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Beamforming using spatial matched filtering with annular arrays (L)

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Abstract

A linear array beamforming method for ultrasonic *B*-mode imaging using spatial matched filtering (SMF) and a rectangular aperture geometry was recently proposed Kim *et al.*, [J. Acoust. Soc. Am. **120**, 852–861 (2006)]. This letter extends those results to include circularly symmetric apertures. SMF applied to annular arrays can improve the lateral resolution and echo signal-to-noise ratio as compared with conventional dynamic-receive delay-sum beamforming. At high frequencies, where delay and sum beamforming is problematic, SMF showed greatly improved target contrast over an extended field of view.

I. INTRODUCTION

Circular aperture transducers, including annular arrays, are widely used particularly at high frequencies to improve focusing with a small number of transducer elements. Above 30 MHz, accurate digital beamforming is challenging for arrays with many elements.¹ Instead, mechanically scanned, circular, single-element transducers geometrical focused at a fixed depth are used to image subsurface structures in the human eye, skin, and inside small animals.² Although the spatial resolution can approach the wavelength in these high-frequency systems, the maximum cross-range resolution is often intentionally reduced to attain adequate depth of focus. Annular arrays with dynamic receive focusing (DRF) can extend the depth of focus at lower *f*-number, however the receive channel count must increase from 1 to about 8, and the improvement is often marginal. Alternative beamforming techniques,^{2,3} including spatial matched filtering (SMF), have the potential to further increase depth of focus using low *f*-number, fixed-focus apertures while also increasing the echo signal-to-noise ratio (eSNR).

Our group recently described a SMF beamformer for one-dimensional linear array imaging with a rectangular aperture.⁴ We found that the SMF approach leads to improved eSNR, lateral resolution, and contrast resolution as compared with conventional DRF provided the pulse-echo impulse response can be accurately determined. The lateral resolution was found to be equivalent to several synthetic-aperture-focusing methods for one-dimensional arrays.⁵

Our goal with this letter is to extend the SMF beamforming strategies to circular symmetric transducers, and then evaluate them through comparison with conventional delay-sum beamforming methods. We examine a large-aperture, fixed-focus technique for pulse transmission in which received signals are matched filtered by the pulse-echo point spread function. While lateral resolution and eSNR improved compared with DRF methods, axial resolution is somewhat compromised due to the two-dimensional (2D) filter. We show that

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spatial matched filtering of individual receive-element signals using annular arrays generates lower sidelobes than does filtering beamformed echo signals. Thus contrast resolution may be improved at the cost of increasing the computational load. Spatial filtering offers the additional advantage of not requiring delay circuits in the beamformer, which simplifies the stringent hardware requirements for imaging with arrays particularly at high frequencies. We describe the conditions under which the SMF method offers an efficient beamforming solution for circularly symmetric apertures.

II. METHOD AND ANALYSIS

A. Conventional fixed-focus beamforming (FF)

Assume we transmit and receive a broadband acoustic pulse using spherically focused circularly symmetric apertures, $a_t(x_0, y_0)$ whose radius of curvature is z_F . The source-detector plane is defined by (x_0, y_0) and (x, y, z) represents field points. The complex pulse-echo pressure field at (x, z) and at radial temporal frequency $\omega = 2\pi f$ can be obtained from the Rayleigh-Sommerfeld diffraction formula. The field, $\bar{\psi}_\omega(x, z)$, at wavelength λ and wave number $k = 2\pi/\lambda$ is

$$\begin{aligned}\bar{\psi}_\omega(x, z) &= C^2 \left[\int_{-\infty}^{+\infty} dy_0 e^{jk\beta y_0^2} \int_{-\infty}^{+\infty} dx_0 e^{-jk(x/z)x_0} a_t(x_0, y_0) e^{jk\beta x_0^2} \right]^2 \\ &= C^2 \left[\int_{-\infty}^{+\infty} dx_0 e^{-jk(x/z)x_0} e^{jk\beta x_0^2} b(x_0) \right]^2 \\ &= C^2 \left[\Im \left\{ b(x_0) e^{jk\beta x_0^2} \right\}_{u=x/\lambda z} \right]^2,\end{aligned}\tag{1}$$

where

$$b(x_0) = \int_{-\infty}^{+\infty} dy_0 e^{jk\beta y_0^2} a_t(x_0, y_0), \quad C = \frac{e^{jkz}}{j\lambda z},$$

