Published in final edited form as:

J Biomech. 2009 June 19; 42(9): 1345–1349. doi:10.1016/j.jbiomech.2009.03.015.

Comparison of Different State Space Definitions for Local Dynamic Stability Analyses

Deanna H. Gates^{1,*} and Jonathan B. Dingwell²

¹Department of Biomedical Engineering, University of Texas, Austin, TX 78712

²Department of Kinesiology & Health Education, University of Texas, Austin, TX 78712

Abstract

Measures of local dynamic stability, such as the local divergence exponent (λ_s^*) quantify how quickly small perturbations deviate from an attractor that defines the motion. When the governing equations of motion are unknown, an attractor can be reconstructed by defining an appropriate state space. However, state space definitions are not unique and accepted methods for defining state spaces have not been established for biomechanical studies. This study first determined how different state space definitions affected λ_s^* for the Lorenz attractor, since exact theoretical values were known *a priori*. Values of λ_s^* exhibited errors < 10% for 7 of the 9 state spaces tested. State spaces containing redundant information performed the poorest. To examine these effects in a biomechanical context, 20 healthy subjects performed a repetitive sawing-like task for 5 minutes before and after fatigue. Local stability of pre- and post-fatigue shoulder movements was compared for 6 different state space definitions. Here, λ_s^* decreased post-fatigue for all 6 state spaces. Differences were statistically significant for 3 of these state spaces. For state spaces defined using delay embedding, increasing the embedding dimension decreased λ_s^* in both the Lorenz and experimental data. Overall, our findings suggest that direct numerical comparisons between studies that use different state space definitions should be made with caution. However, trends across experimental comparisons appear to persist. Biomechanical state spaces constructed using positions and velocities, or delay reconstruction of individual states, are likely to provide consistent results.

Introduction

Nonlinear dynamics methods for determining the local stability of kinematics have gained increased interest in recent literature. For experimental data, the governing equations are typically unknown. During most repetitive movements, the dynamics of such movements can be represented geometrically within an n -dimensional ‘state space’ where n is the number of state variables (Dingwell and Cusumano, 2000; Kantz and Schreiber, 2004). Typically, these experimental data exhibit the structure of an attractor, i.e., a sub-space of the n -dimensional state space to which neighboring trajectories converge (Strogatz, 1994; Dingwell and Kang, 2007).

© 2009 Elsevier Ltd. All rights reserved.

Please address all correspondence to: Deanna H. Gates, M.S., Department of Biomedical Engineering, University of Texas at Austin, 1 University Station, C0800, Austin, TX 78712, Phone: 512-471-4017, Fax: 512-471-8914 Email: E-mail: dgates@mail.utexas.edu.

Publisher's Disclaimer: This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final citable form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

Measurable biomechanical state variables typically include linear and/or angular displacements, velocities, and/or accelerations. State spaces may also be defined using delay embedding, which allows reconstruction from a single scalar recording, once an appropriate time lag and embedding dimension are determined (Kantz and Schreiber, 2004). This can greatly simplify data collection and may be advantageous for situations where data for some measurable dimensions are prone to error. However, since divergence in one dimension may be compensated by contraction in another (Granata and Gottipati, 2008), analyzing one single trajectory may not adequately represent the movement. One proposed way to compensate for this is to perform delay embedding on the Euclidean norm of the three Euler angles at a joint (Granata and Gottipati, 2008)¹. Another option is to analyze a state space composed of all angles and angular velocities at that joint. However, this will typically include redundant information which could negatively impact the results. One way to define state variables with minimum redundancy is to perform a principal components analysis (PCA) on the data (Daffertshofer et al., 2004). Many state spaces have been applied in biomechanical stability analyses (Table 1). To date, however, researchers have not explored how different definitions of these state spaces affect local stability measures.

