Kinetic Output: A Conceptual, Dimensional and Empirical Analysis

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A dimensional analysis permitted behavioral frequency, duration and available time to be collapsed into a singular dimensionless expression called Kinetic output (K_o). An empirical analysis across several behaviors in some human subjects showed that the derived index of behavioral output, K_o , was more likely to exhibit less variability than rate or duration measures of the behavior. As a reduction in variability is synonymous with finding an increased order, the results tentatively support the notion that behavior may exhibit conformity at a previously undetected level. One major implication of the K_o formulation is that divergent frequency and duration performances are interpretable as reflecting alternate forms of the same equivalence.

Key words: kinetic output, quantitative integration, variability, dimensional analysis, response strength

Intrinsic to any nominally persistent behavior are two recurrent attributes, repeatability and extension. An event record pictorially displays the attributes as a sequence of initiations and terminations extending over time. Enumeration of initiations, and summation of the temporal extent across initiations and terminations yields, respectively, measures of behavioral frequency and duration. Enumerations as a function of time provide the transformed frequency measure, rate of the behavior.

Although any behavior may be reported as a rate and duration measure, usually just one dimension is selected. Several factors seem to be responsible for eschewing a bidimensional analysis in favor of a monodimensional analysis. The results of a bidimensional analysis can be awkward to describe and difficult to interpret, especially if one is interested in examining in some detail the behavior of individual subjects. For example, Rast, Johnston, Drum, and Conrin (1981) reported the rate and duration of ruminations across successive sessions, resulting in the need for separate graphic displays

for each variable. Evaluating the results of the investigation requires concomitantly using both displays to determine rate changes and trends, duration changes and trends, and the ongoing nature of the covariation between the two variables. The difficulty in evaluating behavior as a simultaneous function of its rate and duration characteristics may be overcome by collapsing the two measures into a single entity, bout duration, which equals duration divided by frequency. As pointed out elsewhere (Powell, 1984a), this is an unsatisfactory solution as oppositional changes in the numerator and denominator exaggerate the ratio, and tend to make the bout duration an unstable expression. The presence of divergent results may present the greatest difficulty when employing a bidimensional analysis. The necessary relationship between frequency and duration exists only in the singular, for each response includes an onset and some temporal extension. In the aggregate, e.g., across sessions, the two attributes may or may not covary. A number of studies culled from the literature serve to illustrate the kinds of covariation that can be expected (Bitgood, Peters, Jones, & Hathorn, 1982; Harris & Wolchik, 1979; Rast, Johnston, & Drum, 1984; Rast et al., 1981; Rose, 1978; Springer, Brown, & Duncan, 1981). Each of these studies reported the rate

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and duration of a behavior across sessions. Collectively, the investigations revealed instances where the two measures moved in the same direction, or in opposite directions, or one moved while the other remained constant, or both remained constant; all possible outcomes were represented. Failure to find a consistent relationship among constituent measures of behavior represents a long recognized but unresolved issue (Hall, 1976, p. 198; Kimble, 1961, p. 111).

The difficulties associated with a bi-dimensional analysis of behavior are avoided by selecting the single "best" attribute, the best attribute being the one that experience has revealed to be sensitive to a wide variety of experimental conditions. Skinner (1950) argued that rate is the appropriate attribute. "Rate of responding appears to be the only datum which varies significantly and in the expected direction under conditions which are relevant to the 'learning process'" (p. 198). Essentially the same argument is presented elsewhere (Skinner, 1953, p. 62; Skinner, 1969, pp. 109–113). Premack (1971) viewed things differently. "All responses, no matter how different their form, have in common the fact of extension in time. Further, given that certain standard assumptions are met, response probability can be estimated from response duration" (p. 128). Premack's use of the term response probability is convenient in the context of this discussion for it points out that the quantified attribute, be it frequency or duration, serves as an estimator of response probability. Moreover, response probability is itself an estimator of a conceptual entity, response strength, which may be viewed as a presupposed state that represents the organism's moment-to-moment propensity to exhibit overt behavior. The conceptual status of response strength renders it nonmeasurable, but it is necessarily viewed as directly proportional to the quantity chosen to represent it. For example, an observed doubling in the rate or duration of a behavior would not be interpreted as a doubling in the strength of the response, but rather as an increase in response strength; conversely, a dec-

rement in the measured attribute indicates a weakening in the strength of the response. It is at this conceptual level that failure to find a consistent relationship among constituent measures of behavior becomes troublesome. Consider the case where a single measure, either frequency or duration, is applied to an increasing frequency-decreasing duration change state. The measure employed will dictate whether an increase or decrease in response strength is inferred. As a single behavior cannot simultaneously satisfy both states, the failure to find covariation between the two measures appears to allow for the existence of a basic contradiction—the concomitant strengthening and weakening of a behavior. The articles cited above are cogent exemplars for they document the existence of observed change states analogous to the hypothetical one described here.