$\beta = 1/2z - 1/2z_F$, and $\Im\{\cdot\}_u$ is the spatial Fourier transform of the argument evaluated at frequency u . Equation (1) is the narrowband pulse-echo field for a conventional beamformer and one standard for comparison. The goal of the SMF beamformer is to improve cross-range resolution in the near and far field as compared with conventional fixed-focused beamforming in Eq. (1).

B. Spatial matched filtering

In the SMF method, we transmit and receive fixed-focus beams as in Eq. (1). However the received echoes are matched filtered. To simplify the result, we autocorrelate Eq. (1) along the lateral (x -axis) direction to find

$$\begin{aligned}
\bar{\bar{\psi}}_{\omega,S}(x,z) &= \bar{\psi}_{\omega}(x,z) * \bar{\psi}_{\omega}^*(-x,z) = C^4 \int_{-\infty}^{+\infty} d\tau \left[\int_{-\infty}^{+\infty} dy_0 e^{jk\beta y_0^2} \int_{-\infty}^{+\infty} dx_0 e^{-jk(\tau/z)x_0} a_t(x_0, y_0) e^{jk\beta x_0^2} \right]^2 \\
&\times \left[\int_{-\infty}^{+\infty} dy_1 e^{-jk\beta y_1^2} \int_{-\infty}^{+\infty} dx_1 e^{jk[(\tau-x)/z]x_1} a_t(x_1, y_1) e^{-jk\beta x_1^2} \right]^2 \\
&= C^4 \int_{-\infty}^{+\infty} dx_0 \left[e^{-jk(x/z)x_1} e^{jk\beta x_0^2} \int_{-\infty}^{+\infty} d\tau e^{-jk[(x_0-x_1)/z]\tau} \int_{-\infty}^{+\infty} dy_0 e^{jk\beta y_0^2} a_t(x_0, y_0) \int_{-\infty}^{+\infty} dy_1 e^{-jk\beta y_1^2} \int_{-\infty}^{+\infty} dx_1 a_t(x_1, y_1) e^{-jk\beta x_1^2} \right]^2 \\
&= C^4 \left(\frac{4\pi^2}{k} \right)^2 \int_{-\infty}^{+\infty} dx_0 \left[e^{-jk(x/z)x_0} \int_{-\infty}^{+\infty} dy_0 e^{jk\beta y_0^2} a_t(x_0, y_0) \int_{-\infty}^{+\infty} dy_1 e^{-jk\beta y_1^2} a_t(x_0, y_1) \right]^2 \\
&= C^4 \left(\frac{4\pi^2}{k} \right)^2 \int_{-\infty}^{+\infty} dx_0 e^{-jk(2x/z)x_0} [b(x_0)b^*(x_0)]^2 = C^4 \left(\frac{4\pi^2}{k} \right)^2 \mathfrak{S}\{|b(x_0)|^4\}_{u_x=2x/\lambda z}.
\end{aligned} \tag{2}$$

If a rectangular aperture is substituted into Eq. (2), as with linear arrays,⁴ the 2D aperture function is separable into lateral (x -axis) and elevational (y -axis) components, $a_t(x_0, y_0) = a_t(x_0)a_t(y_0)$, and Eq. (2) simplifies to the spatial Fourier transform of $|a_t(x_0)|$.⁴ Because circular apertures are not separable in the same way, the SMF result is proportional to $|b(x_0)|^4$ that includes an integral of the aperture times a quadratic phase factor.

