore, in this paper, we have investigated a method for estimating the 2D RRF. The method relies on the estimation of four symmetric RRFs using a square edge test tool, combining them in a piecewise fashion

FIG. 5. (a) shows the construction of the 2D RRF($\mathbf{r}_{out}$) using the piecewise Gaussian mixture model with rotational interpolation using cubic splines. (b) and (c) are projections of the 2D RRF($\mathbf{r}_{out}$) for two beam incidence angles. (a) The piecewise 2D RRF($\mathbf{r}_{out}$) using a Gaussian mixture/spline model for the $\theta=30^\circ$ incidence angle. (b) The projected 2D RRF($\mathbf{r}_{out}$) for the $\theta=0^\circ$ incidence angle. (c) The projected 2D RRF($\mathbf{r}_{out}$) for the $\theta=30^\circ$ incidence angle.

to generate the asymmetric RRF profiles along the x - y -axis, and obtaining the 2D RRF by interpolation. One of the benefits of this method is that it can be applied to any fixed imaging system without requiring access to the scintillator screen. The experimental technique for estimating the LRF is

FIG. 6. (a) shows an example $\mathbf{H}^T \mathbf{H}$ calculation, while (b) shows the first four eigenvectors and eigenvalues of Eq. (8). (a) The $\mathbf{H}^T \mathbf{H}$ of the 0° incidence angle displayed in log-scale. (b) Example eigenvectors and eigenvalues for the 0° incidence angle.

simple and uses standard imaging system quality control equipment; it can therefore be practically used to evaluate existing clinical imaging systems. Having an analytical model of an imaging system response function is useful for the determination of the imaging system basis functions.

Thus far the basis functions of x-ray projection imaging systems have been assumed to be exponential wave functions. With this paper we demonstrated a method for obtaining an estimate of the actual imaging system basis functions as can be seen in Fig. 7. The shift invariance assumption that Fourier-based methods make imposes an important limitation, that the system basis functions must be symmetric superpositions of exponential wave functions. By determining the system basis by SVD we can predict the object detection performance of a detector system based on the actual basis functions of the system: the system basis functions can be used to predict the detected signal, i.e., image $\mathbf{g}$ shown in Fig. 8 and by dividing with the system noise, the signal to noise ratio (SNR), or detectability can be estimated.

The standard MTF definition uses the detector LRF to estimate the detector transfer properties. The LRF is a sum-

mation over one of the dimensions of the RRF.²² The effect of this summation is that the LRF is much broader than the axis profile of the 2D RRF as we see in Fig. 4. Detector performance measures that make use of the LRF directly will therefore underestimate the absolute performance of an imaging system, as well as disregard any asymmetric performance.

We can see in Fig. 5 that the distance differences between the 0° and 30° RRFs are 0.3, 0.2, 0.142, 0.104, and 0.0662 mm for 75%, 50%, 25%, 10%, and 1% of the RRF height, respectively. The difference is insignificant in the positive x -direction or the y -axis. This difference between the RRFs for the two angles of incidence also affected the orientation and shape of the eigenvectors in Fig. 7.

Different beam qualities were used in order to test the detector response under no, moderate, and heavy beam filtration, as we see from Figs. 3(a) and 3(b). Notice that the Lu/Er filter combination produces a slightly narrower response, however the difference in the measurements is not significant. One possible explanation is that the mean energies of the three spectrums are similar. The consequence of these results is that for the energies we used, the detector RRF is not strongly dependent on the spectrum.

The power of the $\mathbf{H}$ matrix methodology is seen in Figs. 7 and 8. By performing singular value decomposition we can uncover the null space of an imaging system, which provides information about the parts of an object phase-space that cannot be transferred through the system. Such information is very important in the prediction of the object detection uncertainty. By studying Figs. 7(a) and 7(b) we see that the eigenvectors of $\mathbf{H}$ are not simple wave functions since the eigenvectors are not symmetric. Furthermore, the $\theta=0^\circ$ sets of eigenvectors are different from the $\theta=30^\circ$ ones.

To make a connection between the MTF and the system SVD we take an eigenvector from Fig. 7, count the number of dark and bright line-pairs in a given direction, and divide that by the total length of the eigenvector (from Fig. 6 the length is 4 mm). For example, at $\theta=0^\circ$ the first eigenvector with 0.4 eigenvalue has four dark and bright line-pairs. In Fourier terms, the detector can transfer one line pair per mm with 63% transfer magnitude in the vertical detector direction, which is a reasonable number for a detector with a 2.56 mm^{-1} Nyquist frequency.

