Micromixer based on viscoelastic flow instability at low Reynolds number

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We exploited the viscoelasticity of biocompatible dilute polymeric solutions, namely, dilute poly(ethylene oxide) solutions, to significantly enhance mixing in microfluidic devices at a very small Reynolds number, i.e., $Re \approx 0.023$, but large Peclet and elasticity numbers. With an abrupt contraction microgeometry (8:1 contraction ratio), two different dilute poly(ethylene oxide) solutions were successfully mixed with a short flow length at a relatively fast mixing time of $< 10 \mu s$. Micro-particle image velocimetry was employed in our investigations to characterize the flow fields. The increase in velocity fluctuation with an increase in flow rate and Deborah number indicates the increase in viscoelastic flow instability. Mixing efficiency was characterized by fluorescent concentration measurements. Our results showed that enhanced mixing can be achieved through viscoelastic flow instability under situations where molecular-diffusion and inertia effects are negligible. This approach bypasses the laminar flow limitation, usually associated with a low Reynolds number, which is not conducive to mixing. © 2009 American Institute of Physics. [DOI: [10.1063/1.3108462](https://doi.org/10.1063/1.3108462)]

I. INTRODUCTION

A key attribute of flow dynamic in microdevices is their laminar character, which is in sharp contrast to the easily achievable turbulent flow in macroscopic process equipment. As a result, mixing fluids in microchannels is challenging and often an obstacle to achieve good performance in microfluidics systems.¹ The reason for a predominantly laminar flow in microdevices is their microscale dimensions, which lead to a small Reynolds number

$$Re = \rho \bar{V} d_0 / \eta_0, \quad (1)$$

where ρ , $\bar{V}$, d_0 , and η_0 are the fluid density, the average velocity, the characteristic length, and the viscosity, respectively. Viscous effects become dominant in microscale and suppress viscous-inertial flow instabilities. Consequently, mixing of multiple streams in microchannels often relies on molecular-diffusion mechanism and not on the effective mechanism of chaotic/turbulent flow instability.^{2,3} In general, micromixers can be categorized as passive micromixers and active micromixers.⁴ Passive micromixers, which include lamination and chaotic advection micromixers, rely on the diffusive mixing mechanism. They require long and/or complicated channel geometry. Although active micromixers might be employed, they require external actuators. Both methods lead to complex and expensive fabrication processes. A different approach that bypasses the limitation of the low Reynolds number, yet provides efficient mixing, will be a significant improvement for microfluidic system design.

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An alternative method to achieve a short mixing path is by hydraulic focusing.³ This idea has been demonstrated to reduce the effective mixing time to the order of few microseconds.⁵ However, the sample fluids would have to diffuse into the sheath flows and the focusing ratio is constrained by the applied flow rate or pressure, which will impact on the ease of implementation in a microfluidic system. A simpler geometry was demonstrated by Stroock *et al.*⁶ which is commonly known as staggered herringbone mixer (SHM). Using SHM the mixing length was shortened to 1.5 cm compared to the meter long basic T-mixer. The major effect of the herringbone structures is to deflect the flow in such a way that the interface area between the fluids increases and therefore mixing is enhanced. Another approach is to operate in a two-phase manner in which liquid droplets are formed and transported in a continuous flow phase.⁷ In this two-phase flow, heat and mass transfers are enhanced due to the high interfacial surface generated, and the droplets also behave as batch reactors where the products mix and react.

Solutions with trace amount of highly deformable polymers are viscoelastic fluids. These fluids, which are non-Newtonian, have a complex internal microstructure that can lead to counterintuitive flow and stress responses. The stress experienced by these fluids will not immediately become zero with the cessation of driving force and fluid motion but decays with a characteristic time due to its elasticity. By employing these fluids, viscoelastic flow instability can be generated in a microfluidic device at very low Re ($Re < 1$). Burghlea *et al.*⁸ had recently extended the works of Groisman and Steinberg⁹ for viscoelastic mixing in millimeter length scales to micrometer length scales. Chaotic flow instabilities were exploited for the enhancement in mixing at very low Reynolds numbers. Efficient mixing was demonstrated with a relatively long mixing channel at a moderate flow rate. The characteristic mixing times of these viscoelastic fluids were reduced by three to four orders of magnitude as compared to mixing based on the mechanism of molecular diffusion.

