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A Method for Accurate *in silico* modeling of Ultrasound Transducer Arrays

Drake A. Guenther and William F. Walker

Abstract

This paper presents a new approach to improve the *in silico* modeling of ultrasound transducer arrays. While current simulation tools accurately predict the theoretical element spatio-temporal pressure response, transducers do not always behave as theorized. In practice, using the probe's physical dimensions and published specifications *in silico*, often results in unsatisfactory agreement between simulation and experiment. We describe a general optimization procedure used to maximize the correlation between the observed and simulated spatio-temporal response of a pulsed single element in a commercial ultrasound probe. A linear systems approach is employed to model element angular sensitivity, lens effects, and diffraction phenomena. A numerical deconvolution method is described to characterize the intrinsic electro-mechanical impulse response of the element. Once the response of the element and optimal element characteristics are known, prediction of the pressure response for arbitrary apertures and excitation signals is performed through direct convolution using available tools. We achieve a correlation of 0.846 between the experimental emitted waveform and simulated waveform when using the probe's physical specifications *in silico*. A far superior correlation of 0.988 is achieved when using the optimized *in silico* model. Electronic noise appears to be the main effect preventing the realization of higher correlation coefficients. More accurate *in silico* modeling will improve the evaluation and design of ultrasound transducers as well as aid in the development of sophisticated beamforming strategies.

I. INTRODUCTION

Considerable effort has been dedicated to predicting the diffraction patterns produced by ultrasound arrays. While analog circuitry, digital electronics, signal processing, and the graphics display have large effects on the quality of the output image, ultimately it is the ultrasound transducer and beam pattern that determine clinical image quality. Many researchers have spent time developing theory to characterize the pressure field emitted from ultrasound transducers [1-8]. These studies typically use the Topholme and Stepanishen method [1,2], which derives the pressure field from a transducer modeled as a planar piston vibrating uniformly in an infinite rigid baffle. Simulation tools based upon this theory of propagation have been developed to predict the acoustic output from arbitrarily shaped transducers and arrays [9-11]. While these approaches are efficient, theoretically accurate, and help guide system development, there exists a sharp disconnect between precise modeling of a conventional, commercial system and *in silico* characterization. In real world situations medical ultrasound transducers do not completely behave as theorized. Element angular sensitivity, mechanical and electrical coupling between elements, element nonuniformity,

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nonlinearities, and the fixed acoustic lens are some of the factors that limit agreement between theory and experiment. While these transducer characteristics can be modeled, quantifying their overall effects usually requires extensive experimental characterization. Better agreement between theoretical and experimental responses will improve the evaluation and design of ultrasound systems. More precise models will also enable implementation of sophisticated beamforming algorithms [12-18].

Our goal is to develop an accurate and precise *in silico* model in FIELD II [9], which completely characterizes the spatio-temporal response of a single transducer element in a commercial 1D linear array. We use a linear system's approach, which describes the transmitted pressure response as a series of temporal convolutions. Our first step is to determine the optimal element parameters that maximize the correlation between the simulated and experimentally captured responses. The optimization algorithm presents a novel approach to accurately characterizing element angular sensitivity, mechanical crosstalk, and lens effects using existing tools in the FIELD II simulation program. Once the optimal element parameters have been quantified, the transducer element's electro-mechanical impulse response is determined via deconvolution. Similar inverse filter algorithms have been used for decades to reconstruct ultrasound images [19,20]. In [6] a deconvolution approach analogous to ours is used to determine the electro-mechanical response from piston transducers. In this paper we show how this algorithm can be used with elements in a linear array using the spatial impulse responses calculated in FIELD II. This step requires measuring the electrical excitation voltages produced by the ultrasound system. Given the optimal element parameters and electro-mechanical impulse response, we present results showing how the optimized *in silico* model accurately predicts the spatio-temporal response from a multi-element focused aperture excited with an arbitrary waveform.