A DIMENSIONAL ANALYSIS

The above analysis leads to the thesis of this paper, which is it may be possible to describe behavior quantitatively at a level other than that of its individual frequency or duration characteristics. That is, it is conceivable that a change in the level of analysis may provide a new referent that reflects a stability in behavior that is simply not apparent at the frequency and duration level. The search for a new referent must begin with an identification of the variables thought to contribute to it. Once these variables are assembled, a dimensional analysis can be performed to determine if a suitable expression may be derived. A dimensional analysis functions to integrate to some degree the involved variables and, because it does so, it produces a new expression. A name needs to be selected to identify the new expression: Kinetic output (K_o) will do. We begin by assuming that $K_o = f(N, D, T, z)$ where N equals the number of responses, D equals the total duration of the N responses, T equals the time available to exhibit N and D , and z equals a variable that is the composite product of some unspecified number of free parameters. N is dimension-

less, D and T share the temporal dimension and we shall assume that z is dimensionless. The presence of a single dimensional entity, time, greatly restricts the number of possible expressions of K_o . The grouping, $N/(D \times T)$, is permissible for it yields an expression, responses per min^2 , which dimensionally defines response acceleration. Although a dimensionally permissible expression, it is difficult to justify since it requires combining the duration of the behavior and the length of the observation period. This combination is sufficiently strange so as to suggest that it be dismissed from consideration. The other permissible grouping assumes the form, $(N \times D)/T$, which yields a dimensionless expression since the time dimension cancels out. Dimensionless expressions are well represented in the physical world, e.g., an angle measured in radians, scales of hardness, stellar magnitude, earthquake intensity (Richter scale), relative velocity (Mach number), and fluid turbulence (Reynolds number). Also, it is less difficult to justify this grouping for it combines two output measures of behavior and makes the combined result relative to the time available to produce the output. This group represents a less strange notion, and it will be accepted as a possibility. Next, we take the log of this dimensionless group, for it permits a small scale to accommodate large numbers, and then we shall take z times this expression, or $K_o = z \log[(N \times D)/T]$. And finally, because there is no means of initially determining the value of z , it will be assigned a value of 1.00. The final form is, then, $K_o = \log[(N \times D)/T]$. The expression has some predictable properties. If there is no behavior, the expression is undefined; if the ratio of $(N \times D)/T$ is greater (less) than 1.00, K_o will be positive (negative). In this latter respect K_o is like the Richter scale which yields a positive (negative) value for shocks greater than (less than) a standard shock value.

AN EMPIRICAL ANALYSIS

Although the equation has been derived via a dimensional analysis, it is un-

likely that many will rush to embrace it on that basis alone. What is needed is an empirical test where K_o may be directly compared to the two competing quantities, rate and duration. The objective of the comparison should be a determination, if possible, of the expression that yields the most orderly outcome. This is the same as saying that the quantity that exhibits the least variability lays *some* claim to being considered the preferred expression. The suggested comparison requires assessing the variability of three quite different formulations. Fortunately, there is a simple descriptive statistic with a long and continued use in the biological sciences that allows for the direct comparison of the variability in dimensionally different quantities. The statistic is the Coefficient of Variation (V) which equals the $(\text{SD}/\text{Mean}) \times 100$ (see Zar, 1984). An appropriate empirical test thus becomes one of examining a data base of known frequency and duration, deriving K_o from the frequency and duration data, and then calculating and comparing V for the three quantities.

