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Optimizing Multicompression Approaches to Elasticity Imaging

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Abstract

Breast lesion visibility in static strain imaging ultimately is noise limited. When correlation and related techniques are applied to estimate local displacements between two echo frames recorded before and after a small deformation, target contrast increases linearly with the amount of deformation applied. However, above some deformation threshold, decorrelation noise increases more than contrast such that lesion visibility is severely reduced. Multicompression methods avoid this problem by accumulating displacements from many small deformations to provide the same net increase in lesion contrast as one large deformation but with minimal decorrelation noise. Unfortunately, multicompression approaches accumulate echo noise (electronic and sampling) with each deformation step as contrast builds so that lesion visibility can be reduced again if the applied deformation increment is too small. This paper uses signal models and analysis techniques to develop multicompression strategies that minimize strain image noise. The analysis predicts that displacement variance is minimal in elastically homogeneous media when the applied strain increment is 0.0035. Predictions are verified experimentally with gelatin phantoms. For in vivo breast imaging, a strain increment as low as 0.0015 is recommended for minimum noise because of the greater elastic heterogeneity of breast tissue.

I. INTRODUCTION

Elasticity imaging is rapidly developing as a diagnostic technique for detecting and differentiating soft tissue diseases based on changes in local viscoelastic properties [1]–[3]. In breast imaging for lesion detection, compressive strains greater than 1% of the total tissue thickness are often applied to generate sufficient contrast resolution. The stiffness of heterogeneous breast tissues range by more than an order of magnitude [4]. When the range is large, greater deformations must be applied to ensure each region is strained to generate signals within the dynamic range of the technique. To strain the stiffest tissues (e.g., tumors), the softer surrounding regions can become so highly strained that the radio frequency (RF) ultrasonic signals from sequential echo frames decorrelate. Echo decorrelation significantly increases displacement errors generated by correlation-based strain estimators and is a major source of strain image noise. The accumulation of displacements from many small deformations can provide the necessary contrast resolution for large deformations, although the echo noise increments from each step will accumulate to again decorrelate echo signals. To find the combination of system parameters that yield the smallest net noise, we are developing methods for predicting estimation errors from all major sources. Because the estimation bias is relatively small, less than 10% of the mean [5], our focus is on displacement variance.

Variance bounds are predicted from knowledge of the echo coherence function [6]. Coherence functions can reveal how the physics of tissue deformation and associated measurement instrumentation determine displacement errors [7], but only if the signal models accurately represent the echo data. This paper begins with a model of ultrasonic echo signals in deformed media. It leads to the coherence function used to predict the Cramér-Rao lower bound for

displacement variance. The variance bound is the criteria for optimizing the design of multicompression strain imaging methods.

II. BACKGROUND

A. Echo Signals

The following signal model combines features of two models previously described [7], [8], $\mathbf{g}_j$ is an $M \times N$ matrix of RF data samples that defines the j -th recorded echo frame. Each data sample in the frame $g_j[m, n]$ results from a linear transformation of a continuously varying two-dimensional (2-D) scattering field $f_j(\mathbf{x})$ through a linear system characterized by a shift-varying, spatiotemporal impulse response $h(\mathbf{x}, \mathbf{t})$ [8]:

$$\mathbf{g}_j = \int_{-\infty}^{\infty} d\mathbf{x} h(\mathbf{x}, \mathbf{t}) f_j(\mathbf{x}) + \mathbf{e}_j. \quad (1)$$

The $M \times N$ matrix $\mathbf{e}_j$ represents signal-independent echo noise. The vector $\mathbf{x} = (x, y)$ describes continuous 2-D spatial coordinates (axial and lateral), and $\mathbf{t} = t[m, n]$ describes the analogous discrete temporal coordinates (range and cross range), where $0 \leq m \leq M-1$ and $0 \leq n \leq N-1$. Linear transformations of these coordinate systems define spatial deformations of the scattering function and warping of echo signal time series. Data are recorded sequentially in time $t[m, n] = (m+nM)T$ where m is the fast-time index (range) and n is a slow-time index (cross range); T is the RF sampling interval, MT is the pulse repetition interval, and NMT is the frame interval.