Current methods, which experimentally characterize ultrasound arrays, include electrical impedance measurements with impedance analyzers, cross coupling characterization with equivalent circuit models, and finite element analysis [21-23]. Other methods include one-way electro-acoustic characterization with a hydrophone [24], pulse-echo acoustic characterization with a flat target [6], and recently one-way electro-acoustic characterization via laser interferometry [25]. In this paper we attempt to accurately model the spatio-temporal response of a single transducer element in a commercial 1D linear array. One important feature of our approach is that the required experiments can be performed using equipment common in most ultrasound research laboratories and no specific knowledge of the transducer's physical dimensions or specifications is needed. Furthermore, this method can be extended to characterize arbitrary transducers in transmit, receive, and transmit-receive modes.

The rest of this manuscript is organized as follows. Section II presents the underlying theory behind the linear systems model used to describe the pressure field emitted from an ultrasonic transducer. A brief explanation of the deconvolution algorithm for determining the electro-mechanical impulse response is also discussed. Sections III and IV present in detail the methods used to create the *in silico* model. Specifically, section III describes the algorithm for determining the optimal element parameters and presents the computed results. Section IV derives the electro-mechanical impulse response for the investigated linear array and compares the agreement between the simulated response and experimentally captured response for arbitrary excitation functions. Section V presents results using the *in silico* model to accurately predict the emitted pressure waveform from a multi-element focused aperture. This section is followed by a brief discussion (Section VI) and conclusions (Section VII).

II. THEORY

Following the Tupholme-Stepanishen model, we assume that the single piezoelectric element in the linear array behaves as a planar piston transducer mounted in an infinite rigid baffle

[1]. Ignoring multiple scattering and nonlinear propagation, the incident pressure, $p(\vec{x}_o, t)$, at a location $\vec{x}_o$ in three dimensional space can be computed as:

$$p(\vec{x}_o, t) = e(t) *_t v(t) *_t h^{SIR}(\vec{x}_o, t), \quad (1)$$

where $e(t)$ is the electrical excitation waveform, $v(t)$ is the element's electro-mechanical transfer function, $h^{SIR}(\vec{x}_o, t)$ is the element's spatio-temporal impulse response, and $_t^*$ denotes convolution in time. A term relating to the angular sensitivity of the element can be included in $h^{SIR}(\vec{x}_o, t)$ as was done in [26] and attenuation can also be modeled. Note that our general approach to describing the emitted pressure waveform as a series of convolutions is consistent with the ultrasound literature [6,7,9,10].

In this paper we experimentally measure the pressure field of a single element of a linear array transducer using an 85 μm diameter capsule hydrophone (Model HGL-0085, ONDA Corporation, Sunnyvale, CA). The limitations of hydrophone measurements are well documented in the literature, and numerous papers suggest methods for removing the spatial averaging, the angular sensitivity, and finite amplitude distortion caused by the hydrophone [24,27,28]. Spatial averaging effects are ignored since the diameter of the hydrophone's active element is almost 3 times smaller than the investigated acoustic wavelength. Furthermore, the system setup satisfies the guidelines set forth in [24], in that the effective diameter of the hydrophone, d_h , should be less than $0.5z\lambda / d_s$, where z is the source-to-hydrophone distance, λ is the acoustic wavelength, and d_s is the greatest dimension of the source transducer. A term accounting for the hydrophone's spatio-temporal impulse response is included so that the pressure waveform measured by the hydrophone is:

$$p(\vec{x}_o, t) = e(t) *_t v(t) *_t h^{SIR}(\vec{x}_o, t) *_t D(\vec{x}_o, t) \quad (2)$$

where $D(\vec{x}_o, t)$ is a term describing the effects of the hydrophone as a function of space and time. Due to the wideband frequency response of the hydrophone its impulse response is assumed to be a delta function. Therefore, the hydrophone scales the output voltage/pressure waveform [27]:

$$D(\vec{x}_o, t) = \alpha \delta(t - \tau) \quad \text{where} \quad \alpha = \frac{2J_1(ka_h \sin\theta)}{ka_h \sin\theta} \cos\theta. \quad (3)$$

$J_1(ka_h \sin\theta)$ is the Bessel function of first order, k is the wavenumber ($2\pi/\lambda$) of the impinging pressure wave, a_h is the hydrophone's radius, and θ is the angle between $\vec{x}_o$ and the transducer. The additional cosine term is included as a soft baffle condition [26].