The complex phase factor $\exp(jk\beta y_0^2)$ converts the normally constant aperture function a into a rapidly oscillating function. This feature contributes to sidelobe energy and beam spreading that correspond to high frequency components in the spatial Fourier domain. Despite these limitations, the SMF method improves in-plane, cross-range resolution (often called lateral resolution) over an extended depth of field. Equation (2) shows that spatial matched filtering improves lateral resolution by eliminating the lateral quadratic phase factor $\exp(jk\beta x_0^2)$ from the transmit-receive field at all depths. The inseparability of the aperture in the (x_0, y_0) plane does not provide the closed-form solution that allows us to clearly predict the effects of filtering with circular apertures. Nevertheless, the numerical and experimental results presented in the following show that matched filtering clearly improves beam quality.

It is significant that the spatial frequency variable for the SMF beamformer in Eq. (2) is scaled by a factor of 2 as compared with the FF aperture in Eq. (1). The factor of 2 means that the SMF applied to beamformed echo data has a narrower lateral main-lobe width, like synthetic aperture focusing techniques.⁵

C. Annular array imaging using spatial matched filtering

Rf data from the entire aperture are being filtered in the previous section. In this section, however, we filter receive-channel echoes from annuli before summation. A focused array aperture $a_t(x_0, y_0)$ transmits sinusoids at frequency ω . The filtered pulse-echo field from the n th element, $\psi_w(x, z; n)$, becomes

$$\psi_w(x, z; n) = \phi_w(x, z) \phi'_w(x, z; n), \tag{3}$$

where $\phi_w(x, z)$ represents the transmit field using the whole aperture and $\phi'_w(x, z; n)$ is the receive field from just the n th element. We now matched filter the receive fields along the x axis. The field is evaluated at the corresponding element before summation and without applying time delays. The pulse-echo field for the SMF beamformer, where the echoes are filtered and then summed, can be expressed by

$$\bar{\bar{\psi}}_{\omega,S}(x, z) = \sum_{n=1}^N \psi_w(x, z; n) * [\psi_w(-x, z; n)]^*, \tag{4}$$

where N is the number of receiver elements in the annular array.

To compare the characteristics of SMF before and after summation, we rewrite the pulse echo field of SMF of Eq. (2) as

$$\begin{aligned}
 \bar{\psi}_{\omega,S}(x,z) &= \left[\sum_{n=1}^N \psi_w(x,z;n) \right] * \left[\sum_{n=1}^N \psi_w(-x,z;n) \right]^* \\
 &= \sum_{n=1}^N \psi_w(x,z;n) * [\psi_w(-x,z;n)]^* \\
 &\quad + \sum_{l=1}^N \sum_{m=1, m \neq l}^N \psi_w(x,z;l) * [\psi_w(-x,z;m)]^* \\
 &= \bar{\psi}_{\omega,S}(x,z) + \sum_{l=1}^N \sum_{m=1, m \neq l}^N \psi_w(x,z;l) * \\
 &\quad \times [\psi_w(-x,z;m)]^*.
 \end{aligned} \tag{5}$$

Comparing Eqs. (4) and (5), we see that if the echoes are filtered before summation, the cross terms between different elements are eliminated, which can be expected to concentrate beam energy and improve spatial resolution.

III. SIMULATION AND EXPERIMENTAL RESULTS

The above-mentioned predictions of beamformer performances were validated using rf echo simulations from Field II⁶ for an annular array transducer composed of eight elements, each with equal area. The center frequency of the transmitted pulse was 10 MHz and the -6 dB bandwidth was 6 MHz. And diameter and geometric focal length of the array are 30 and 45 mm, respectively, resulting in an $f/1.5$ geometric aperture. Simulation parameters were the same as those of the associated experiment.

In the following results, “FF” beam plots describe patterns obtained when transmit and receive apertures are both focused at 45 mm depth. “DRF” plots describe results for a fixed 45 mm focal length on transmit and a dynamically focused receive aperture. “SMF(A)” and “SMF(B)” are results for SMF after the summing and SMF before element summation, respectively.