Subjects and Procedure

The data base to be reported on, generated at a private educational facility for disturbed youth, was previously reported relative to the issue of observer agreement (Powell, 1984b). For purposes of this report some pertinent aspects of this study are as follows. Ten persons ranging in age from 11 to 17 who were diagnosed as autistic or autistic-like were observed for 17 to 25, 30-min sessions in a classroom environment. Stereotypic and attending behaviors were observed and recorded. The stereotypic topographies were diverse: rubbing, twirling or pulling hair; body rocking; hand waving; finger tapping; staring at a hand or object held in front of the face; rubbing an arm or leg, etc. During observation periods the students were involved in tasks such as putting together large puzzles; sorting objects according to color, shape, size and texture; stringing beads in various sequences; and working on a Language Board to promote communication skills.

TABLE 1

A comparison of the variability in the rate, duration and Kinetic output for two behaviors

Ss	Stereotypic						Attending					
	r/min	V^a	min	V	K_o^b	V	r/min	V	min	V	K_o	V
1	0.7	71	3.7	50	0.35	84	0.9	70	10.4	50	0.87	40
2	1.9	48	10.2	45	1.21	30	1.4	47	14.0	32	1.22	23
3	1.3	61	5.5	59	0.75	60	1.1	88	22.3	18	1.37	23
4	1.6	43	14.2	35	1.35	20	1.4	78	16.2	43	1.20	32
5	2.3	62	8.7	43	1.19	33	1.4	57	16.4	28	1.40	30
6	3.0	48	11.4	40	1.46	21	1.3	62	12.7	37	1.15	34
7	1.0	54	3.5	49	0.47	85	0.8	70	21.2	17	1.17	26
8	2.5	59	15.3	26	1.48	24	1.4	75	4.8	67	0.68	89
9	3.0	59	19.5	24	1.67	20	1.2	74	17.8	28	1.27	31
10	1.2	62	6.0	64	0.76	62	0.6	59	16.1	33	1.05	26

^a $V = (SD/Mean) \times 100$.

^b $K_o = \text{Kinetic output} = \log[(N \times D)/T]$.

Active involvement in these activities constituted attending.

A direct observation procedure, described in detail in the Powell article, was used to generate rate and duration measures for both types of behavior. Briefly, in a predetermined sequence each student was observed using a 5 s observe–5 s record format to provide a total of 30 observations per session per student: a prerecorded cassette signalled the observe and record periods. An observer viewed a subject for 5 s, and in the subsequent 5 s recorded for both behaviors (a) whether the behavior had been initiated within the preceding 5 s interval, and (b) whether the behavior was ongoing exactly at the end of the preceding interval. The initiation and end-of-interval scores were transformed into rate and duration measures as described in the above article. Prior to beginning the formal collection of data, two experienced observers were given extensive practice in the recording procedure. As reported previously, a mean occurrence agreement of 90% or greater for initiations and continuations for both behaviors was achieved during formal data collection procedures.

Results and Discussion

The results are contained in Table 1. For each subject and behavior, the table

lists the mean rate of the response, the mean duration of the response and the mean of the derived value for each session, K_o . The adjacent column for each of these quantities gives the corresponding V value in percent. In evaluating V it was arbitrarily assumed that no difference existed between any two values if the absolute difference between the two was 5 or less. This difference strategy was initially adopted to counteract any possible sampling or rounding errors, but, as will be seen shortly, the strategy was unnecessary for its use did not appreciably influence the results or the conclusions drawn from them. Also, as V is the product of a ratio, a comparison of V values should properly be restricted to expressions of greater than or lesser than. The restricted comparison is more easily accomplished by using the term, variability, as a synonym for V .

First consider stereotypic responding. For two subjects (S-1, S-7), K_o variability exceeded the variability observed in both the rate and duration measures. For two other subjects (S-3, S-10), K_o and the rate and duration measures showed no difference in variability; in an additional two cases, there was no difference in variability between K_o and the duration measure (S-8, S-9). For all other subjects and cases, K_o exhibited less variability than either the rate or duration measure. A summary for the stereotypic behavior re-

veals that the derived expression, K_o , relative to the rate and duration measures, exhibited greater variability in 4 instances, the same variability in 6 instances and less variability in 10 instances. Next consider the behavior of attending. For S-8, K_o variability exceeded that observed in the rate and duration measure, and, in another case, K_o variability exceeded duration variability (S-7). In four cases, K_o and duration showed no difference in variability (S-3, S-5, S-6, S-9). For all other subjects and cases, K_o exhibited less variability than either the rate or duration measure. A summary for the attending behavior reveals that K_o relative to the rate and duration measures exhibited greater variability in 3 instances, the same variability in 4 instances and less variability in 13 instances. Considering the stereotypic and attending behaviors together, the total number of available comparisons was 40; the results showed that in 33 of these 40 comparisons (82.5%) the variability in K_o was the same or less than the variability exhibited by the rate and duration measures. If the requirement is dropped that an absolute difference of 5 must be observed to register a difference between two V values, the results were that K_o variability was less than rate or duration variability on 28 of 39 occasions (72.0%).