Assuming a perfect model for the element's spatial impulse response, $h^{SIR}(\vec{x}_o, t)$, electrical excitation waveform, $e(t)$, and that the emitted pressure waveform is measured directly on the central axis of the element (i.e. $D(\vec{x}_o, t) = 1$), one can solve for the electro-mechanical impulse response, $v(t)$. This response has no spatial dependence, so the deconvolved waveform is constant with range (note that a new $h^{SIR}(\vec{x}_o, t)$ is computed for each range). Using a discrete form for the above convolutions, the output from the hydrophone for this ideal measurement case is:

$$p = EHv, \quad (4)$$

$$E = \begin{bmatrix} e_1 & 0 & \cdots & 0 \\ e_2 & e_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_N & e_{N-1} & \cdots & e_1 \end{bmatrix}, \quad (5)$$

$$H = \begin{bmatrix} h_1^{SIR} & 0 & \cdots & 0 \\ h_2^{SIR} & h_1^{SIR} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_N^{SIR} & h_{N-1}^{SIR} & \cdots & h_1^{SIR} \end{bmatrix}, \quad (6)$$

$$v = \begin{bmatrix} v_1 & v_2 & \cdots & v_N \end{bmatrix}^T. \quad (7)$$

From the above expressions, p is a vector corresponding to the sampled hydrophone output, E and H are convolution matrices made with the sampled electrical excitation waveform, $e(t = iT)$, and sampled spatial impulse response, $h^{SIR}(\vec{x}_o, t = iT)$ respectively. T represents the sample period and $i = 1, 2, 3, \dots, N$. Solving (4) for the element's electro-mechanical impulse response yields:

$$v = E^{-1} H^{-1} p. \quad (8)$$

In order to avoid any ill-conditioning when computing the inverse of the E and H matrices, we simply normalize the E matrix by e_1 and the H matrix by h_1^{SIR} . Since both these triangular matrices have 1's on their main diagonals, their determinants are both equal to 1. We know from basic linear algebra, if the determinant of a matrix is greater than zero, that matrix is invertible. The deconvolution approach to determining the electro-mechanical impulse response is similar to that presented in [6,19,20], but we do not use it here to reconstruct an image, rather to develop a useful model for a pulsed single transducer element.

In this manuscript full advantage is taken of FIELD II's ability to efficiently and accurately compute spatial impulse responses. However, as discussed before, simply using the probe's physical parameters *in silico* often produces results that do not correlate well with those observed experimentally. Accounting for element angular sensitivity, mechanical and electrical coupling, and the fixed acoustic lens will produce a more accurate FIELD II spatial impulse response. The novel optimization procedure explained in the next section determines the element parameters, which maximize the correlation between the *in silico* model and experiment. Given an electrical excitation waveform and the optimal element parameters, the element's electro-mechanical impulse response can be computed using equation (8). From there, arbitrary aperture geometries as well as excitation signals can be used to further investigate transducer performance and simulate various beamforming schemes.

III. OPTIMIZATION OF ELEMENT PARAMETERS

Our goal is to maximize the correlation between the measured pressure field of a single pulsed element and the response simulated using FIELD II. The spatio-temporal response of a pulsed single element in the L14-5/38mm (Prosonic, Gyeongbuk, Korea) 128 element linear array was measured in a water tank. This array is used with the Ultrasonix Sonix RP (Ultrasonix Medical Corporation, Burnaby, BC, Canada) imaging system. The transducer was affixed to a two-axis tilt and rotation stage (Model M37 with micrometers, Newport Co., Irvine, CA), employed to align the transducer to the movement of the motion stage (Newport Motion Controller MM3000, Newport Co., Irvine, CA) so as to keep the image plane parallel to the translation plane. The hydrophone (Model HGL-0085, Onda Corporation, Sunnyvale, CA) and transducer were immersed in a tank of deionized water. The output voltage signal from the hydrophone was preamplified prior to digitization and stored using a LeCroy oscilloscope (Model LC334A, LeCroy Corporation, Chestnut Ridge, NY). Care was taken to align the transducer with the hydrophone using the two-axis tilt and rotation stage such that the spatio-temporal response measured from the hydrophone was symmetrical in azimuth and elevation.