Indeed, to exploit effectively the viscoelasticity of fluids for chaotic flow instability, and thus mixing, sharper and smaller geometries should be employed.¹⁰ Stress singularities developed at such corners have been the source of elastic instabilities in many macroscale experiments for fluids with viscosity in at least $O(\text{Pa s})$, while rounded corners tend to suppress elastic behavior.^{11,12} The distinct similarities of these nonlinear flow phenomena at microlength scale experiments were demonstrated qualitatively in microdevices with dilute viscoelastic fluids, with viscosity in $O(\text{mPa s})$, for instance, in converging entry flow via a planar contraction microgeometry at a relatively low Reynolds number ($Re < 10$) (Ref. 13) and for flow in a flip-flop memory devices.¹⁴ The elastic effects were demonstrated to be dominant and they induced flow instability in an abrupt convergent/divergent microlength scale flow system.

The promotion of viscoelastic instability by utilizing microchannels with abrupt contraction/expansion geometry for micromixing was recently exploited by us.^{15,16} We have reported our experimental observations of streak flow in silicon-glass microchip.¹⁶ Our mixing experiments have very high values of elasticity numbers (a ratio of elastic to inertial forces of the system), which are approximately 932 and 5070. At such a high elasticity number, inertial effects were negligible. Thus, Reynolds number (the ratio of inertial to viscous forces of the system) is no longer relevant in describing the flow behaviors. The flow dynamics and thus mixing of two dissimilar viscoelastic fluid streams were mainly governed by the competition of the viscous and elastic forces in the flow field. As such, the Deborah number, which is the ratio of elastic to viscous forces of the system, would be the dominant governing parameter.

In the current investigation, we analyze the flow dynamics of mixing two dissimilar viscoelastic fluids through a converging/diverging flow system based on the measurements of both velocity flow field and fluorescence concentration field. Dilute poly(ethylene oxide) (PEO) fluids were employed for its inherent viscoelastic effects and its biocompatibility such that any residual should not induce inflammatory response. In addition, PEO is water soluble, avoiding the usage of potentially toxic organic solvents for dissolving it.¹⁷ The development of viscoelastic instability with an increase in flow rate and the Deborah number is monitored by the increase in velocity fluctuation, which is normally not expected for a stable laminar flow.

FIG. 1. Planar contraction/expansion micromixer. (a) Silicon-glass microchip and jig holder and (b) microgeometry configuration of the microchannel.

II. MATERIALS AND METHODS

A. Experimental setup

A simple abrupt contraction/expansion silicon-based microchannel geometry of 1000:125:1000 μm was used to introduce the convergent/divergent flow (see Fig. 1). The microdevice had a uniform depth of 150 μm and a contraction channel length of 1000 μm . Side streams were introduced via two side channels, each at either side of the main channel, into the central main stream. The intersection of the main and the side channels was located 3000 μm upstream from the center line of the contraction. Both the main and the side channels are 1000 μm in width. The microdevice was made of silicon wafer and Pyrex glass wafer. A jig holder was fabricated as the microfluidic to macrofluidic interface. Details on device fabrication are contained in Ref. 18.

The two sample fluids were primed by a precision syringe pump (Lomir Biomedical Inc.). The desired flow rate ratio was achieved by driving simultaneously three different syringes on one pump. The total flow rate was $\dot{Q}$ with the main stream flow rate at $0.5\dot{Q}$ and both side stream flow rates at $0.25\dot{Q}$. The investigated $\dot{Q}$ were 0.5, 5, and 10 ml/h.