Arbitrary deformations of continuous objects or sampled data can be modeled as a set of affine transformations of the coordinates $\mathbf{x}$ or $\mathbf{t}$ that are applied to segmented regions of the object, Ω_i [7]. Smooth deformations allow coarse segmentation. A physical compression of the object is specified by a translation vector $\mathbf{a}$ and scaling matrix $\mathbf{A}$ for that region. Similarly, the local companding parameters $\mathbf{b}$ and $\mathbf{B}$ allow us to warp echo signals similar to the physical deformation. Assume we record echo frame j before object deformation, frame $j+1$ after, then apply local companding to the j -th frame such that $\mathbf{b} \approx \mathbf{a}$ and $\mathbf{B} \approx \mathbf{A}$. For echo signals in the region $t[m, n] \in \Omega_i$, there will be $M'N'$ ($M' \leq M$, $N' \leq N$) companded precompression samples given by:

$$\mathbf{g}_j = \int_{-\infty}^{\infty} d\mathbf{x} h(\mathbf{x}, \mathbf{B}^{-1}\mathbf{Q}^{-1}\mathbf{t} - \mathbf{b}) f_j(\mathbf{x}) + \mathbf{e}_j(\mathbf{B}^{-1}\mathbf{Q}^{-1}\mathbf{t} - \mathbf{b}), \forall t[m, n] \in \Omega_i. \quad (2)$$

Corresponding echo samples in the postcompression frame are given by:

$$\mathbf{g}_{j+1} = \int_{-\infty}^{\infty} d\mathbf{x} h(\mathbf{x}, \mathbf{t}) f_j(\mathbf{A}^{-1}\mathbf{x} - \mathbf{a}) + \mathbf{e}_{j+1}. \quad (3)$$

The matrix:

$$\mathbf{Q} = \begin{bmatrix} 2/c & 0 \\ 0 & MT/Y \end{bmatrix},$$

defines the space-time transformation $\mathbf{t} = \mathbf{Q}\mathbf{x}$ and Y is the spatial increment along the cross range axis. Expressions (2) and (3) are more realistic than those reported previously [7] because they are sampled in time and include the spatially varying spatiotemporal impulse response of the instrument.

B. Coherence Function

Let $G_j[k, \ell]$ and $G_{j+1}[k, \ell]$ be the 2-D discrete Fourier transforms of $g_j[m, n]$ and $g_{j+1}[m, n]$, respectively. The complex coherence function [7]:

$$\gamma_g[k, \ell] = \frac{E\{G_j^*[k, \ell] G_{j+1}[k, \ell]\}}{\sqrt{E\{|G_j[k, \ell]|^2\} E\{|G_{j+1}[k, \ell]|^2\}}}, \quad (4)$$

is a frequency-domain measure of similarity for the two data sets. $E\{\cdot\}$ denotes expectation and G^* is the complex conjugate of G . The range of the magnitude squared coherence (MSC) is $0 \leq |\gamma_g|^2 \leq 1$. One advantage of this measure is its ability to be separated into the MSC for the scattering function $|\gamma_f|^2$, which defines properties of the physical deformation, and a net signal-to-noise ratio (SNR), which define properties of the instrumentation:

$$|\gamma_g[k, \ell]|^2 = \frac{|\gamma_f[k, \ell]|^2 \text{SNR}[k, \ell]}{1 + \text{SNR}[k, \ell]}. \quad (5)$$

The SNR is a function of the channel signal-to-noise ratios S/N_j and S/N_{j+1} for the warped-pre- and post-compression echo waveforms, respectively:

$$\text{SNR}[k, \ell] = \frac{S/N_j[k, \ell] S/N_{j+1}[k, \ell]}{1 + S/N_j[k, \ell] + S/N_{j+1}[k, \ell]}. \quad (6)$$

Each function was described in detail previously [7].

C. Variance Bound

Another advantage of the MSC function is its relationship to the Cramér-Rao lower bound of displacement variance [9]. For uniaxial compression of a 2-D incompressible medium that generates strain ϵ , the lower bound on variance for displacements estimated along the beam axis, τ , is given by [7]:

$$\text{var}(\tau) \geq \frac{(cM'T)^2}{32\pi^2(1-\epsilon)^2} \left(\sum_{k, \ell \in \Omega_i} k^2 \frac{|\gamma_g[k, \ell]|^2}{1 - |\gamma_g[k, \ell]|^2} \right)^{-1}, \quad (7)$$

where c is the longitudinal sound speed and $E\{\tau\} = 0$.