This study determined how including different state variables and/or different numbers of variables in the state space altered short-term local divergence exponents, λ_s^* , a measure of local dynamic instability (Dingwell and Cusumano, 2000). Data for which the true λ_s^* was known *a priori* were analyzed to determine how different state space definitions affect the resulting values of λ_s^* . Experimental data were then analyzed to determine how different state space definitions might affect the answers to research questions attempting to quantify differences in stability between conditions. The state space definitions examined here were chosen primarily from previously published studies (Table 1).

Methods

The Lorenz attractor (Strogatz, 1994) was used to test the effect of different state space definitions on λ_s^* . The Lorenz system is defined by three coupled nonlinear differential equations:

$$\begin{aligned}\dot{x} &= \sigma(y - x) \\ \dot{y} &= x(\rho - z) - y \\ \dot{z} &= xy - \beta z\end{aligned}\tag{1}$$

where σ , ρ , and β are fixed parameters that were set to $\sigma = 16$, $\rho = 45.92$ and $\beta = 4$ (Rosenstein et al., 1993). These equations were integrated in Matlab using a fourth order Runge-Kutta method (ode45) to generate 10 trials of 155 seconds at 100 Hz. The first 5 seconds were removed to eliminate transients. The first derivative of each trajectory was estimated using a three point difference formula. Uniformly distributed random noise was added to attain a minimum signal-to-noise ratio of 100:1, which is considered ‘moderate’ (Rosenstein et al., 1993). Because the Lorenz attractor for these parameter values is chaotic, λ_s^* in this case defines the maximum finite-time Lyapunov exponent, which should be 1.50 (Rosenstein et al., 1993).

We defined 9 different state spaces for this Lorenz attractor. The “Full” state space was defined by the three $x(t)$, $y(t)$, and $z(t)$ trajectories (Eq. 1). A “Redundant” state space was defined from these three trajectories and their derivatives:

¹The Euclidean norm of 3 Euler angles is *not* an appropriate measure of the “distance” between two angular orientations, as it would be for linear displacements. This measure should be interpreted here merely as a weighted average of the 3 joint angles.

$$\text{Redundant SS}(t) = [x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t)] \quad (2)$$

We performed principal components analysis (Daffertshofer et al., 2004) on both the Full and Redundant state spaces. In cases where state variables had different units, we normalized each to unit variance. To explain $\geq 95\%$ of the variance, 2 principal components were needed for the Full state space and 4 were needed for the Redundant state space. However, since this system is 3-dimensional (Eq. 1), we tested state spaces composed of both 3 and 4 principal components (see Supplementary Material).

Four additional state spaces were defined using delay embedding (Kantz and Schreiber, 2004) of $x(t)$, $y(t)$, $z(t)$ and $r(t) = \sqrt{x^2 + y^2 + z^2}$ (Granata and Gottipati, 2008). Multi-dimensional state spaces were reconstructed from each original time series and its time-delayed copies (Takens, 1981):

$$Q(t) = [q(t), q(t+T), q(t+2T), \dots, q(t+(d_E - 1)T)] \quad (3)$$

where $Q(t) \in \{X(t), Y(t), Z(t), R(t)\}$ was the d_E -dimensional state vector, $q(t) \in \{x(t), y(t), z(t), r(t)\}$ was the original 1-dimensional data, T was the time delay, and d_E was the embedding dimension (Fig. 1). Time delays were calculated as the first minimum of the average mutual information function of each signal (Fraser and Swinney, 1986).

Maximum local divergence exponents (λ_s^*) were calculated from the exponential rates of divergence of small perturbations in state space (Rosenstein et al., 1993; Dingwell and Cusumano, 2000). Positive exponents indicate local instability. Larger exponents indicate greater sensitivity to local perturbations. Short-term λ_s^* were estimated from:

$$\lambda_s^* = \frac{1}{\Delta t} \langle \ln[d_j(t)] \rangle \quad (4)$$

where $\langle \ln[d_j(t)] \rangle$ represents the mean log divergence for all pairs of nearest neighbors, j (Rosenstein et al., 1993), over $0 \leq t \leq 1$ seconds. To test the effects of embedding dimension, we calculated λ_s^* for $X(t)$, $Y(t)$, and $Z(t)$ for $d_E \in \{2, 3, 4, 5, 6, 7, 15, 20\}$. Kennel et al. (1992) recommend using a d_E that is twice the true dimension (i.e., $d_E = 6$). Rosenstein et al. (1993) suggest using a d_E equal to the true dimension (i.e., $d_E = 3$).