B. Fluid preparations

In this study we focused on dilute and semidilute liquids of PEO with mean molecular weight (M_{wt}) (The Dow Chemical Co.) of approximately 2×10^6 g/mol. The sample fluid in the dilute concentration regime was prepared by mixing the PEO powder (at concentration of 0.1 wt % PEO) in de-ionized (DI) water with 3 μm red fluorescent microsphere liquid. For brevity, it was denoted as 0.1% PWS. The sample fluid in the semidilute concentration regime has a higher viscosity and elasticity than the dilute regime. The sample fluid was made of 1.0 wt % of PEO dissolved in a mixture of 55 wt % glycerol in DI water with green fluorescent dye. For brevity, it was denoted as 1 wt % PGWF. A fluorescence dye (fluorescein disodium salt $\text{C}_{20}\text{H}_{10}\text{Na}_2\text{O}_5$, Sigma Aldrich) was added at a weight ratio of 3×10^{-4} :1. The liquid of dispersed 3 μm red fluorescent microspheres (Duke Scientific Co.) was added at a volume ratio of 0.1:1. The fluorescence green dye with a mercury lamp was used for concentration measurement for the evaluation of mixing efficiency. The red microspheres, which served as tracers, were used in microparticle image velocimetry (μ -PIV) with a double pulsed twin laser system for flow velocity evaluation.

TABLE I. Fluid properties. Measurements were recorded at 25 ± 1 °C.

Fluid	Zero-shear viscosity η_0 (mPa s)	Density ρ (kg/m ³)	Relaxation time λ (ms)
0.1% PWS	2.24	997	1.5
1% PGWF	1020	1130	220

C. Rheological property measurements

The ARES (Advanced Rheometric Expansion System) rotational rheometer (Rheometrics scientific) with a parallel plate measuring system (plate diameter=50 mm and gap=1 mm) was used to characterize the semidilute fluid in term of its zero-shear viscosity, dynamic properties, and relaxation properties. The rheological measurements of 1% PGWF in steady and dynamic modes were carried out at room temperature of 25 °C with a shear rate range of $0.1 \leq \dot{\gamma} \leq 100$ s⁻¹ and frequency of $0.1 \leq \omega \leq 100$ rad/s, respectively. After loading the sample fluid and setting up the measuring parameters, approximately 5–10 min was taken for the system to reach equilibrium before the start of each test. The elastic energy storage modulus (G') and the viscous dissipation modulus (G'') obtained from the dynamic frequency oscillation tests were used to compute the Maxwell relaxation times (λ).¹⁹ Due to the low viscosity of 0.1% PWS, its viscosity was measured by the viscometer Contraves LS 40 at controlled rate mode. However, its λ could not be determined with our existing facility, and the value was taken from Ref. 13.

Since the diffusion coefficient is inversely proportional to the sample viscosity,²⁰ we estimated that $D = 1.47 \times 10^{-12}$ m²/s for 1% PGWF based on the diffusion coefficient of the dye in water [$D = 1.5 \times 10^{-9}$ m²/s (Ref. 21)]. A small D indicates that mixing based on diffusion mechanism is negligible. All the fluid properties were determined with the additives and the measured rheological properties of the fluid are contained in Table I.

D. Dimensionless parameters

For mixing in laminar flow, diffusion is an important mechanism. Peclet number (Pe) indicates the relative importance of advection to diffusion and is given by

$$Pe = \dot{Q}L_{\text{char}}/D(w_c d), \quad (2)$$

where L_{char} is the upstream channel width, D is the diffusion coefficient, and $\dot{Q}$ is the total volumetric flow rate. The higher the Pe, the less significant is diffusion.

For an aqueous or a viscous solution, the onset of flow instability can be characterized by the Reynolds number (Re), which measures the relative importance of inertial and viscous effects in a fluid flow. It can be defined as

$$Re = 2\rho\dot{Q}/\eta_0(w_c + d), \quad (3)$$

where d is the channel depth and w_c is the contraction width. ρ and η_0 are the fluid density and viscosity, respectively.

For a given geometry, the viscoelastic effects of a fluid flow can be characterized by the Deborah number (De). The Deborah number is a dimensionless parameter that typifies the relative importance of the elastic stresses of the fluid with the time scale of the flow system.¹⁰ Generally, a smaller dimension results in a higher characteristic deformation rate for the same flow rate, resulting in higher elastic effects and a higher De. The Deborah number can be expressed based on the characteristic shear rate,

$$De = \lambda \dot{\gamma}_{\text{char}} = \lambda(2\dot{Q}/w_c^2 d) = 2\lambda\dot{Q}/w_c^2 d. \quad (4)$$

FIG. 2. Optical setup for fluorescent concentration and PIV measurements.