Figure 1 shows broadband, 2D, in-plane, pulse-echo point spread functions for each method. Simulation and experimental results were very similar except for noise. After beam formation, the point spread functions are envelope detected, the amplitudes are normalized to the peak values, and then log compressed with 50 dB dynamic range. The SMF results are from a 2D spatial matched filter applied in the x - z plane.

As seen from Fig. 1, the broadband pulse energy is more spatially compact with the SMF method compared to the FF and DRF method in the near field and far field. The PSFs were all very similar at the transmit focal depth of 45 mm, except for the expected loss of axial resolution for the SMF methods from 2D filtering. There is generally more background noise in the experimental results, and the SMF(B) methods are most effective at suppressing that noise.

Complete comparisons of the three methods must extend beyond point reflectors to include scattering fields that generate speckle and have low-contrast targets. Figure 2 shows experimental results using a cyst phantom with 2D anechoic targets, where the center of the anechoic region was positioned anywhere from 35 to 55 mm. The SMF(B) approach provides the greatest lesion visibility.

These results differ from those of a linear array.⁴ In our previous study using commercial ultrasound system, we could not completely control aperture growth. Thus the impulse response function was only approximately known. The associated errors degraded lateral resolution when SMF was applied before beamforming. Moreover, the assumption of a shift-invariant impulse response along the lateral axis of the image was violated in practice for linear arrays because the length of the subaperture varies with lateral position. Such limitations were not a factor with annular arrays because we had fine control over each parameter and the aperture was truly fixed. Thus we could obtain more accurate impulse response functions for filtering, which led to measurement results being more consistent with theoretical and simulation results.

One promising application of SMF can be high frequency imaging. At transmission frequencies over 50 MHz, it is very difficult to implement digital dynamic focusing since very high-speed digital circuits are required. SMF requires only 2D FIR filters for focusing, so that receive focusing can be quickly and efficiently implemented. To evaluate the performance of spatial filtering for high frequency application, we employed Field II simulations for a 70 MHz (45% BW), single element, circular transducer with a focal depth of 7 mm.

Figure 3 shows computer generated cyst phantom images that were processed using conventional FF and SMF method at each depth. Simulations were performed using two apertures which have different f -numbers: $f/1.5$ and $f/3.5$. The SMF method provides much improved spatial and contrast resolution in the near- and far-field regions compared with fixed focusing, suggesting greater depth of focus without compromising lateral resolution.

IV. CONCLUSIONS

SMF can effectively focus the ultrasound beam. The resulting pulse-echo point spread function yields superior lateral resolution compared to conventional FF or DRF except near the transmit focal length where they are comparable. Also, in the case of array transducers, applying the SMF before summation enhances lateral resolution more than applying it to beamformed data.

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FIG. 1.

Impulse response estimates are imaged using different beamformers. Simulation results (a)–(c) and experimental results (d)–(f) for a broadband, pulse-echo, 2D point spread functions are compared for the FF, DRF, and SMF beamformers at (a), (d) 25 mm depth, (b), (e) 45 mm depth, and (c), (f) 65 mm depth. The transmit focus is fixed at 45. Images are normalized to the individual peak values and displayed with 50 dB dynamic range.

FIG. 2.

Experimental images using a cyst phantom that were processed using FF and SMF methods. The transmit focus is fixed at 45 mm. The cyst diameter is 1.5 mm and the medium has a constant speed of sound. All images are displayed with 50 dB dynamic range. Depths of cyst centers are indicated in the left column. CNR values appearing below each image are computed

using $CNR = |\langle S_i \rangle - \langle S_o \rangle| / \sqrt{\sigma_o^2 + \sigma_i^2}$, where $\langle S_{i,o} \rangle$ and $\sigma_{i,o}^2$ are the mean and variance of image pixels inside and outside the target.

FIG. 3.

The images are from echo simulations of a cyst phantom at 70 MHz center frequency that were processed using FF and SMF methods. The transmit focus is set to 7 mm. The cyst diameter is 0.5 mm and the medium has a constant speed of sound. All images are displayed with 50 dB dynamic range. Depths of cyst centers are indicated on the left.