Experimental Data

Twenty healthy subjects (age 25 ± 2 years) participated after providing institutionally approved written informed consent. Subjects completed a repetitive sawing-like task for five minutes before and after performing a repetitive sagittal plane lifting task. This was designed to test the effect of local fatigue of the shoulder flexors on shoulder stability. The 3-D positions of 19 reflective markers placed on the right arm and trunk were recorded continuously during the sawing task at 120 Hz using an 8-camera Vicon system (Oxford Metrics, Oxford, UK). Marker data were filtered using a 5th order Butterworth filter with a cutoff frequency of 15 Hz. Rotational motions of the shoulder were defined using Euler angles (Wu et al., 2005) (see Supplementary Material).

We defined six state spaces for shoulder movement. The “Full” state space was composed of the three rotational angles and their angular velocities. Each state was then normalized to unit

variance and PCA was performed. Four principal components explained > 95% of the data variance. The second state space consisted of these four principle components. Four state spaces were defined using delay embedding. Global false neighbors analyses (Kennel, 1992) suggested $d_E = 5$ for all three rotational angles. Therefore, we tested $d_E \in \{3, 4, 5, 6, 7\}$. Short-term local divergence exponents (λ_s^* , Eq. 4) were calculated over $0 \leq t \leq 1$ cycle (Dingwell and Marin, 2006). Stability estimates are sensitive to time series length (Granata and England, 2006; Bruijn et al., In Press). Therefore, each time series was first normalized to exactly 36,000 points. This resulted in a < 1% change in the length of each time series. Pre- and post-fatigue λ_s^* from each trial were compared using paired t-tests for each state space.

Results

For the Lorenz attractor, λ_s^* differed depending on which state space was used (Fig. 2A; Table 2). Redundant state spaces containing derivatives performed the poorest (errors > 10%). Taking only three principal components of the redundant state space decreased this error from 20.4% to 7.53%. All methods correctly indicated that the Lorenz attractor is locally unstable ($\lambda_s^* > 0$).

As the embedding dimension was increased, λ_s^* decreased (Fig. 2B). We expected that a minimum of 3 (Rosenstein et al., 1993) to 6 (Kennel et al., 1992) states would reconstruct the state space with minimal error. Large errors were obtained when either too many or too few states were used.

For the experimental data, λ_s^* values also changed across different state spaces. Shoulder motion was always locally unstable, both pre- and post-fatigue. For all state spaces, λ_s^* tended to decrease post-fatigue (Fig. 3A). This decrease was statistically significant for 3 of the 6 methods (Fig. 3A). As with the Lorenz attractor (Fig. 2B), increasing the embedding dimension caused λ_s^* to decrease (Fig. 3B). However, pre- versus post-fatigue differences remained.

Discussion

The only formal requirement for a valid state space is that it uniquely defines the state of a system at all points in time (Kantz and Schreiber, 2004). Thus, any proposed state space is not unique. Here, we explored how several different state spaces affect local dynamic stability (λ_s^*) calculations. The formulations presented here are by no means exhaustive. It is quite possible that other state space definitions might perform as well or better.

Reasonably accurate results were obtained when PCA was performed on the full Lorenz state space and when the correct number of principal components was chosen from the redundant state space. PCA may overestimate the true dimension of the system (Clewley et al., 2008) and poor results were in fact obtained when too many principal components were included (Table 2). PCA also requires prior normalization of states that have different units. These normalizations are not unique and can affect the value of λ_s^* . We therefore do not recommend using PCA to define the state space.