Had a larger detector area been used for the determination of the $\mathbf{H}$ matrix the system eigenvectors could have been estimated with finer samples, allowing for higher order eigenvector estimation. Therefore the choice of the detector area to be used should consider the object to be evaluated.

VII. CONCLUSIONS

We have presented an extension of the slanted-edge technique that can be used to determine an estimate of the 2D detector RRF.

We have applied the SVD analysis of the $\mathbf{H}$ matrix for the evaluation of an imaging system given the RRF. Although the example imaging system we used for demonstrating the methodology was a detector used in x-ray projection radiog-

FIG. 7. (a) and (b) provide a comparative presentation of the singular value decomposition of $\mathbf{H}^T \mathbf{H}$ for the two beam incidence angles. The eigenvectors were separated into bins based on their eigenvalue magnitude. Only the first 20 eigenvectors are shown. (a) Eigensystem histogram for the 0° incidence angle. (b) Eigensystem histogram for the 30° incidence angle.

raphy, the methodology is general and can be used for cone-beam and standard CT, as well as x-ray tomosynthesis, since all modalities rely on x-ray projections. The $\mathbf{H}$ SVD methodology can be used to evaluate a complete imaging system including the patient and an object of interest by including the effects of the focal spot, as we have shown in the past.¹³

For our future work we intend to investigate the imaging system basis function's dependence on the system geometry, and the x-ray beam quality parameters. The system basis functions will allow us to estimate the transfer of any given object through an imaging system, making it a step closer to a task specific imaging system performance assessment.

FIG. 8. (a) Comparison between the *measurable* and *null* parts of a test crosspattern for 0° in (a) and 30° in (b) angle of incidence. (a) The object phase-space and its image, compared to the *measurable* and the *null* parts of the object phase-space for the 0° angle of incidence. (b) The object phase-space and its image, the *measurable* and *null* parts of the object phase-space for the 30° angle of incidence.

APPENDIX A: TYPEFACE AND SYMBOL CONVENTIONS

In this paper we use the following conventions with regards to the typeface style:

- Serif typeface symbols denote continuous variables and functions (e.g., z , g , f).
- The calligraphic typeface is used to denote continuous-to-continuous operators (e.g., $\mathcal{B}$, $\mathcal{H}$).
- The typewriter typeface is used to denote continuous-to-discrete operators ($\mathcal{D}$, $\mathcal{W}$).
- Sans-serif typeface is used to distinguish discrete quantities such as vectors ($\mathbf{g}$, $\mathbf{f}$), matrices, and discrete to discrete operators ($\mathcal{H}$).
- Greek symbols and the Fraktur typeface are used to extend the available range of symbols (e.g., φ , $\mathfrak{d}$).
- By changing the weight of a symbol to bold we denote that the symbol is either a vector, matrix, or a matrix operator (i.e., $\mathbf{r}$, $\mathbf{g}$, $\mathbf{f}$, $\mathcal{B}$, or $\mathbf{g}$, $\mathbf{f}$, $\mathcal{H}$).

- The hat ($\hat{\cdot}$) over a bold symbol is used to denote unitary vectors.

APPENDIX B: CONSTANTS OF THE PIECEWISE FIT

The constants determined by a nonlinear fit to the model of Eq. (16a) using the data of Fig. 4 are, for $\theta=0^\circ$,

$$\alpha_{i,j} = \begin{pmatrix} 0.81 & 0.833 & 0.814 & 0.825 \\ 0.554 & 0.553 & 0.559 & 0.547 \\ 0.177 & 0.133 & 0.143 & 0.128 \end{pmatrix}, \quad (B1)$$

$$\beta_{i,j} = \begin{pmatrix} 6.698 & 6.105 & 6.883 & 6.706 \\ 3.53 & 3.126 & 3.495 & 3.438 \\ 1.667 & 1.286 & 1.386 & 1.328 \end{pmatrix},$$

and for $\theta=30^\circ$

$$\alpha_{i,j} = \begin{pmatrix} 0.931 & 0.836 & 0.839 & 0.846 \\ 0.362 & 0.532 & 0.531 & 0.522 \\ 0.054 & 0.104 & 0.099 & 0.108 \end{pmatrix}, \quad (B2)$$

$$\beta_{i,j} = \begin{pmatrix} 4.59 & 5.90 & 4.18 & 6.60 \\ 2.26 & 2.98 & 2.25 & 3.23 \\ 0.385 & 1.01 & 0.914 & 1.18 \end{pmatrix}.$$

$i=(1,2,3)$ is the index of the terms in the Gaussian mixture model, and $j=(1,2,3,4)$ corresponds to the LRFs on the positive x - up to the negative y -axis counterclockwise obtained from the four edges of the Dy plate.

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