The crucial finding above is the one demonstrating that the variability of the dimensionally derived expression, K_o , can exceed the variability exhibited by either a rate or duration measure of behavior. The finding is crucial for it demonstrates that there is nothing about K_o that axiomatically demands that it exhibit less variability than the other two quantities. That it often did is intriguing for it lends support to the notion that behavior may exhibit stability at a heretofore undetected level.

GENERAL DISCUSSION

A dimensional analysis was performed that resulted in the integration of the frequency of the behavior, the duration of the behavior and available time into a dimensionless index of behavior output

called K_o . An empirical analysis revealed that K_o was more likely to exhibit less variability than a rate or duration measure of behavior. The basis of the comparison, degree of expressed variation, is a time-honored concept embedded in the history of the scientific enterprise. Reduction of variability is synonymous with finding an increased order; when variability is understood, order prevails; order permits the formulation of either fixed or statistical laws.

The search for a new quantitative expression was based on the general finding that constituent measures of behavior often tend not to be in accord. Following a discussion on this point, Hall (1976) concluded,

In summary, there is no simple solution to the problem of what response measure should be used. Perhaps all the investigator can do is to use response measures demonstrated in earlier experiments to be reliable, and which reflect consistent differences as a function of basic learning variables that may be manipulated. (p. 199)

Evidence was presented earlier that frequency and duration measures of behavior may or may not covary. A lack of correspondence between frequency and duration measures would be nonproblematic if a literature were available that identified mutually exclusive domains for the two measures. Neither Premack nor Skinner delimited any such domains, nor has anyone else. If one limits the discussion to an examination of behavior under conditions where both frequency and duration are free to vary, current practices indicate that an investigator simply exercises an independent judgment as to which measure is appropriate for the problem at hand. If the results of this investigation are to be believed, it carries the suggestion that both measures are equally important and, moreover, that both measures are actually members of an intersecting set that we have called K_o .

The suggestion that behavior is integrated at a level different than its isolated frequency or duration characteristics does not appreciably alter existing practices, for it requires only that both measures rather than just one be accumulated. What is drastically altered, however, is

Figure 1. Two hypothetical event records where the transition from the upper to lower record represents a doubling of the frequency and a halving of the duration.

the interpretation of what constitutes an orderly outcome. Figure 1 provides a convenient means of visually presenting this altered interpretation. If the focus were placed on the frequency and duration characteristics, the two event records contained in Figure 1 would most assuredly be viewed as representing dissimilar outcomes. The derived expression, K_o , however, forces the interpretation that the two outcomes actually represent alternate forms of the same equivalence, for in constructing the two records the frequency was doubled while the duration was halved. It strikes a discordant note to suggest that a "high frequency-low duration" and a "low frequency-high duration" outcome represent equivalent outputs. This is, nevertheless, precisely what is implied by the expression, K_o . Should K_o or some variant of it prove useful, it would accomplish the scientifically important goal of finding an underlying commonality in seemingly disparate phenomena.

The premise that frequency and duration are members of an intersecting set is not without precedent, for there are suggestions in the literature that one attribute may on occasion serve as a substitute for the other. For example, if one assumes that each response is of the same duration, then the matching law holds whether rate or duration is the dependent variable (Baum & Rachlin, 1969). And, in this same article, the authors argue that a uniform sequence of responses may be treated as a continuous measure, i.e., a duration measure. In applied behavior analysis this latter approach is routinely practiced. For instance, as reported in this paper, individual finger taps were subsumed into episodes of finger tapping; episodes of finger tapping were then reported as both a rate and duration measure. In addition, if the analysis is con-

fined to a single observation period, a dependency exists between the two attributes. A lower limit is set by the circumstance where a single response endures the entire observation period. Subsequent increases in frequency require that the duration of the responses and/or the interresponse times (IRTs) decrease. An upper limit is set by the nature of the response and the capacities of the organism. High frequencies can only be exhibited at the expense of lower durations and shorter IRTs. In summary, current theory and practice as well as a natural dependency between frequency and duration independently provide some support for the results arrived at via a dimensional analysis.