The element was excited with a 2 cycle 6.67 MHz pulse, and the acoustic pressure field was measured by scanning the hydrophone in the elevation-azimuth plane at a fixed range. The response was measured on a 60×100 grid of spatial samples separated in elevation and azimuth by $250 \mu\text{m}$ at ranges of approximately 1.5 and 2.5 cm. At every spatial location, 2000 pulses were averaged to improve the signal-to-noise ratio prior to storing the waveforms at a sampling rate of 500 MHz. The measured hydrophone waveforms were later bandpass filtered offline in MATLAB (The Mathworks Inc., Natick, MA) using a 101st order bandpass finite impulse response (FIR) filter with cutoff frequencies at 4 and 12 MHz.

The pulsed single element experimental setup was modeled in FIELD II using a sampling frequency of 1 GHz for all simulations. The pulse emitted from the *in silico* element was the pulse measured at the output of the hydrophone on the central axis of the element. A finer grid ($50 \mu\text{m}$ by $50 \mu\text{m}$ in azimuth and elevation) around the center of the element was scanned in order to accurately measure this response. We assume that the spatial-impulse response of the element, $h^{SIR}(\vec{x}_o, t)$ and hydrophone, $D(\vec{x}_o, t)$ are delta functions at this measurement location. This is a reasonably valid assumption given the measurement depth (2.5 cm), location of the hydrophone to the element's central axis, and the computational element size used in the FIELDII program [11]. The spatial impulse response of the element will have a larger effect on the pulse shape off the central axis.

The transducer element was subdivided into a 7×15 grid of computational elements in the FIELD II simulation (7 lateral, 15 elevation). Element width, element height, lens focus (applied as a time delay in elevation to the computational elements), angular sensitivity (applied by amplitude weighting the computational elements), offsets in the azimuth and elevation sampling grid, range, sound propagation speed, and element rotation relative to the hydrophone were all variables used to compute the simulated response. The correlation between the experimental response and simulated response was maximized using MATLAB's constrained minimization function, `fmincon`. Since `fmincon` minimizes the user defined function, we minimized the inverse of the computed correlation value ($1/\text{corr}$) between simulation and experiment in order to maximize the correlation. This optimization function uses all the previously mentioned parameters as input variables. Upper and lower bounds for each parameter were explicitly defined per the function's definition. Since we cannot guarantee the convexity of the problem, and to avoid local minima, we ran the `fmincon` optimization, which employs a gradient search algorithm, using 200 different initialization points.

Refraction by and attenuation within the lens were ignored. The lens in the model thus only affects the delays between the individual computational elements in elevation. These delays are computed using the fixed focus of the lens and the assumed sound speed in the propagating medium (i.e. 1487 m/s in water). The optimized element parameters and the published probe specifications [29] are shown in Table I. These correspond to the response measured at 2.36 cm in range (optimized range value).

The results in Table I are intriguing. Clearly the published probe specifications yield sub-optimal agreement with experiment. It is interesting to note that the optimal element width is actually larger than the probe's pitch (300 μm). Although a 313.6 μm element width is physically impossible given the probe's 300 μm pitch, this value suggests that due to mechanical crosstalk and element angular sensitivity, the probe's elements are behaving as if they were 14% wider than their physical dimensions. The optimized element height is also 130 μm larger than the published element height. The geometry and apodization of the optimized element is shown in figure 1. The published element dimensions are also shown as the dashed black line slightly inside the borders of the optimized element. Note that the optimized element's apodization decreases at the top and bottom edges in elevation. This suggests that the elements are less sensitive at their upper and lower edges, which could be an effect of the lens (refraction) or rigid baffle.

