Measurement of the Mechanical Properties of Bone

A Recent History

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Abstract Much progress has been made in the last 50 years in our understanding of bone's mechanical properties, and the reasons it has these properties and not others. The question is to what extent these advances have arisen from an increase in the techniques available for the study of bone, and how much stems from an increased understanding of the basic processes involved. Although considerable enlightenment has come from the transfer of ideas from the physical sciences, in particular materials science, the author argues that most increases have come from the vastly increased power and resolution of the observational and mechanical techniques available. Even so, the remarkably hierarchical nature of bone's structure makes it an almost uniquely difficult material to understand properly, and much remains to be done to marry explanations at the macro-, micro- and nanolevels to obtain a full understanding of bone mechanics.

Introduction

In 1970, reviewing bones' mechanical properties in this journal [10], I ended on a rather grandiloquent note: "Much of the basic knowledge is now available... To me it now seems that the study of bone mechanics is entering the

beginning of an extremely interesting second phase; we are now in a position to start to understand bone." I think I was wrong, by about 10 or 15 years. Although the study of bone mechanics advanced in the 1970s, it was not until the beginning of the 1980s, or a bit later, that its study really began to take off. And this was driven not by deep theoretical insights, but by advances in techniques. "Technik ist alles" as the great biochemist Warburg (among many others) said, and the techniques that have become available over the last 20 years are now enabling a transformation of our understanding of bone.

There are three aspects of analysis of bones' mechanical properties that have grown more powerful over the years.

First is the increasing precision and power of the actual physical and mechanical tests themselves. Apart from the general increase in sophistication of testing devices, the precision of what we can say has increased because of new methods, for instance finite element analysis. Second is the increasing realization of the importance of features of the testing method that were previously considered unimportant or were even ignored, such as size effects. Third is our increasing ability to characterize the bone and to combine differences in bone structure and differences in mechanical properties. Some of this comes from the increased sophistication of old methods such as transmission and scanning electron microscopy, but also there are entirely new techniques (as applied to bone). These include all kinds of spectroscopy, and scanning confocal microscopy.

In the early years biologists and mechanical people tended to work on their own and not interact enough. Julian Vincent and I, describing a symposium we organized in 1980 [58] called "The Mechanical Properties of Biological Materials" wrote, "...it was clear that the biologists who work on the mechanical properties of materials need much more to call upon the materials scientists for guidance on

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what is known already about the phenomena they are studying; on the other hand it was equally clear that the materials scientists had little idea of the richness of biological diversity and of the pervasive ability of natural selection to provide the optimum solutions to very complex problems.” Nowadays, unfortunately, the situation has, if anything, worsened because many biologists have quit the scene, being frightened by the mathematics and mechanics often needed, and the materials scientists, left on their own, are barely more knowledgeable about the diversity of natural phenomena.

People like to do tests themselves, and think it is best to repeat things using up-to-date methods. For instance, Evans’s 1973 book [15] *Mechanical Properties of Bone* lists about 170 references, the great majority of which refer to straightforward measurements of bone’s mechanical properties. Yet very few of these earlier works are referred to in recent papers, even in catch-all introductions.

An idea of progress over 55 years can be gained by reading or skimming Evans [14, 15], Yamada and Evans [62], Roesler [41, strongly historical], Vincent [57], Biewener [5], Turner and Burr [54], An and Draughn [1], Cowin [8], Currey [11], and Cowin and Doty (not for the faint-hearted!) [9]. All of these books or papers have their strengths and weaknesses, and should be treated with circumspection as, indeed, should all scientific literature. Some interesting insights into how bone compares mechanically with other materials are given by Ashby [2].

In this brief review I shall deal with some of these points. It is not possible to mention everything, and this review is necessarily eclectic and has a strong Anglo-Saxon slant, but I hope it will give a flavor of the changes that have come about over the last 30 years or so. Often, in various sections, I quote a couple of recent papers, not because they are the “best” but because they give some idea of where we are now.

Testing Methods, New and Old

Testing machines are now driven by computers, and many people think this a great advance. I am not so sure, having often been told that such and such a test is not possible because of the computer. This shows a lack of good programs, but someone has to write the programs, and this takes time and a great deal of money. Perhaps the greatest advances in old methods are in ultrasonic measurements that were once matters of great controversy, particularly in relation to cancellous bone, which seem now to have settled down. Nowadays it seems that it might be possible to distinguish different histological types of bone from their ultrasonic signature [44]. Also, fatigue experiments are now much easier to perform.