The characteristic shear rate is $\dot{\gamma}_{\text{char}} = \bar{v}_c / (w_c/2) = (\dot{Q}/w_c d) / (w_c/2) = 2\dot{Q}/w_c^2 d$, λ is the relaxation time of the viscoelastic fluid measured in shear, and $\bar{v}_c$ is the average flow velocity.

A smaller channel has smaller flow characteristic length and time. Thus, Re is smaller and it is difficult to have inertia/viscous flow instability. Conversely, De becomes larger and it is easier to have elastic/viscous instability. The relative dominance of elastic to inertial effects is typified by the elasticity number (El), i.e., the ratio of fluid elasticity to fluid inertia. El is expressed as

$$\text{El} = \text{De}/\text{Re} = \frac{2\lambda \eta_0 (w_c + d)}{\rho w_c^2 d}. \quad (5)$$

El is a function of the given fluid and geometry. It is dependent on the fluid properties and the inverse of the characteristic cross-sectional area.

E. Imaging techniques

The experimental setup, for both measurements of the velocity and the concentration fields, is depicted schematically in Fig. 2. Basically, the setup consists of four main components: an illumination system, an optical system, image acquisition devices, and a control system. Two different illumination sources, a double pulsed Q -switch (quality switched) Nd:YAG (yttrium aluminum garnet) laser and a mercury lamp, were used for the measurements of the velocity field and the concentration field. The optical system consisted of an inverted microscope (model ECLISPE TE2000-S) with a set of epifluorescent attachments. The image acquisition devices consists of an

interline transfer charge-coupled device (CCD) camera (HiSense MkII), a FlowMap system hub with last-in-first-out capability, and a commercial camcorder (Sony, model DCR-DVD803E). The resolution of the CCD camera is 1344×1024 pixels with 12 bit grayscale. The recorded images were digitally transferred to a personal computer for further analysis.

For each flow rate, the flow field images in the same experiment were visualized using the green fluorescent dye (main stream) and the red fluorescent microspheres (side streams). The images of the streams, at the same area of interest, were captured by switching the epifluorescent attachments of the microscope. The green fluorescent dye has an excitation wavelength and an emission wavelength of 490 and 520 nm, respectively. The epifluorescent attachment of type Nikon B-2A (excitation filter for 450–490 nm, dichroic mirror for 505 nm, and an emission filter for 520 nm) was used for the green dye. The red microspheres (Duke Scientific Co.) with an excitation wavelength of 540 nm and a maximum emission wavelength of 610 nm were imaged with the epifluorescent attachment of type Nikon G-2E/C (excitation filter for 540 nm, dichroic mirror for 565 nm, and an emission filter for 605 nm). The relatively large red microspheres help to minimize the diffusive effect in the experiments. Both green and red filters have a bandwidth of 25 nm.

III. RESULTS AND DISCUSSIONS

We analyzed the multiple-stream flow dynamics of two dissimilar viscoelastic fluids, 1% PGWF with 0.1% PWS, based on the measurements of velocity flow field. With reference to 1% PGWF (main stream fluid) properties, investigation was conducted over ranges of $21.73 < De < 434.6$ and $0.001 < Re < 0.023$. For the same device and fluids, the elasticity number ($El = 19.3 \times 10^3$) was constant in our investigation. With the large Peclet number ($Pe > 1.78 \times 10^7$) for our experiments, mixing due to molecular diffusion was negligible.

A. Measurement of the velocity field

As fluorescent microspheres were added only to the side streams (0.1% PWS) for μ -PIV investigations, only the flow field of the side streams would be measured. The microspheres have a maximum excitation wavelength close to the characteristic wavelength of Nd:YAG laser. The measurement was carried out with a $4\times$ objective lens and two 30 mJ laser pulses with a delay time of 4 ms. At least 50 images were collected for every flow condition. The fluorescent microsphere solution was not added to the higher viscosity 1% PGWF. This would ensure that any velocity measured was related to particles that originated from the side streams. This would allow us to easily evaluate the penetration of the side streams to the main stream. In addition, if the solution was added to the main stream, it would compromise the viscosity of the main stream due to the exponential relationship of viscosity to the concentration of the polymeric solution, particularly the solvent glycerol.