While there were quantitative differences in λ_s^* for the different experimental state spaces (Fig. 3), all methods demonstrated whether trajectories were locally more or less stable. Differences between experimental conditions persisted, although the statistical significance of these differences did vary. Similarly, previous work demonstrated qualitatively similar changes in walking stability with changes in speed (Dingwell and Marin, 2006; England and Granata, 2007; Kang and Dingwell, 2008), despite using different state space definitions. Thus, while it may be difficult to make direct numerical comparisons between studies that use different state space definitions, it seems that qualitative trends persist. Overall, our findings suggest that

biomechanical state spaces constructed using positions and velocities, or delay reconstruction of individual states, are likely to provide more consistent results than those constructed using principal components. Efforts should also be made to avoid using redundant information wherever possible.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

ACKNOWLEDGMENTS

Funding for this project was provided by grant #EB003425 from the National Institutes of Health.

REFERENCES

- Bruijn SM, van Dieën JH, Meijer OG, Beek PJ. Statistical precision and sensitivity of measures of dynamic gait stability. *Journal of Neuroscience Methods* In Press, Corrected Proof
- Buzzi UH, Stergiou N, Kurz MJ, Hageman PA, Heidel J. Nonlinear dynamics indicates aging affects variability during gait. *Clinical Biomechanics* 2003;18(5):435–443. [PubMed: 12763440]
- Clewley RH, Guckenheimer JM, Valero-Cuevas FJ. Estimating effective degrees of freedom in motor systems. *IEEE Transactions on Biomedical Engineering* 2008;55(2):430–442. [PubMed: 18269978]
- Daffertshofer A, Lamoth CJC, Meijer OG, Beek PJ. PCA in studying coordination and variability: A tutorial. *Clinical Biomechanics* 2004;19(4):415–428. [PubMed: 15109763]
- Dingwell J, Robb R, Troy K, Grabiner M. Effects of an attention demanding task on dynamic stability during treadmill walking. *Journal of NeuroEngineering and Rehabilitation* 2008;5(1):12. [PubMed: 18426571]
- Dingwell JB, Cusumano JP. Nonlinear time series analysis of normal and pathological human walking. *Chaos* 2000;10(4):848–863. [PubMed: 12779434]
- Dingwell JB, Cusumano JP, Sternad D, Cavanagh PR. Slower speeds in neuropathic patients lead to improved local dynamic stability of continuous overground walking. *Journal of Biomechanics* 2000;33(10):1269–1277. [PubMed: 10899337]
- Dingwell JB, Kang HG. Differences between local and orbital dynamic stability during human walking. *Journal of Biomechanical Engineering* 2007;129(4):586–593. [PubMed: 17655480]
- Dingwell JB, Marin LC. Kinematic variability and local dynamic stability of upper body motions when walking at different speeds. *Journal of Biomechanics* 2006;39(3):444–452. [PubMed: 16389084]
- England SA, Granata KP. The influence of gait speed on local dynamic stability of walking. *Gait & Posture* 2007;25(2):172–178. [PubMed: 16621565]
- Fraser AM, Swinney HL. Independent coordinates for strange attractors from mutual information. *Physical Review A* 1986;33(2):1134–1140. [PubMed: 9896728]
- Granata KP, England SA. Stability of dynamic trunk movement. *Spine* 2006;31(10):E271–E276. [PubMed: 16648732]
- Granata KP, Gottipati P. Fatigue influences the dynamic stability of the torso. *Ergonomics* 2008;51(8):1258–1271. [PubMed: 18608477]
- Jordan K, Challis JH, Cusumano JP, Newell KM. Stability and the time-dependent structure of gait variability in walking and running. *Human Movement Science* 2009;28(1):113–128. [PubMed: 19042050]
- Kang HG, Dingwell JB. A direct comparison of local dynamic stability during standing and walking. *Experimental Brain Research* 2006a;172(1):35–48.
- Kang HG, Dingwell JB. Intra-session reliability of local dynamic stability of walking. *Gait & Posture* 2006b;24(3):386–390. [PubMed: 16413784]
- Kang HG, Dingwell JB. Effects of walking speed, strength and range of motion on gait stability in healthy older adults. *Journal of Biomechanics* 2008;41(14):2899–2905. [PubMed: 18790480]