Strain variance is approximated from displacement variance by propagation of error [10]:

$$\text{var}(\epsilon) \simeq \frac{2\text{var}(\tau)}{X\Delta X}, \quad (8)$$

where X is the correlation window length and ΔX is the strain pixel size. Both parameters are measured in units of length. Relative strain error is defined as $\text{std}(\epsilon)/\epsilon = \text{var}^{1/2}(\epsilon)/\epsilon$.

III. METHODS

A. Controlled Phantom Deformation

Data were acquired from a 5-cm cubic block of gelatin in which fine graphite scattering particles were randomly suspended [11]. The graphite concentration is 3.26% by weight. The gelatin block was compressed uniformly from above by a transducer-embedded compressor plate, and the opposite surface was held fixed. The cube was confined on two additional surfaces by lubricated rigid plates to approximate a plane-strain deformation in the scan plane with free slip boundaries. Two opposing surfaces normal to the scan plan were free to expand under compression. The experimental geometry is said to be in a plane-strain state because displacements were confined to the imaging plane. Out-of-plane motion is avoided to minimize decorrelation noise.

The 5-cm block was compressed with a motion controller by displacing its top surface downward at a rate of 0.8 mm/s while RF data were acquired from a linear array at 29 frames per second. The first $C = 96$ frames were used for variance estimation. The top surface was displaced 2.70 mm in 3.38 s for a total strain of $\epsilon_T = 0.054$ or 5.4%. The applied strain per step was $\epsilon_j \approx 0.055\%$. Displacement variance was found for one-step compression by analyzing echo frames 1 and 49 with 2.7% strain. A two-step multicompression technique summed displacements computed between frames 1 and 25 to that computed between frames 25 and 49. Using this method, displacement and strain variances were found by applying 2.7% strain in $C' = 1, 2, 3, 4, 6$, and 8 steps (Fig. 1). Forty-eight pairs of echo frames with the same relative deformation [e.g., frames (1,49), (2,50), ... (48,96)] were examined to compute means and standard deviations of the variance estimates. The standard deviations are plotted as error bars in the plots below.

B. Displacement Estimation

Displacements are measured using standard 1-D cross-correlation techniques. Displacement fields are computed, spatially registered, and summed before calculating the variance. Because the applied strain was known and the medium is elastically homogeneous and incompressible, spatial registration was accomplished by scaling and translating estimates using global companding techniques [12]. Displacements increased approximately linearly with depth, as illustrated in Fig. 2(a). We needed to compute the local mean for variance estimation. Therefore, we fit the 1-D displacement estimates to a first-order polynomial to measure the mean axial displacement field, which is subtracted from the result. But there are still displacement nonstationarities in the lateral direction, so we fit the displacement laterally using a fourth-order polynomial, and we subtracted it. Then we computed the mean square displacement as a variance estimate. Fitting was necessary to eliminate small irregularities, such as gelatin heterogeneities, nonuniform boundary effects, nonstationarities, and shift-varying impulse responses. Fourth order was the lowest order polynomial for which results did not vary with gel sample or experimental trial. An example of displacements after subtraction of the local mean is found in Fig. 2(b).

The analysis region was rectangular and centered at the transmission focus. Displacement variance was computed from every fourth A-line to minimize correlation between individual estimates.

For the three windows compared in Figs. 2(c) and (d), the largest sizes produced the lowest variance. However, we must consider a balance between image noise and spatial resolution.

Therefore, we select a window length of 4.93 mm as a compromise. With this window length, the variance closest to the bound occurs when the analysis region is 5.0 mm (range) $\times$ 2.4 mm (cross range). These parameters are used below in the phantom data processing to verify predictions. The center frequency was 6.67 MHz, the -6 dB fractional bandwidth was 0.6, the sampling rate was 40 Msamples/s, and the line density was 66.75 lines/cm.