Figure 3 shows the typical results obtained from the μ -PIV measurement upstream of the contraction. For a viscoelastic two-fluid flow at $\dot{Q} = 10$ ml/h ($De = 434.6$ and $Re = 0.02$), asymmetrical vortices were observed and confirmed with postanalysis (streamlines and velocity vector field) using a commercial PIV software (FLOWMANAGER, FlowMap). The formation of these asymmetrical vortices based on empirical observations was discussed in our previous works.^{15,16}

Figure 4 presents the consecutive velocity fields of viscoelastic two-fluid flow at $\dot{Q} = 10$ ml/h ($De = 434.6$ and $Re = 0.02$). Salient and large corner vortices with viscoelastic “whipping” at upstream of the contraction were evident. Successive snapshots show a large fluctuation of both magnitude and direction of the velocity vectors. This fluctuation indicates the highly unstable nature of the viscoelastic flow. The results illustrated the chaotic competition of the two fluids at the contraction and across the downstream channel. These results were in good agreement with our earlier observations. Significant mixing of the two viscoelastic fluids was expected and observed.

Figures 5–7 show the statistical analysis of the velocity flow profile across the channel (both upstream and downstream) for three different flow rates, 0.5, 5, and 10 ml/h, respectively. Each

FIG. 3. Typical results of two dissimilar viscoelastic fluid flows ($\dot{Q}=10$ ml/h, trace microspheres are in the side streams): (a) raw particle image, (b) evaluated streamlines of the flow field, (c) evaluated velocity field, and (d) surface plot of the velocity magnitude.

plot represents a statistic over 50 image pairs (μ -PIV double frames imaging) and the standard deviation of velocity magnitude fluctuation was illustrated as an error bar in the plots. At $\dot{Q}=0.5$ ml/h ($De=21.73$ and $Re=0.001$), the entire flow field was stable and steady. The flow was laminar and the measured results show a small variance in velocity (minimal velocity variation) (see Fig. 5). For the upstream velocity profile shown in Fig. 5(a), no data points were shown at the central portion due to the lack of fluorescent microspheres in the main stream. The downstream velocity profile [Fig. 5(b)] shows bigger variances at the central portion of the channel. This phenomenon was caused by the penetration of fluorescent microspheres from the side streams into the main stream. This penetration was intermittent but consistent. However, in general, the entire flow field shows little flow instability.

As the flow rate increased (i.e., De increased) to $\dot{Q}=5$ ml/h, salient corner vortices were formed upstream of the contraction. For $\dot{Q}=5$ ml/h ($De=217.3$ and $Re=0.01$), the upstream vortices were larger than in $\dot{Q}=0.5$ ml/h ($De=21.73$ and $Re=0.001$) but remained rather constant in size. The entire flow field was still rather laminarlike. This fact was supported by the symmetrical velocity profile plot with an increasing, but still relatively small variance (see Fig. 6). As shown in Fig. 6(a) upstream of the contraction, there was a region at the center portion of the channel with much higher velocity. This indicated that the side stream particles had penetrated into the main stream and their velocity had increased due to the transfer of momentum from the main stream to these side stream particles. As shown in Fig. 6(b) downstream of the contraction, one can observe more penetrations of the side streams into the central portion of the channel with increasing fluctuation of the flow field at the center portion of the channel.

At the highest flow rate investigated, i.e., $\dot{Q}=10$ ml/h ($De=434.6$ and $Re=0.02$), the upstream vortices were the largest and unstable in nature. “Elastic whipping” of the central main stream was observed upstream of the contraction, with interpenetration of the main stream and side streams. As shown in Fig. 7(a), the results show a competition between the main and the side streams at the entrance of the contraction with significant velocity fluctuation. The entire flow field

FIG. 4. Consecutive PIV measurements for viscoelastic fluids flow ($\dot{Q}=10$ ml/h, trace microspheres are in the side streams): (a) $t=0$ s (arbitrary), (b) $t=2.4$ s, and (c) $t=4.8$ s.

became unstable and chaotic. The whipping effects allowed the side streams to penetrate deeply into the central portion downstream [see Fig. 7(b)]. Extensive flow instability occurred downstream of the contraction, as indicated by the large velocity variance. Indeed, the well defined parabolic average velocity profile (solid line) in Fig. 7(a) was much less obvious in Fig. 7(b), suggesting the possibility of the onset of turbulent flow. In addition, with deep penetration of the side streams into the main stream, and with flow instability, it is expected that effective mixing would result in the microchannel.