Consider the commonplace circumstance where conspecifics are exposed to the same condition and produce different outputs. The observed intersubject variation is seen as the product of variables operating at some unknown level of specificity. Attempts to discover variables that are functionally related to these observed individual differences is an important area of study (see Harzem, 1984). This paper makes no contribution relative to the identification of variables that influence individual differences. It is possible, though, that a quantitative estimate that reflects individual differences can be developed using the K_o formulation. In order to pursue this line of inquiry, it is necessary to return to an important aspect of a dimensional analysis.

If all the physical variables thought important to the problem under consideration are assembled to form a dimensional group, and a subsequent dimensional analysis exhausts all the assembled physical variables in the group to create a dimensionless expression, then the new dimensionless expression functions as a constant (see McMahon & Bonner, 1983, p. 78). Just such a result occurred in the construction of K_o where N , D , and T were assimilated into a single dimensionless quantity. Thus, a dimensional analysis predicts that K_o should be a constant, but the results presented in this report clearly showed no such constancy. If it is assumed that K_o should at least

theoretically equal a constant, and the variable z is reintroduced into the equation, a means of resolving the apparent paradox becomes available. Digressions from the expected constant output can be attributed to variations in z . Furthermore, the ratio of the expected output to the observed output can yield a quantitative value for z . Two pertinent questions are immediately raised. Can the proposition that output should be constant be justified, and, if so, how can the value of the constant be determined?

To postulate that a subject exposed to a carefully circumscribed and unchanging set of experimental conditions should produce an unwavering behavioral output is to assume that the experimental configuration perfectly constrains the subject to produce a singular outcome. This conceptualization is in accord with a behavior-analytic approach where variation is assumed to be the product of environmental factors rather than idiosyncratic differences. Within this framework, variability is considered to be imposed rather than intrinsic (Sidman, 1960), and the investigator searches for means to reduce or eliminate the sources of the imposed variation. To assume that behavioral output should theoretically be nonvariant, then, is substantially in accord with a behavior-analytic model. Deriving a quantity that represents the presumed constant output for a particular behavior within a particular circumscribed experimental space is the same as specifying a standard behavior-space value. Once a standard has been determined, all subsequent outcomes are expressed as deviations from the standard. The standard selected may in fact be arbitrarily chosen, but it should be a value that permits a wide range of subsequent comparisons: the arithmetic mean is just such a value. The general outline of a method to determine z values can now be proposed.

The approach requires running a large number of subjects over an extended period of time under conditions where frequency and duration measures are easily accumulated; deriving the quantity, $\log[(N \times D)/T]$, across subjects; finding

the mean value of this quantity and then designating this mean as the constant output demanded by the experimental environment. The value z can then be determined, for it is simply the ratio of the expected output to the actual output. If the two outputs are the same, the ratio would equal 1.0; the greater the disparity between the two outputs the greater will be the deviation from 1.0. The value z functions as an individual difference index, and its derivation and examination have a potentially useful purpose. If additional relevant variables can be incorporated into a more general expression for K_o , the result should be the production of more stable z values. For instance, the inclusion of some direct or surrogate expression for the work or effort expended by the behaving organism, e.g., relative body weight, might well reduce the variability observed in K_o across subjects, a result that would be reflected in z values more closely approaching a value of 1.0. In general, the process being explicated is one of extracting free parameters from the composite variable z and explicitly referencing them in an expanded output equation.

The proposition advanced in this paper that behavior displays stability at a level not previously reported will clearly require considerable confirmation. At the present time it is not possible to delimit the circumstances where this order may or may not prevail. There does not appear to be anything particularly unusual about the behaviors examined in this study, for they seem representative of many naturally occurring behaviors in that they possessed meaningful frequency and duration characteristics. A dimensional analysis does not address the issue of what variables are to be assembled to perform the analysis. It may well be the case that the current formulation will need to be elaborated on or modified, perhaps substantially. The important point to be made is that a "new" way of looking at behavior has been proposed: this new view may or may not prove viable. Commitment to a particular theoretical perspective or frame of reference has been known to impede exploration,

but ultimately the mathematical relationship between quantities—the language of science—will be the final arbitrator as to the important or lack thereof of something called Kinetic output.

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