C. Variance Estimation

The sample displacement variance for a C' -step multicompression technique is [13]:

$$\begin{aligned} \text{var}_{C'}(\hat{\tau}) &= \text{var}(\hat{\tau}_1 + \hat{\tau}_2 + \dots + \hat{\tau}_{C'}) \\ &= \sum_{j=1}^{C'} \text{var}(\hat{\tau}_j) + \sum_{j=1}^{C'} \sum_{j' \neq j}^{C'} \text{cov}(\hat{\tau}_j, \hat{\tau}_{j'}), \end{aligned} \quad (9)$$

where $1 \leq C' \leq C$ and $\text{cov}(\hat{\tau}_j, \hat{\tau}_{j'})$ is the covariance between the j th and j' th compression steps. If the estimates to be summed are uncorrelated and the compression steps are equal:

$$\text{var}_{C'}(\hat{\tau}) = C' \text{var}(\hat{\tau}_j), \forall j \text{ where } 1 \leq C' \leq C. \quad (10)$$

D. Point Spread Function

Measurement of the temporal impulse response [i.e., point spread function (PSF)] is an important part of MSC and variance-bound computations. The PSF was approximated by scanning a taut human hair (~ 0.04 mm diameter) embedded in clear congealed agar, oriented normal to the beam axis, and positioned 2 cm from a 6.67 MHz linear array, near the elevational focus. Digital RF echo data were recorded using the Ultrasound Research Interface (URI) on a Siemens Antares system at 40 Msamples/s (Siemens Medical Systems, Ultrasound Group, Issaquah, WA). The PSF was found by fixing the position of the hair at $\mathbf{x} = \mathbf{x}_0$ and averaging echo samples from 20 data frames to minimize echo noise. The 2-D PSF $h(t[m], n, \mathbf{x}_0)$ and the magnitude of its temporal Fourier transform, the system response function $|H(u[k], \ell, \mathbf{x}_0)|$, are shown in Fig. 3. A Gaussian bandpass filter was applied to suppress a small direct current (DC) value. The hair phantom is a line source rather than a point source of scatter, but the approximation seems reasonable given the results presented in the next section.

A hydrophone was used to scan the transmitted pressure field of the linear array. The elevational focus of the transducer was located at a depth of 2 cm. The in-plane transmit focus was set to 2 cm, and all data acquisitions were centered about this range.

E. Echo Signal-to-Noise Ratio

If the random processes are wide-sense stationary and the impulse response is invariant over time and space for all $m, n \in \Omega_i$, then the echo SNR (eSNR) is given by [14]:

$$\text{eSNR} = \frac{\rho}{1 - \rho}, \quad (11)$$

where ρ is the correlation coefficient; (11) is derived in the Appendix for our signal model. Scattering f and noise e functions are assumed to be independent, identically distributed, Gaussian random processes given by, respectively, $\mathcal{N}(0, V)$ and $\mathcal{N}(0, 1)$. V is the ratio of variances, $\text{var}(f)/\text{var}(e)$, computed from eSNR and the impulse response:

$$V = \frac{\text{var}(f)}{\text{var}(e)} = \frac{\text{eSNR}}{\int_{-\infty}^{\infty} d\mathbf{x} h^2(\mathbf{x}, t_0)}. \quad (12)$$

IV. RESULTS

A. Variance Bounds

We used (7) and Monte Carlo techniques [7] to predict the lower variance bound for displacement variance. From phantom measurements we determined eSNR (43 dB), then we used eSNR along with the PSF to compute the variance ratio, which is found to be $V = 398$. The predicted results are an average of 100 trials. For (7) to hold, we must assume uniaxial compressive strain, Poisson's ratio of ~ 0.5 , and an average displacement of zero; $\mathbf{a} = \mathbf{0}$.

Experimental results are plotted together with predicted values for single-compression strain estimation in Fig. 4(a). Extrapolating to lower strain values, we see that the variance changes very little for applied strains less than 0.1%. In this range, electronic noise dominates displacement variance. Above 0.1%, strain-induced decorrelation dominates variance. Strains larger than 2% generate very large displacement errors. Because the experimental data match the predictions, we find that cross correlators are efficient estimators of displacement.

We selected the largest measured strain in Fig. 4(a), 2.7%, as the total strain value for the multicompression studies. Predicted and measured values are plotted in Fig. 4(b). We found a 78% reduction in displacement variance for a six-step multicompression technique relative to a single compression technique. It seems that the predictions break down for strains above 2% in both plots in what we believe is a violation of the small-strain assumption inherent in the Cramér-Rao approach.