Lens effects clearly dominate improving the correlation between the measured and simulated responses. Using the optimized values for range, sound speed, sampling grid shift, element rotation, and lens elevation delays with the original probe specs increases the correlation from 0.846 to 0.985. Allowing for mathematical element apodization and variable element dimensions in the optimization algorithm further increases the correlation to 0.988. This suggests that modeling the lens is extremely important. Applying the optimal parameters to the data collected at the shallower range (1.36 cm), resulted in correlations of 0.965, 0.821, and 0.955 for the optimal, L14-5 published specs, and L14-5 w/ optimal lens delays respectively.

Six different time instances for the measured and computed response at the 2.36 cm range are shown in figure 2. These images show the good agreement between the simulated and measured responses. One possible limit on correlation is the fact that SNR decreases considerably away from the central axis of the element due to the element's angular sensitivity. Indeed the responses measured at the fringes of the 60 $\times$ 100 spatial sample grid contained more noise and smaller signal amplitudes. The SNR was about 45 dB at the peak response of element and fell below 7 dB at the edges. Calculating the correlation between the simulated and measured response in a 1.0 cm diameter about the middle of the element increased the correlation above 0.994. SNR is related to the maximum achievable correlation, r , between two signals by the following expression [30]:

$$r = \frac{SNR}{\sqrt{SNR^2 + 1}} \quad (9)$$

Using the mean SNR over the entire sampling grid of 18 dB (SNR = 7.94 in equation (9)) yields a maximum possible correlation of 0.992 between the simulated and measured responses. This is not much greater than the maximum correlation achieved of 0.988.

Furthermore, it was assumed that the spatial sampling grid is strictly uniform at 250 μm sample spacing. The optimization procedure only allows for gross shifts in the elevation and azimuth dimensions. Hysteresis in the motion stage occurs, and will obviously induce errors in the

assumed sample locations. Nonetheless, the optimized element parameters provide a more accurate model for the commercial transducer than the published specifications.

IV. ESTIMATION OF THE ELEMENT ELECTRO-MECHANICAL IMPULSE RESPONSE

In this section a detailed method is described for estimating the electro-mechanical impulse response of the single transducer element in the Prosonic L14-5/38mm probe. The Sonix RP applies user defined bipolar electrical excitations on transmit at a temporal resolution of 12.5 ns. Inputting the bipolar pulse: “++++-----++++-----” excites the transducer element with a 2 cycle 10 MHz square waveform. A series of electrical excitations were measured from five different transmit channels using the LeCroy oscilloscope with 1 GHz sampling. Measured excitations are shown in figure 3. The top row of plots in figure 3 shows the electrical excitation waveforms when applying positive and negative square waveforms of different duty cycles: 12.5 ns, 25 ns, 37.5 ns, 50 ns, 62.5 ns, 75 ns, and 100 ns. All waveforms show a positive or negative overshoot when the transmit level returns to ground as well as an extended decay of the overshoot voltage. The overshoot increases as the length of the square wave increases. The bottom two plots in figure 3 show the excitation waveforms for 2 cycle 6.67 MHz pulses with opposite polarity. The overshoot at the end of the waveforms is still evident, but not as pronounced as the overshoot in the monopolar waveforms. The transmit circuitry was excited with the default power level for the system's general B-mode imaging setting in all experiments.

In order to compute the electro-mechanical impulse response of the L14-5/38mm probe's element, the element's pressure response at multiple ranges was measured for varying electrical excitations. Responses were averaged 2000 times at each location to increase SNR. The ideal excitation (for characterization) would be a short voltage spike to emulate a delta function, however, the transmit circuitry control does not allow for this. Pressure responses were measured for every square wave excitation from 12.5ns to 100ns duty cycle. Care was taken to align the hydrophone and transducer using the two-axis tilt and rotation stage. The pressure waveforms were measured in range from 1.0 cm to 4.0 cm in 0.5 cm increments. The electromechanical impulse responses were calculated using equation (8). The optimal element parameters derived in the previous section were applied when computing the element spatial impulse response.