Testing a smooth-surfaced specimen of bone gives a good idea of Young’s modulus, but the relevance of the calculated strength to real life is less clear. Real bones have complicated shapes, rough surfaces, and are more or less tough. The whole science of “fracture mechanics” has developed to deal with such problems. Fracture mechanics attempts to characterize the tendency of a material to fracture as a material property, independent of the geometry of the specimen. It also attempts to determine how the actual geometry of the specimen, including the presence of flaws, will affect fracture behavior.

In 1920, Griffith [20] realized that whether a crack in a brittle material would run was a matter of the balance between the surface energy needed to extend the crack and the increased availability of strain energy caused by the crack extension. Since 1920, of course, great advances have been made. Bone is, however, particularly difficult to deal with from a “pure” fracture mechanics position because it is so extraordinarily hierarchical. There is no level at which one can say that one is dealing with “bone.”

There have been many attempts to produce fracture mechanics values for bone. Melvin [30] wrote an early review. Zioupos [64] emphasized the importance of distinguishing the toughening events that lead up to the initiation of crack extension, and the toughening occurring during crack extension. Both are “toughening,” but bone behaving in a brittle fashion will show neither the microcracking (see below) that leads to the delay of the onset of a fatal crack, nor any of the postyield toughening mechanisms that exist. Zioupos also emphasized the importance of a piece of bone being brittle or tough according to circumstances: temperature, strain rate, the velocity of the crack and so on [64]. One interesting recent development is the concept of the “R-curve” [28]. This enables one to distinguish the crack initiation toughness from the growth of the crack [34]. For instance, Nalla et al. [33] showed that crack initiation stress in older people was about 40% lower than in younger people but the toughness associated with crack growth was almost absent in older people.

A technique that has come to be much used recently is nanoindentation. This involves pressing a tiny probe onto the bony surface and measuring simultaneously the load and the deformation. A seminal 1992 work by Oliver and Pharr [37] cited 4400 times by late November 2008 and updated in 2004 [38], described a method for determining the Young’s modulus of very small parts of materials. This method was enthusiastically adopted by people who wanted to learn about differences in the elastic modulus of small volumes of bone. Previously the elastic modulus of only quite large pieces of bone could be measured (the smallest were single trabeculae, and the results were and are treated with skepticism by many). Since, for instance,

the mineral content and anisotropy of bone can vary considerably over a few microns, this size restriction was a severe limitation. Nanoindentation allows the determination of differences in the elastic modulus in objects as small as individual lamellae of bone. As an example Gupta et al. [21] used nanoindentation to examine the changes in modulus as a function of mineral content in the small transition region of calcified cartilage to bone in the human patella.

Many applications of Oliver and Pharr's method have been uncritical, and people often ignore the fact that, for a highly anisotropic material like bone, the modulus they report is only some kind of "average" modulus. Some have proposed quite complex modifications of the method, mainly relating to the fact that bone is to some extent viscoelastic [46]. Some consider that nanoindentation is actually not well-suited for relatively compliant, anisotropic materials like bone [45]. Nevertheless, I think that this method, or modifications of it, will be important in the future in determining difference in stiffness properties over very small distances.

An enormously valuable technique, effectively originating in the 1970s when computers became powerful, is finite element analysis (FEA). FEA is a computer method for determining (in our case) strains in loaded models. FEA programs range from those allowing only two Young's moduli in the whole system, and assuming that the bone material is isotropic, to those allowing many Young's moduli, anisotropy, and so on. The more variation you want, the heavier it all becomes in computer time.

FEA is particularly useful for determining strains in bones of complex shapes, like skulls and cancellous bone. Ideally one takes a shape, digitizes it in some way, turns it into an FEA model, attributes elastic material properties to each element (this step is often not possible except in a global way), loads the model with any loading one wishes, and sees the distribution of strains produced in the model. Each one of these steps makes worrying assumptions, and much work has been done over the last decade or two to justify the assumptions. Richmond et al. [40] is a simple introduction to FEA in bone which, quite rightly, emphasizes the problems associated with it.

The advance in technique allowing FEA to progress so much in bone studies was computer-aided tomography (CT). In micro-CT, the specimen is scanned in virtual slices with an xray beam, and a clever algorithm allows the density of the various parts of the object to be calculated. The slices are then amalgamated to produce a 3-D density image of the object. Each part of the object has both its density and 3-D position rendered objectively in numbers, and these data can be turned, almost directly, into an FEA model. There are again many assumptions that are required to produce the results. An example: what is the relationship

between the xray density of a part (voxel) and its Young's modulus? Opinions differ sharply.

Nevertheless, micro-CT allows the detailed characterization of very complex shapes such as cancellous bone and then, using FEA, a prediction of the strains that will be produced by given loads. However, at the moment, FEA of complex micro-CT images is very, very computer intensive. For instance, notable recent work by Verhulst et al. [56], comparing the heads of two femora, needed more than 5 weeks of many processors of a supercomputer.