B. The Importance of Covariance

In this section, we evaluate the importance of displacement covariance in the estimation of multicompression variance given by (9). Covariances were included in the data of Fig. 4. These results are reproduced in Fig. 5(a), in which we also include displacement variance estimates assuming between-step covariances are all zero. The analysis region for this phantom data was $2.4 \text{ mm} \times 5.0 \text{ mm}$ (range $\times$ cross-range) and was centered about the focal length of the transducer. The correlation window is 4.93 mm with 93% overlap. For this small analysis region, the covariance is relatively small. The 6×6 covariance matrix for the six-step multicompression technique shown in Fig. 5(b) has off-diagonal elements that are positive and negative.

This analysis was repeated after increasing the region of interest in the phantom to $5.0 \text{ mm} \times 12.3 \text{ mm}$ about the focal length; see Figs. 5(c) and (d). We find an order of magnitude increase in both individual variance and covariance terms, and the latter are all positive. The fact that the variance terms depend on the region of interest despite our efforts to subtract the mean described in Fig. 2 suggests that the nonstationary properties of the medium deformation that interact with a shift-varying system impulse response contribute significantly to displacement errors and ultimately strain noise. We conclude that covariance terms cannot be ignored in (9), particularly for large regions of interest.

C. Optimal Strain Increment

Fig. 4 and Fig. 5 validate the analysis and show us situations in which the variance bound predicts the measured variance. With this guidance, we now use (9) to design multicompression techniques that yield minimum measurement variance.

The family of curves plotted in Fig. 6(a) predicts how displacement variance depends on the number of compression steps when the total compression varies. The optimal number of compressions for each total applied strain value is summarized in Table I. Expression (8) indicates that the strain variance is proportional to the displacement variance if the correlation window and the shift are fixed. We then plotted strain standard deviation divided by the mean in Fig. 6(b). The data in both figures suggest that multicompression techniques can reduce errors when the total applied strain is greater than 0.5%. Minimum displacement and strain errors occur in elastically homogeneous media when the increment of applied strain is approximately 0.35%.

D. Experimental Parameters Influencing Strain Errors

The model predicts that echo noise and the system impulse response ultimately limit strain noise. To illustrate this point, relative strain errors were computed from displacement variances using (8), in which correlation window length is $X = 4.93$ mm with 93% overlap, $\Delta X = 0.31$ mm. In Fig. 7, relative strain errors are plotted for single compression (left column) and multicompression (right column) techniques in which eSNR, bandwidth, or beam width are varied, but the other two parameters are held fixed.

In elastically homogenous media scanned with a commercial imaging system in which eSNR = 43 dB, we found that the minimum strain noise was obtained by limiting the applied strain per compression step to approximately 0.35%. The optimal strain increment is reduced significantly when analyzing clinical data acquired with the same imaging conditions. The reason for this is straightforward. To avoid decorrelation noise, the data from Table I suggest that we keep the maximum strain less than about 0.35% everywhere in the imaging field. When the stiffness throughout the object is uniform, the local strains are uniform and approximately equal to the applied value. When the object stiffness varies with location, however, stiffer tissues cause the surrounding regions to strain more. Thus we must lower the average applied strain to keep the maximum strain per compression step at or below 0.35%.

Relative strain errors decrease with the amount of applied strain up to between 1 and 2% for typical system parameters. Varying the eSNR produces the greatest effect on strain errors. Fig. 7(a) shows that increases in eSNR of 10 dB yield reductions in relative strain errors by as much as 10 dB. The reduction in error is much less at larger applied strain values in which decorrelation noise dominates. Low echo noise [see eSNR = 66 dB curve in Fig. 7(b)] reduces strain errors overall, and it suggests that we should form strain images by summing many tiny compressions to reach the minimum point. However, in high echo noise conditions (eSNR = 36 dB) minimum strain error is found by using just one or two compressions. The total applied strain is 2% for all multicompression in Fig. 7(b).

Changes in the bandwidth or beam width of the pulse merely scale the strain error curves in predictable ways; Figs. 7(c)–(f) show that errors are monotonically reduced by increasing the bandwidth and more tightly focusing the beam. Increasing the bandwidth or decreasing the beam width reduces the PSF volume, which reduces the amount of strain on that spatial scale and thus reduces echo decorrelation. These data show that pulse dimensions, normally important for spatial resolution, also significantly influence strain noise.