The derived electro-mechanical impulse responses for one element in the L14-5/38mm probe, for four different ranges are shown in figure 4. These responses were calculated using the positive and negative 12.5 ns square wave pulses. Using more and/or temporally longer square wave pulses or pseudo-random noise sequences did not improve the correlation between the measured responses and the derived pulses. From figure 4 it is clear that the electro-mechanical impulse response remains relatively constant with range, as expected. The derived response at 1.0 cm in range is the most dissimilar of the group. The spatial averaging of the hydrophone at small ranges and misalignment between the hydrophone and transducer are possible reasons for the different response at 1.0 cm.

The measured electrical excitation waveforms and derived electro-mechanical response were used to predict the pressure response at multiple ranges for the single element excited with a 1 cycle and 2 cycle 6.67 MHz excitation. The mean of the derived electro-mechanical impulse responses for all ranges was used as the true response. Figure 5 shows the measured and modeled 2 cycle 6.67 MHz pulse waveforms emitted from the element at four different ranges. The pulses are most dissimilar in the tails. The correlations between the measured and derived pulses for four different excitation functions are shown in table II. For the majority of ranges and pulses, the correlation between the measured and simulated response is above 0.98.

V. PREDICTION OF THE OUTPUT SIGNAL FROM A MULTI-ELEMENT FOCUSED APERTURE

In the previous sections a robust simulation model was developed for a single element in the Prosonic L14-5/38mm probe attached to the Ultrasonix Sonix RP imaging system. That model is extended to synthesize a multi-element focused aperture. This assumes that all the elements in the array behave similarly to the derived model. Using the optimized element parameters, the derived electro-mechanical impulse response, and an arbitrary excitation voltage, a 64 element transmit aperture with a transmit focus at 3.0 cm was modeled. Transmit focal delays were quantized at 80 MHz. The experimental pressure response from a 64 element focused aperture was also acquired for comparison. The hydrophone was scanned laterally at three different ranges to measure the emitted pressure field from the focused aperture. The measured and derived responses are shown in figure 6. Notice the excellent agreement between the two responses.

VI. DISCUSSION

A new method for accurately modeling ultrasound arrays was presented. The procedure begins by maximizing the correlation between the observed and simulated response from a pulsed single element in a 1D linear array. The optimization algorithm allows for varying element size and sensitivity and accounts for the diffraction effects of the fixed acoustic lens in elevation. It was seen that the elevation focusing of the lens is more important to model than the element's effective dimensions. A deconvolution technique was presented to determine the electro-mechanical impulse response of the element. With the electro-mechanical impulse response, optimal element characteristics, and arbitrary system excitation signals, a multi-element focused aperture was synthesized *in silico* that accurately predicts the emitted acoustic pressure field from a commercially available linear ultrasound array. The developed *in silico* model can be used to investigate transducer performance, arbitrary aperture geometries, beamforming algorithms, and pulse sequences. The model can be extended to describe the behavior of the element in reception, so that the two-way response can be well characterized.

The model ignores nonlinearities in the system circuitry, nonlinearities in the electromechanical transfer function of the element, nonlinear propagation, and multiple scattering. Attenuation can be readily included, but was not investigated in this paper. Extending the model to include crosstalk from adjacent elements and lens refraction and attenuation will further improve accuracy. Modeling propagation through the lens using an equivalent circuit model was well described in [23] and could be included. However, extensive lens modeling will require knowledge of the physical dimensions of the lens as well as the material properties of the lens, piezoceramic, and any matching layers. Such data is not readily available for commercial ultrasound probes.

The proposed method provides a high degree of accuracy for characterizing the behavior of commercial ultrasound arrays *in silico*. The experiments can be performed using equipment common in most ultrasound research laboratories and no specific knowledge of the transducer's physical dimensions or specifications are needed. Although more sophisticated modeling may be required in some applications, this method improves the simulated performance of ultrasound systems and will enhance the development of ultrasound transducers and beamforming algorithms.

