Dimension in Action: A Commentary on Powell and Dickie

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... whether or not a thing is measurable is not something to be decided *a priori* by thought alone, but something to be decided only by experiment.

—Richard P. Feynman

Powell and Dickie's paper is very stimulating. It evokes some quite fundamental issues regarding how we are to construct a science of behavior. Any such science depends upon consistent modes of measurement which, in turn, are founded explicitly or implicitly upon a conceptual framework—a theory—about what constitutes the basic units or dimensions of analysis. As the authors point out, there are prior theoretical commitments to response rate or duration as measures of "response strength." Their strategy is to: (1) construct a dimensionless set comprised of rate and duration—Kinetic output, K_o ; (2) test this measure on some extant data; and (3) select a criterion of order based on the smallest coefficient of variation to compare rate, duration, and Kinetic output.

The results are intriguing, if not satisfying. First of all, they are not as clear as the authors indicate. The average coefficients of variation for the duration measures and Kinetic output are nearly the same, both being less than that for rate. There are, moreover, complications in using variability alone to indicate orderliness. The real issue is how a measure concomitantly varies with manipulation of other variables, *i.e.*, whether the measure effectively enters into functional relations of interest. While baseline variability may be relatively large (an operant level might be an example), appropriate control procedures may yield consistent effects allowing for effective prediction. On the other hand, behavioral measures

may show relatively little within-subject variability, *e.g.*, those related to certain respondent or species-specific behaviors, and yet be difficult to modify by variable manipulation.

The application of dimensional analysis to the problem of appropriate measures is a bold and exciting move. Powell and Dickie do a service in introducing the method to the behavior analyst. I have some concerns about its present treatment in the context of behavior analysis, however. Dimensional analysis can be a powerful tool for the determination of sometimes complex functional relations holding between sets of variables whose measures are specifiable by products of a small number of fundamental units or dimensions. In physics a common set is length (L), mass (M), and time (T). Many physical variables can be expressed as products of powers of this basic set. Thus force may be expressed as:

$$F = MLT^{-2}$$

There is actually a deep field here with relations to group theory and to the solution of partial differential equations, but virtually every engineering student learns how to derive useful formulae by simple use of the method. Is there a role to play in behavior analysis? Certainly one has to be dimensionally consistent in writing equations describing behavioral events; and this necessity may be useful in generating the relations in the first place. In guessing about possible functions, one does not want to end up equating responses with reinforcers. But there are profound difficulties not shared with physics or engineering. In those fields the fundamental units are agreed upon with explicit methods of measurement and the classes of variables entering into functional relations are relatively small. These happy conditions emerge from effective theories which help one to specify reasonably well most, if not all, those vari-

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ables likely to enter into a functional relation. To take a simple and common example, we can use dimensional analysis to derive a relationship for the period of a simple pendulum. Elementary theory allows us to guess that l , the length of the pendulum, g , the acceleration of gravity, and θ , the angle of the pendulum bob from the vertical, might be important variables. Suppose, however, we wanted to derive a relation that would predict how many responses would be emitted (or how long responding would occur) during extinction after an extended history of responding under a given variable-interval schedule of reinforcement. What would we do? At present, not very much. We simply do not know enough about the dynamics of behavior, never mind the appropriate units of behavior to derive such point predictions. If there is anything that sets us apart from the physical sciences, it is a lack of consistent, fundamental measures or units of behavior with which one can formulate a general analytic framework.

The authors are attempting to construct a measure based upon response rate and duration. This is a different and more fundamental task than generating a functional relation, say between some characteristic of behavior and a set of predictor variables:

$$B = F(x_1, x_2, x_3, \dots, x_n).$$

Powell and Dickie are asking, in part, what should be the dimensional structure of B in order to provide for effective functional relations? If there is sufficient analytic development, entities or measures may *emerge* because they enter into functional relations commonly and consistently. Such is the case with the concepts of Kinetic and potential energy. But we have no such analytic development in behavior analysis, so one is left to play with common measures in the hope that some combination might prove useful.

There are problems with the Kinetic output measure suggested by Powell and Dickie. For example, the issue of the presumed constancy of K_0 is clouded by the considerable burden borne by the free parameter, z . The quantity inside the log-

arithm *must* be dimensionless to make mathematical sense. That expression contains a single dimension, time. According to Buckingham's theorem (McMahon & Bonner, 1983), if there were only a single independent dimension, then no dimensionless constant could be made from the set of variables. Thus, z must contain at least *two* other independent dimensions in order to provide a dimensionless constant. What could those dimensions be? What is more, the variables comprising z are combined so that z is itself dimensionless—a major assumption. Under these conditions it is hardly surprising that K_0 is not a constant.

The expected value of the variance of K_0 should theoretically be estimated from the variances of the distribution of rate and duration, but that turns out to be difficult mathematically because one has to deal with the integral of the products of correlated distributions. I haven't a clue as to how to do this, but it is possible to get an intuitive idea why it is difficult to guess what the variance of K_0 is from knowing the variances of the rate and duration. The total time, T , will be the sum of the response durations, D_i , and their interresponse times, IRT_i .

$$T = \sum_i^N D_i + \sum_i^N IRT_i$$

$$T/N = D_{av} + IRT_{av}$$

$$N/T = \text{rate} = R = 1/(D_{av} + IRT_{av}).$$

Thus, the rate, R , is a function of both the durations and the interresponse times, and changes in either or both can affect R . Of course, both could change in such a way that the rate would remain constant. This same logic would apply to duration or to interresponse time as the dependent variable. The correlation between rate and duration (consider rate as the dependent variable) will depend upon the variance of the durations and the IRTs and also their values relative to the total time. For example, the correlation would be zero if the response duration was constant and short relative to the total time.

This paper should provoke consider-

able comment and controversy; for that reason alone it would merit attention. Clearly, a good deal of careful thought went into this paper and *Behavior Analyst* readers should benefit in their wrestling with it.

REFERENCE

- McMahon, T. A., & Bonner, J. T. (1983). *On size and life*. New York: Scientific American Library.