The mechanism of this viscoelastically induced flow instability is mainly attributed to the deformation of polymer coils through a planar contraction/expansion flow. The velocities of the viscoelastic fluids have to increase significantly as they flow from upstream into the contraction. This results in elongation flows, and the viscoelastic fluids (the polymer coils within them) are stretched with elastic energy stored within them. The main stream 1% PGWF fluid is much more elastic (with more polymer coils per unit volume) than the side stream 0.1% PWS fluid. Due to its relatively high elasticity, the main stream fluid was extensively elongated as the upstream flow approaches the contraction due to the flow competition mentioned previously [see Fig. 7(a)]. Thus, the main stream fluid has a substantial amount of elastic energy and deformation entering the contraction. There is little time for relaxation, and thus decrease in elastic energy, within the contraction due to the high flow speed [$O(10^1 \text{ mm s}^{-1})$] within it.

This substantial amount of stored elastic energy of the main stream fluid would be released comparatively quickly (due to the elastic recovery of the polymer coils) with a sudden loss of

FIG. 5. Velocity profile across the channel at $\dot{Q}=0.5$ ml/h (each plot represents a statistic over 50 image pairs with the standard deviation of velocity magnitude fluctuation shown as an error bar): (a) upstream of the contraction and (b) downstream of the contraction.

physical constraint at the exit of the contraction. Hence, downstream of the contraction, the quick release of this much larger elastic energy from the main stream as compared to the side streams induces a strong transverse flow and displaces the side streams next to it. Due to this quick elastic recovery of the stretched polymer coils within the main stream, the flow could become unstable with random penetration of the main stream into the side streams. This would result in chaotic cross-stream flow instability at the downstream flow field [see Fig. 7(b)].

B. Measurement of concentration field

Concentration measurements were conducted to evaluate the efficiency of mixing. The measured area was illuminated by a mercury lamp. The main stream with green fluorescent dye was employed as the reference for evaluating the mixing performance. After recording the CCD images on a PC [Figs. 8(a) and 8(b)], the concentration profiles were evaluated using a customized program written in MATLAB. The noise in the measured image was removed with an adaptive noise-removal filter. For each pixel, a local mean value is calculated with a window of 5×5 pixels. The noise distribution is assumed to have a Gaussian characteristic.

The mixing performance of the mean flow was evaluated from the probability density function (PDF) of the intensity images, i.e., concentration measurements. The intensity of the gray scale in the recorded images is proportional to the green dye concentration. Details on quantifying mixing efficiency ϵ_{eff} from the PDF of intensity images are contained in Ref. 15. The mixing efficiency ϵ_{eff} is quantified based on the following definition:

FIG. 6. Velocity profile across the channel at $\dot{Q}=0.5$ ml/h (each plot represents statistic over 50 image pairs with standard deviation of velocity magnitude fluctuation shown as an error bar): (a) upstream of the contraction and (b) downstream of the contraction.

$$\varepsilon_{\text{eff}} = \left[1 - \frac{\sum_{C_i=0}^{C_i=1} |C_i - C_{\infty}| P(C_i)}{C_{\infty}} \right] \times 100\%, \quad (6)$$

where C_i is the observed concentration on the normalized scale, C_{∞} is the concentration on the normalized scale for perfect mixing, and $P(C_i)$ is the PDF. In our investigation, for equal flow rate of the main and the side streams, perfect mixing concentration is $C_{\infty}=0.5$. With Eq. (6), $\varepsilon_{\text{eff}}=0\%$ for no mixing and $\varepsilon_{\text{eff}}=100\%$ for perfect mixing.

Figure 8 compares the progression of ε_{eff} along the flow path for the Newtonian fluids (DI water) and the viscoelastic fluids at the same total flow rate of $\dot{Q}=10$ ml/h. It was demonstrated by evaluating the ε_{eff} across the channel at various locations along the flow path for both upstream and downstream channels. The size of analysis window is an area of $30 \times 950 \mu\text{m}^2$ across the channel. The analysis windows were distributed evenly at a spacing of $100 \mu\text{m}$ except at the proximity of the contraction plane, where the spacing interval is halved [see Figs. 8(a) and 8(b)].