E. Preliminary Clinical Studies

In vivo breast data from four patients were studied for noise minimization using the multicompression technique. Compression to the patient breast was applied by free hand with a transducer-embedded compressor plate. Three sets of data, case Nos. 1–3 in Table II, were acquired using the Siemens Elegra ultrasound system with a 7.5 MHz linear array (7.5L40) transmitting 7.2 MHz broadband pulses. The RF signals were digitized at 36 Msamples/s, and the effective RF data density was 72 Msamples/s for processing with a bandwidth of 18 MHz. The frame rate is 16 frames per second, and the correlation window length is 10.9 mm. Case No. 4 was acquired using the Siemens Antares system with a faster 52 frames per second rate. The total applied compression is 1.4%. We needed a larger decorrelation window length of 19.7 mm to smooth images and lower strain noise. But the spatial resolution is also reduced.

For each set of data, the image with the highest number of compression steps (usually the smoothest one) is regarded as the mean. The mean is subtracted from other images with fewer compression steps, and the standard deviations of the strain are computed and compared with the predicted lower bound. Fig. 8 illustrates the data from case No. 4, showing how the strain standard deviation changes with the number of multicompression steps for in vivo breast imaging. Minimum strain error occurs at 12 compression steps corresponding to a strain increment of 0.12%.

Also from data in [15], we find the optimal strain increment is reduced to between 0.10%–0.20%, which is less than half of the homogeneous phantom results (0.35%). The greater heterogeneity of breast tissue requires a larger number of small compression steps to minimize noise for clinical elastography. An average strain increment of 0.15% per step would be a better choice for in vivo breast imaging.

V. DISCUSSION

When the applied strain is small ($<0.1\%$), displacement estimates are strain independent and limited by echo noise as shown in Fig. 4(a). For large applied strains ($>2\%$), the large loss of echo coherence dominates estimation errors. Furthermore, the variance bound is not able to predict measured values as assumptions are violated. For $0.1\% \leq \epsilon_T \leq 2\%$, we can rely on (7)–(9) to predict the number of compression steps that minimize strain noise for small homogeneous regions of interest.

The optimal strain increment value of 0.35% was found for the parameters of the Antares system and gelatin phantom: 6.67 MHz, 48% bandwidth (–6 dB), eSNR = 43 dB, and 0.74 mm beam width (–6 dB). Others have measured the optimal strain increments in phantoms to be in the range 0.33–0.4% via 1-D simulated echo data [16]. So we entered their system parameters into our analysis: 5.0 MHz, 60% bandwidth (–3 dB) or 85% bandwidth (–6 dB), and eSNR = 40 dB. To convert their analysis results into our 2-D equations, we assumed an $f/2$ aperture. Our analysis predicts the optimal strain increment for their parameters to be 0.49%. The difference in results points out the need to carefully consider the dimensionality of the analysis when making predictions.

VI. CONCLUSIONS

We found that the optimal strain increment under typical breast imaging conditions is approximately 0.35% if the medium is relatively homogeneous. This value is reduced as the heterogeneity of the tissue increases and as the eSNR increases, suggesting that 0.15% strain per step is recommended for a large dynamic range and in high eSNR conditions. These results were found in phantom data and in vivo breast scans. The equations given in this report, which

assume a uniformly stiff medium, allow one to design minimum variance multicompression methods provided that the strain increment is less than 2%.

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Appendix

APPENDIX A

The following is a derivation of eSNR in terms of a correlation coefficient ρ between a region Ω_i in two echo frames. The result is equivalent to that found in [14].

Assume 1-D sequences $g_j[m]$ and $g_{j'}[m]$ are recorded from two echo frames without compressing the object. A time-averaged cross correlation estimate $\bar{\phi}$ from samples in Ω_i is:

$$\bar{\phi}_{g_j g_{j'}}[m'] = \frac{1}{M'} \sum_{m \in \Omega_i} g_j[m] g_{j'}[m - m']. \quad (13)$$