In the 1960s, the only feasible ways of measuring the deformations in samples of bone being tested were by using strain gauges, or measuring the cross-head movement, making allowance for machine deformation (though this was often not done) or using extensometers clipped to the specimen. The extensometers and cross-head movement measured only global deformations, and one did not know whether different bits of the specimen had different strains. Strain gauges, being glued to the surface, made measurements only of the part to which they were glued.

In noncontact extensometry, optical methods are used to determine the deformation of different parts. The specimen is not connected to any clips or gauges. Furthermore, movements of all points in the face of the specimen facing the measuring device can be determined relative to each other allowing, for instance, Poisson's ratio to be determined directly. There are two main methods. Digital image correlation uses a computer to capture, recognize and map points on the surface, and then to map the changes that occur when the specimen is loaded [27, 51]. This method can be used to measure strains within tissues, such as the extra strain round osteocyte lacunae caused by their strain-concentrating effect [35]. Another method is interferometry (ESPI, or electronic speckle pattern interferometry). This can measure extremely small displacements, but at the moment suffers from considerable noise, so statistical techniques must be used to determine strains [63].

Factors Recently Achieving Prominence

It was realized 130 years ago [39] that the mechanical properties of bone were very different depending on whether the bone was wet or dry, and a systematic approach to this was undertaken by Evans and Lebow [16]. People have in general tested bone wet. However, many of the earlier results performed on wet bone are suspect because people did not realize how rapidly the outer layers of bone dry out, so that specimens that were notionally wet were actually drying in the important outer layer.

One tissue poses a particular difficulty: the antler bone of deer. Its mineral content is rather low, resulting in the bone being much tougher when wet than when dry. Then, it

is unclear whether the antler is wet or dry when it is being used. When males fight the periosteum has gone, and the bony tissue is exposed to the air. Formerly it was thought the antlers were dry, but there is evidence that there is liquid blood remaining in the interior, and may even still be alive [42]. Whether this means that the outer layers are damp when used is unknown. In hot Spain probably not, in cool Finland just possibly yes.

Another complication in determining the probability of failure of a real-life specimen is the “size effect.” Usually the likelihood of failure of a specimen is considered to be determined by the stress, which is calculated in the usual way, normalizing for size. (This itself may be a complex process.) However, the specimen will not, like the “wonderful one horse shay,” disintegrate everywhere at the same moment. A few volumes will be weaker than average, many will be around average, and some will be stronger. In a fairly brittle material, such as much bone, the fatal crack will initiate in some weak volume. Large specimens will have more weak volumes, and are likely to have more very weak volumes than small specimens. The strong and very strong volumes are, unfortunately, of no help in resisting fracture. Therefore, large specimens may fail at lower stresses than small specimens. This is the so-called “size effect.” This general phenomenon had been known for some time and its application to bone has been explored recently by Taylor et al. [47–50]. One method of analysis, increasingly used, is to measure the distribution of strengths of specimens from a particular bone. Given this distribution and the mean value of strength, one can use the Weibull equation to determine the size effect [60]. Weibull [60], cited 2436 times by late November 2008, produced a distribution that had two material constants that could be obtained from the distribution of strengths which in the simplest form has a power law coefficient m . Essentially, the smaller m is, the looser the strength values are around the mean, that is, the different volumes have a large spread of strengths. Therefore, the lower the value of m the more important the size effect is likely to be. There are other ways in which a volume effect may be produced, apart from the “weakest link” described above, possibly particularly relevant for semibrittle materials like bone, and interested readers should read Bažant [3] or Bažant and Pang [4].

The size effect is not trivial, particularly in fatigue. Bigley et al. [6] found that increasing the volume of specimens by a factor of four reduced the number of cycles to failure at a particular stress by a factor of greater than two.

Bone undergoes damage before it completely breaks, and this was realized many years ago by Frost [18]. The appearance of compressive microcracking in vivo was described by Tschantz and Rutishauser [53], but nothing

much more was done about damage until the early 1990s [66]. Recently papers devoted to microcracking have increased enormously, aided considerably by the development of scanning confocal microscopy (see below). Such studies have shown that the “plasticity” or postyield deformation that bone undergoes is not true plasticity, but is damage to the bone that increases its compliance [52]. The development of microcracks during fracture is strongly affected by such things as age (in humans) [13, 32] and loading rate. The realities of microdamage are still far from well-known. There is still considerable argument, for instance, about whether the “diffuse microdamage” often reported is really totally diffuse or consists of cracks that cannot yet be resolved optically [31, 65].