- Kantz, H.; Schreiber, S. *Nonlinear Time Series Analysis*. Vol. 2nd ed. Cambridge, UK: Cambridge University Press; 2004.
- Kennel MB, Brown R, Abarbanel HDI. Determining embedding dimension for phase-space reconstruction using a geometrical construction. *Physical Review A* 1992;45(6):3403–3411. [PubMed: 9907388]
- Lee H, Granata KP. Process stationarity and reliability of trunk postural stability. *Clinical Biomechanics* 2008;23(6):735–742. [PubMed: 18304711]
- Manor B, Wolenski P, Li L. Faster walking speeds increase local instability among people with peripheral neuropathy. *Journal of Biomechanics* 2008;41(13):2787–2792. [PubMed: 18706561]
- Rosenstein MT, Collins JJ, DeLuca CJ. A practical method for calculating largest Lyapunov exponents from small data sets. *Physica D: Nonlinear Phenomena* 1993;65:117–134.
- Segal AD, Orendurff MS, Czerniecki JM, Shofer JB, Klute GK. Local dynamic stability in turning and straight-line gait. *Journal of Biomechanics* 2008;41(7):1486–1493. [PubMed: 18405902]
- Slota GP, Granata KP, Madigan ML. Effects of seated whole-body vibration on postural control of the trunk during unstable seated balance. *Clinical Biomechanics* 2008;23(4):381–386. [PubMed: 18093708]
- Stergiou N, Moraiti C, Giakas G, Ristanis S, Georgoulis AD. The effect of the walking speed on the stability of the anterior cruciate ligament deficient knee. *Clinical Biomechanics* 2004;19(9):957–963. [PubMed: 15475129]
- Strogatz, SH. New York, NY: Addison-Wesley; 1994. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*.
- Takens, F. Detecting strange attractors in turbulence. In: Rand, D.; Young, LS., editors. *Dynamical Systems and Turbulence*, Warwick 1980. Vol. 898. Berlin: Springer-Verlag; 1981. p. 366–381.
- Wu G, van der Helm FC, Veeger HE, Makhsous M, Van Roy P, Anglin C, Nagels J, Karduna AR, McQuade K, Wang X, Werner FW, Buchholz B. ISB recommendation on definitions of joint coordinate systems of various joints for the reporting of human joint motion--part ii: Shoulder, elbow, wrist and hand. *Journal of Biomechanics* 2005;38(5):981–992. [PubMed: 15844264]
- Yoshino K, Motoshige T, Araki T, Matsuoka K. Effect of prolonged free-walking fatigue on gait and physiological rhythm. *Journal of Biomechanics* 2004;37(8):1271–1280. [PubMed: 15212933]

Figure 1.

A) Data for the Lorenz attractor is shown with $\sigma=16$, $\rho=45.92$, and $\beta=4$. A 3-D was created from the $x(t)$, $y(t)$ and $z(t)$ data. **B)** Using the method of delays, a state space can be created by reconstruction of a single trajectory, $x(t)$. The time delay, T , is determined as the first minimum of the average mutual information function. A 3-D was created using the original time series, $x(t)$, and its time delayed copies, $x(t+T)$ and $x(t+2T)$. While the state spaces for the two methods appear different, their dynamics should be the same.

Figure 2.

Modeling Results. **A)** The short-term local divergence exponent, λ_s^* , was calculated for 10 simulations with nine different state space definitions. Error bars represent the mean $\pm 95\%$ confidence interval for each method defined in Table 2. The dotted line represents the expected value of 1.50. **B)** λ_s^* decreases with embedding dimension, d_E and then levels off with relatively constant error at higher d_E . ‘*’ is λ_s^* for the delay embedding of $x(t)$, ‘x’ is the delay embedding of $y(t)$, and ‘o’ is the delay embedding of $z(t)$. Error bars represent $\pm 95\%$ confidence interval for each method.