The time-average correlation function is found from the estimate by taking the expected value:

$$\begin{aligned} \bar{\phi}_{g_j g_{j'}}[m'] &= \frac{1}{M'} \sum_{m \in \Omega_i} E\{g_j[m] g_{j'}[m - m']\} \\ &= \frac{1}{M'} \sum_{m \in \Omega_i} \phi_{g_j g_{j'}}[m, m - m'] \\ &= \phi_{g_j g_{j'}}[m'], \end{aligned} \quad (14)$$

where the last form is the ergodic property that hold only if the zero-mean object and noise processes are both widesense stationary (WSS). Practically, we have access only to (13). Therefore, the correlation coefficient is approximated by:

$$\rho \simeq \frac{\bar{\phi}_{g_j g_{j'}}[0]}{\sqrt{(\bar{\phi}_{g_j g_j}[0] \bar{\phi}_{g_{j'} g_{j'}}[0])}}. \quad (15)$$

Expanding (14) using (1) and assuming the impulse response is linear and time invariant over Ω_i , we find:

$$\begin{aligned} \bar{\phi}_{g_j g_{j'}}[m'] &= \frac{1}{M'} \sum_{m \in \Omega_i} \int_{-\infty}^{\infty} d\mathbf{x} h(mT - 2|\mathbf{x}|/c) \\ &\cdot \int_{-\infty}^{\infty} d\mathbf{x}' h(mT - m'T - 2|\mathbf{x}'|/c) E\{f_j(\mathbf{x}) f_{j'}(\mathbf{x}')\}. \end{aligned} \quad (16)$$

If the random scattering medium is WSS, then $E\{f_j(\mathbf{x}) f_{j'}(\mathbf{x}')\} = \text{var}(f) \delta(\mathbf{x} - \mathbf{x}')$. Consequently:

$$\begin{aligned}\bar{\phi}_{g_j g_{j'}}[m'] &= \frac{\text{var}(f)}{M'} \sum_{m \in \Omega_{g_j}} \int_{-\infty}^{\infty} dx h^2(mT - 2|x|/c), \\ \bar{\phi}_{g_j g_{j'}}[0] &= \text{var}(f) \int_{-\infty}^{\infty} dx h^2(2|x|/c),\end{aligned}\quad (17)$$

and:

$$\begin{aligned}\bar{\phi}_{g_j g_j}[0] &= \text{var}(f) \int_{-\infty}^{\infty} dx h^2(2|x|/c) \\ &+ \text{var}(e) = \bar{\phi}_{g_{j'} g_{j'}}[0].\end{aligned}\quad (18)$$

Combining (15)–(18), we find the expression for eSNR in (11). eSNR can be defined only this way for WSS random media in which the system impulse response is linear and invariant over space time. The results are approximate as the noise terms in (16)–(18) only cancel completely when temporal averages approach ensemble averages, i.e., (14). Also because eSNR values vary with the scattering source, data from uniformly random phantoms having tissue-like properties and acquired in an isoplanetic region satisfy these assumptions.

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Biographies

Huini Du received her B.S degree from Shanghai Jiaotong University, P.R. China in 2002 and her M.S degree from the Biomedical Engineering department of the University of California, Davis, Davis, CA, in 2004. Her thesis was on multicompression method in breast elasticity

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Claire Pellot-Barakat obtained a Ph.D. in Biophysics from the University of Paris XI, Paris, France, in 1991. She has been a researcher at INSERM (French National Institute of Health and Medical Research) since 1993. She came to the University of California, Davis, Davis, CA, in 1995, first as a visiting scientist in the Department of Radiology and since 2003, as a research assistant professor in the Department of Biomedical Engineering. Her research interests for

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Michael F. Insana (M'85) received his Ph.D. in medical physics from the University of Wisconsin-Madison, Madison, WI, in 1983. His dissertation topic was acoustic scattering and absorption in biological tissues. He was a research physicist at the FDA from 1984–1987, where he developed methods for describing tissue microstructure from the statistical properties of echo signals. From 1987–1999 he was with the Department of Radiology at the University of Kansas Medical Center. There he directed an imaging research lab that applied ultrasonic imaging to the evaluation of progressive renal failure and breast cancer. Between 1999 and 2004 Mike was Professor of Biomedical Engineering at the University of California, Davis, Davis, CA, where he directed the graduate program. He is currently Professor of Bioengineering at the University of Illinois at Urbana-Champaign, Urbana, IL. His research includes the development of novel ultrasonic instrumentation and methods for imaging soft tissue microstructure, viscoelasticity, and blood flow. The goal is to understand basic mechanisms of tumor formation and responses to therapy. He is also interested in the principles of imaging system design and performance evaluation, signal processing, detection and estimation. He is a member of the IEEE and ASA, Fellow of the Institute of Physics, and Associate Editor for *IEEE Transaction on Medical Imaging*.