VII. CONCLUSIONS

Current simulation tools precisely predict the theoretical pressure response emitted from an ultrasound transducer. However, in real world situations the transducers do not always behave

as theorized. This manuscript presented a new procedure for accurately modeling the spatio-temporal response of ultrasound arrays *in silico*. The model accounts for element angular sensitivity, mechanical crosstalk, and diffraction through the fixed acoustic lens in elevation. The presented method uses an optimization algorithm and FIELD II to address these unwieldy aspects of real world system characterization. Using the optimized *in silico* model for the Ultrasonix Sonix RP scanner, the correlation value between simulation and experiment was significantly increased from 0.846 to 0.988. Electronic noise appears to be the limiting factor preventing the realization of correlation coefficients above 0.99. The procedure outlined in this manuscript can be extended to model arbitrary ultrasound transducers in transmit, receive, and transmit/receive modes. More precise *in silico* models will further the development of ultrasound transducers and aid in the evaluation of sophisticated beamforming algorithms and pulse sequences.

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Fig. 1.

Optimized element physical dimensions and sensitivity for the Prosonic L14-5/38mm linear array. The dashed line outlines the published probe dimensions. From the optimization, the element behaves as if it were wider, and also less sensitive along the top and bottom edges than in the middle. The element was subdivided into a 7×15 grid of computational triangular subelements in the optimization procedure using FIELD II. Both triangles in each subelement were given the same weighting.

Fig. 2.

Measured and simulated pressure responses from a pulsed single element. The images show the response at different instants in time measured over an area 1.5 cm in elevation and 2.5 cm in azimuth. The response has an elliptical shape due to the element's physical dimensions and because the lens acts to focus the majority of the acoustic energy along the azimuthal axis.

Fig. 3. Electrical excitations measured from the Ultrasonix Sonix RP system sampled at 1 GHz using the LeCroy LC334A oscilloscope. The top two plots show positive (left) and negative (right) square wave excitations. The bottom two plots show 2 cycle 6.67 MHz excitations with opposite polarity. All waveforms show an overshoot voltage and decay.

Fig. 4.

Derived electro-mechanical impulse responses with range. The responses are quite similar with range, as expected. The response from 1.0 cm was the most dissimilar in the group. The electro-mechanical impulse response used in the focused aperture model (below) was the average of all seven derived responses.

Fig. 5. Derived (solid) and measured (dashed) pressure responses with range from a pulsed single element excited with a 2 cycle 6.67 MHz electrical excitation (+-- polarity). The simulated and measured responses are quite similar, achieving a correlation value above 0.985 at all ranges. Note that the time axis in each plot shows the 2 μ s gated window from the oscilloscope and does not include the time of flight between the transducer and the hydrophone.

Fig. 6. Simulated (left) and measured (right) pressure fields emanating from a 64 element focused aperture using the L14-5/38mm probe attached to the Ultrasonix Sonix RP imaging system. The focal delays are quantized at 80MHz with a transmit focus of 3.0 cm. Using the *in silico* model of the transducer element, the derived element electromechanical impulse response, and the measured electrical excitation, the simulated 64 element full aperture accurately predicts the observed pressure response measured with a hydrophone.

TABLE I
Comparison of optimized and published element parameters

	Correlation	C _{H2O}	Element width (μm)	Element Height (mm)	Lens Focus (mm)
Optimal Parameters	0.988	1486.9	313.6	4.13	28.5
L14-5 Specs	0.846	1486.9	275	4.00	14.0
L14-5 Specs w/ opt. lens	0.985	1486.9	275	4.00	28.5

TABLE II
Correlation between derived and captured responses with varying range AND EXCITATION WAVEFORM

	1.0 cm	1.5 cm	2.0 cm	2.5 cm	3.0 cm	3.5 cm	4.0 cm
1 cycle (+-)	.982	.986	.983	.982	.983	.984	.984
1 cycle (-+)	.967	.982	.974	.973	.977	.980	.982
2 cycle (++-)	.987	.992	.988	.987	.987	.986	.986
2 cycle (-++)	.983	.990	.981	.980	.982	.984	.984