Improvements in Old Methods for Characterizing Bone

Transmission and scanning electron microscopy have been around since before the beginning of the “new” bone mechanics (roughly 1970) but their increased sophistication allows us to see things we could not before. For example, Landis et al. [26] visualized the packing of apatite crystals in collagen fibers and Weiner et al. [61] visualized the arrangement of adjacent lamellae in lamellar bone, producing vital information for the analysis of bone behavior. In scanning microscopy, for example, the increasing prevalence of environmental chambers and the ability to perform mechanical tests while examining the specimen by scanning allow us to see the effects of different humidities on bone behavior.

Ordinary histology has benefited particularly from its increasing ability to quantify information through image analysis, transforming our ability to obtain information from images quickly, and apparently objectively. However, images can be tweaked in ways the observer wishes, to enhance contrast, for example, and the temptation to tweak the images improperly has not always been resisted.

New Methods for Characterizing Bone

Many spectroscopic methods such as Fourier transform infrared spectroscopy (FTIR), Raman spectroscopy, and NMR spectroscopy have been eventually adopted by the bone community. For instance, FTIR was first developed in the 1940s, but used on bone in the late 1980s [7]. FTIR can be used to detect compositional differences in mineral/organic ratios, and also features like crystal size. Fuchs et al. [19] used FTIR to examine the time course of the mineralization of Haversian systems. Raman spectroscopy does not require the tissue to be dried and therefore has the potential, not yet fully realized, for in vivo examination

[29]. NMR spectroscopy measures the amount of water in different compartments in the bone, bound and mobile particularly. For example, Nyman et al. [36] found that in men's bones the amount of bound water decreased with age, and the toughness decreased concomitantly. Synchrotron radiation can be used in various modes, either as a source of light to examine strains in the different components of bone [22] or for more straightforward tomography, producing images of relatively large specimens [52]. Apart from the capital and running costs of a synchrotron facility, synchrotron radiation is ideal in many ways for doing a whole range of mechanical and characterization studies.

The development of scanning confocal microscopy, starting in the early 1990s in the bone field, has allowed the visualization of microcracks in at least the superficial layers of damaged bone, and the examination of the 3-D structure of these microcracks [13, 31, 66]. This has given us new insights into the failure process in bone and mineralized tissues [55, 67].

Analysis at the Nano Level

One of the things people have been trying to do for many years is to explain the properties of bone analytically in terms of its mineral, collagen and water content. There has been progress over the years [17, 24, 25, 43, 59]. Recently these efforts have taken on new importance because it is becoming possible to measure the mechanical properties of very small pieces of bone, so the relationship between theory and reality is gradually becoming clearer, though it has not yet arrived [21–23].

Whole Bone Properties

Prediction of whole bone properties from the behavior of individual test specimens and, for small bones, the prediction of material bone properties from whole bone behavior has not proceeded very far, because it is so difficult. Beam theory is still often used, the bone being considered to be a hollow cylinder made of bone material of uniform mechanical properties. Such assumptions are rarely correct [16].

Predicting accurately the behavior of a whole bone involves knowing the kinds of loads that are put on the bone, their directions and relative magnitudes, knowing the 3-D structure of the bone in some detail, and finally knowing the mechanical properties of the material throughout the bone. Compared to all this, determining mechanical properties of individual specimens is child's play. The structure of whole bones is fiendishly complex. The lovely photographs in De Panafieu and Gries [12] give

some idea of the range and complexity of bones, quite unanalyzable except by the use of CT with FEA, and even then only with enormous difficulty.

Discussion

The remarkably hierarchical nature of bone's structure makes it an almost uniquely difficult material to understand properly, and much remains to be done to marry explanations at the macro-, micro- and nanolevels to obtain a full understanding of bone mechanics. This is the major gap in our understanding at the moment. Whole bone properties (which in the end is what we should be interested in, presumably) will develop quickly when FEA becomes increasingly rapid as computers intensify in power, particularly when paired with an improved understanding of the mechanical properties of little subvolumes of the bones themselves. At the nano level much remains to be done, though a good start has been made.

I would like merely to reiterate my initial suggestion that our recent vastly increased understanding of the mechanical properties of bone, and of the reasons for differences in these properties, have far more to do with the increasing power of the techniques available than with deep intellectual advances. Of course, what is discovered through new techniques has led to changes in our thinking, and I am not denying that many clever people have thought long, hard, and effectively about bone. Nevertheless, if new techniques had not become available, we would not be much further advanced than when I made my mistaken assertion in 1970. This time I shall make no statement about our being in a New Golden Age, though in my heart I think we are.

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