Fig. 1. Frame selection for multicompression estimates with varying number of steps but constant total strain ϵ_T .

Fig. 2.

(a) Spatially registered displacement field before subtraction of the mean. (b) Displacement field after subtraction of the mean. (c) and (d) Show that violation of the stationary, shift invariance assumptions lead to increases in variance estimates with field of view compared to the predicted variance bound. In (c), the lateral dimension on the field is fixed at 2.4 mm, and the axial dimension (abscissa) varies between 1.2 mm and 18 mm. Results depend on the correlation window size: 2.46 mm, 4.93 mm, and 9.86 mm. In (d), the axial dimension is fixed at 5 mm, and the lateral dimension is varied between 0.6 mm and 9 mm.

Fig. 3. Point spread function of the linear array (top) and the magnitude of its 2-D Fourier transform (bottom) are shown.

Fig. 4.

(a) Displacement variance for single compression estimates as a function of strain. Correlation window size for experimental data is 4.93 mm. The axial dimension of strain pixel is 0.31 mm so that the window overlap is 4.62 mm. (b) Displacement variance for multicompression technique in which $\epsilon_T = 0.027$. The lowest variance is found using six steps. Error bars indicate one standard deviation.

Fig. 5.

(a) The circles represent the same data as the one plotted in Fig. 4(b), in which the covariance was included in the calculation. Ignoring the covariances led to an overestimation of the displacement variance. (b) Covariance matrix for a six-step, multicompression technique. (c) Computing variance at 12.3 mm by 5 mm image area. (d) Covariance matrix for six-step, multicompression choosing a larger area than in (a) and (b).

Fig. 6.

Predicted displacement variance bounds (a) and relative strain errors (b) are shown using the 2-D PSF acquired from the Antares system and the gelatin phantom: 6.67 MHz, 48% bandwidth (−6 dB), 0.74 mm beam width (−6 dB) and eSNR = 43 dB (converted to variance ratio $V = 398$), correlation window 4.93 mm. If the total strain is less than 0.5%, single compression gives the smallest variance. For $\epsilon_T > 0.5\%$, the concave curves indicate that multicompression techniques reduce errors.

Fig. 7.

Plots in the left column show predicted strain errors versus applied strain using a single compression technique. Plots in the right column show strain errors versus number of compressions using a multicompression technique in which the total applied strain is 2%. The center frequency of the Gaussian pulse spectrum is 5 MHz and the sampling rate is 50 MHz. (a), (b) The -6 dB bandwidth (3.5 MHz) and -6 dB lateral beam width (0.12 mm) are held constant while varying eSNR. (c), (d) eSNR ($V = 1000$) and beam width (0.12 mm) are held constant while varying bandwidth. (e), (f) eSNR ($V = 1000$) and bandwidth (3.5 MHz) are held constant while varying beam width.

Fig. 8.

Strain standard deviation is plotted versus the number of multicompression steps for in vivo breast data obtained from a patient (solid line). The minimum value occurs at 12 compression steps. The dashed line displays the predicted error bound for a homogeneous medium, which is minimum at four compression steps.

TABLE I

NUMBER OF STEPS IN MULTICOMPRESSION TECHNIQUE THAT YIELD MINIMUM DISPLACEMENT VARIANCE.

Total strain (%)	Number of compressions	Strain increment (%)
0.1	1	0.10
0.2	1	0.20
0.3	1	0.30
0.4	1	0.40
0.5	2	0.25
1.0	3	0.33
1.5	4	0.38
2.5	8	0.31
3.5	12	0.35
4.5	12	0.38
5.5	16	0.34
6.5	19	0.34
7.5	20	0.38

TABLE II

MULTICOMPRESSION TECHNIQUE FOR MINIMUM STRAIN STANDARD DEVIATION IN CLINICAL STUDIES.

No.	Total strain (%)	Steps	Strain increment (%)	Patient status
1	2.0	12	0.16	healthy
2	4.0	15	0.26	healthy
3	4.7	36	0.13	benign fibroadenoma
4	1.4	12	0.12	carcinoma