Figure 3.

Results Using Experimental Data. A) Short-term local divergence exponents, λ_s^* , for pre- and post-fatigue conditions for six state space definitions: the full state space ('Full SS') defined by $[\theta_1, \theta_2, \theta_3, \theta'_1, \theta'_2, \theta'_3]$ the first four principle components of the full state space ('PCA'), and the delay embedding of $\theta_1, \theta_2, \theta_3, r_\theta$ (where $\sqrt{\theta_1^2 + \theta_2^2 + \theta_3^2}$, and. Symbols 'x' and '•' represent pre- and post-fatigue trials, respectively. While the numerical values differed depending on the state space definition, λ_s^* always tended to decrease from pre to post fatigue. This decrease was statistically significant when for Full SS ($p=0.03$), and the delay embeddings of θ_1 ($p=0.04$) and θ_3 ($p=0.04$). B) λ_s^* pre- and post-fatigue are plotted versus embedding dimension, d_E , for the delay embedding of θ_1 (Top), θ_2 (Middle), and θ_3 (Bottom). Symbols are the same as in A. λ_s^* decreased as embedding dimension increased, similar to the Lorenz data (Fig. 2). This did not appear to affect the differences in λ_s^* pre and post fatigue. In all plots, error bars represent between-subject $\pm 95\%$ confidence intervals for each method.

Table 1**Example State Space Definitions**

A variety of state space definitions have been used in previous work for tasks including standing, walking, running, and repetitive lifting.

State Space Definition	References
All angular and linear motion and velocities of a segment	Kang and Dingwell, 2006a,b Kang and Dingwell, 2008
All angles and angular velocities at a joint	Lee and Granata, 2008 Slota et al., 2008
Delay embedding of sagittal plane angles	England and Granata, 2007 Manor et al., 2008 Segal et al., 2008 Stergiou et al., 2004
Delay embedding of linear motion of a single marker	Bruijn et al., In Press Buzzi et al., 2003 Dingwell and Marin, 2006 Dingwell et al., 2008 Jordan et al., 2009
Delay embedding of linear acceleration	Dingwell et al., 2000 Yoshino et al., 2004
Delay embedding of the Euclidean norm of three Euler angles at a joint	Granata and England, 2006 Granata and Gottipati, 2008

Table 2

Percent error between the known local divergence exponent for the Lorenz attractor, $\lambda_s^* = 1.5$, and that calculated from nine different state spaces. The trajectories $x(t)$, $y(t)$, and $z(t)$ define the system. PC1 – PC4 are the first principle components of the Full and Redundant states spaces. The last four state spaces are reconstructed from a single time series using the method of delays.

Method	State Space	% Error
Full SS	$S = [x(t), y(t), z(t)]$	3.40
Redundant SS	$S = [x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t)]$	10.17
PCA of Full SS	$S = [PC1(t), PC2(t)]$	1.25
PCA of Redundant SS	$S = [PC1(t), PC2(t), PC3(t), PC4(t)]$	20.40
PCA of Redundant SS	$S = [PC1(t), PC2(t), PC3(t)]$	7.53
Delay X	$X(t) = [x(t), x(t+T), x(t+2T), \dots, x(t+6T)]$	8.75
Delay Y	$Y(t) = [y(t), y(t+T), y(t+2T), \dots, y(t+6T)]$	1.41
Delay Z	$Z(t) = [z(t), z(t+T), z(t+2T), \dots, z(t+6T)]$	5.87
Delay R	$R(t) = [r(t), r(t+T), r(t+2T), \dots, r(t+6T)]$ Where $r(t) = \sqrt{x^2 + y^2 + z^2}$	6.85