Figure 8(c) shows the progression of ε_{eff} . Upstream of the contraction, for the flow of DI water, there was insignificant mixing (approximately 12%, on average). As there is negligible elasticity in DI water, the flow is laminar with no stored elastic energy or elastic deformation within the fluids. For the viscoelastic fluids flow, on average, more than 15% of mixing was found along the upstream channel. Maximum mixing of 30% was observed at a distance of approximately 1.2 mm (section 11 upstream) away from the contraction plane. Indeed, one would expect that ε_{eff} could only increase as flow progressed. Thus, it was interesting to observe that as flow progressed from sections 11 to 1 upstream, ε_{eff} decreased. The surge of ε_{eff} at section 11 was due

FIG. 7. Velocity profile across the channel at $\dot{Q}=10$ ml/h (each plot represents a statistic over 50 image pairs with the standard deviation of velocity magnitude fluctuation shown as an error bar): (a) upstream of the contraction and (b) downstream of the contraction.

to the relatively large fluctuation in the flow field or viscoelastic whipping as compared to the region near to the contraction. The scale of whipping was constrained by the converging flow path when approaching the contraction plane. In addition, close to the contraction, the main stream and fluids next to the main stream, which consisted of a mixture of main stream and side streams, were “compressed” and flowed at a higher velocity. As such, more of the unmixed side streams would dominate spatially, resulting in lower mixing efficiency ϵ_{eff} .

Downstream of the contraction, the mixing performance for the DI water flow had improved due to the expansion flow effects. The mixing region at the interface of the two streams was stretched out. ϵ_{eff} had improved from 10% to 30%. Highly concentrated main and side streams could still be identified easily from the CCD images [see Fig. 8(a)]. This low efficiency of mixing was as expected for Newtonian flow at this low Reynolds number ($Re \approx 20.2$) and large Peclet number ($Pe \approx 98.8 \times 10^3$).

For viscoelastic fluids flow downstream of the contraction, viscoelastic flow instability, together with the expansion flow effects, had promoted mixing effectively. Next to the contraction, ϵ_{eff} was approximately 50%. This was mainly caused by the inhomogeneous volume of fluids discharged through the contraction intermittently due to the frantic competition of the two streams upstream of the contraction. As flow progressed further downstream, mixing was further promoted through viscoelastically induced flow instability, and ϵ_{eff} increased steadily from 50% to 70% [see Fig. 8(c)]. This mixing performance is in good agreement with the flow mechanism discussed in Sec. III A. A strong transverse flow was induced due to the release of the relatively large elastic energy of the main stream fluid. Random penetration of the main stream into the side streams was indeed captured in Fig. 8(b) as extensive overlapping of the two fluids.

FIG. 8. Progression of mixing along the flow path for both DI water and viscoelastic fluids at upstream and downstream channels. [(a) and (b)] The rectangular box represents the analyzed window for the evaluation of mixing efficiency at a specific location. (c) Progression of mixing performance along the flow path at $\dot{Q}=10$ ml/h.

Thus, despite the very low Reynolds number ($Re \approx 0.023$) and a large Peclet number ($Pe \approx 101 \times 10^6$), significant mixing was achieved through the chaotic cross-stream flow instability at the downstream flow fields.

IV. CONCLUSIONS

Viscoelastic flow instability was achieved by employing the viscoelastic of biocompatible fluids with simple convergent-divergent channel geometry. This flow regime has a negligibly small Reynolds number but a very high Deborah number and Peclet number. Flow at a high Deborah number introduces viscoelastic flow instability, which is evident from the increase in flow velocity fluctuation as the flow rate increases. This induced viscoelastic flow instability, in turn, enhances mixing. This mixing approach bypasses the limitation of a low Reynolds number in microfluidic

applications. This micromixer can potentially be implemented in a lab-on-a-chip platform with minimum requirements for design and fabrication. However, further improvement is required to minimize the pressure drop in achieving a fast flow rate to elastically induce flow instability.

A limitation of this mixing approach is that polymer (PEO) has to be added to an aqueous solution for it to behave viscoelastically. It is envisaged that this is not a significant limitation as PEO is biocompatible. In addition, further investigation is required if biological fluids would behave viscoelastically naturally, and thus no addition of PEO is required.

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