Published in final edited form as:

J Biomech. 2007 ; 40(7): 1559–1566. doi:10.1016/j.jbiomech.2006.07.019.

Minimal Step Length Necessary for Recovery of Forward Balance Loss with a Single Step

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Abstract

Although the boundary conditions necessary to trigger a step in reaction to a forward balance loss have been predicted in previous research, the relationship between minimal step length needed for balance recovery with this single step and the center of mass (COM) motion state (i.e., its position and velocity) remains unknown. The purpose of this paper was to present a theoretical framework within which the minimal step length needed for balance recovery can be estimated. We therefore developed a simplified four-segment sagittal model of human body stepping for balance recovery. The work-energy principle of the Newtonian mechanics was employed in the simulation to determine the amount of excess mechanical energy that can be absorbed as a function of step length and the corresponding eccentric joint work that can be generated in a single step. We found that an increase in initial forward velocity and a greater forward shift of the COM require a corresponding increase in the minimal step length needed for balance recovery. Furthermore, the minimal step length is also a function of the muscle strength at the ankle: the lower the muscle strength, the greater the minimal step length required. Our theoretical framework reduces the complexity associated with previous studies relying on forward dynamics and iterative optimization processes. This method may also be applied to study aspects of balance control such as the prevention of balance loss in the posterior or mediolateral direction.

Keywords

protective stepping; braking; forward momentum; stability region; work-energy principle

INTRODUCTION

Following the initiation of a fall, a range of effective recovery strategies can be employed for balance recovery. For instance, balance can be restored by grasping, by hip and ankle motion (ankle/hip strategy) (Nashner et al., 1989; Runge et al., 1999), or by protective stepping (Maki

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et al., 2003). One obvious limitation of the grasping strategy is the potential lack of any “graspable” fixtures where the fall occurs. Even though the correction generated by ankle/hip movement can be achieved when the disturbance is of great magnitude, it is often completely insufficient for preventing a fall. Greater disturbances of balance can seldom be restored without the subject’s taking a step (Pai, 2003). Thus the protective stepping response is of unique importance in fall prevention.

Up to the present, the usual approach to quantifying upright standing stability has been to identify the thresholds beyond which balance cannot be restored without resorting to stepping. It has been thought that the projection of the center of mass (COM) should be within the base of support (BOS) in order to maintain balance (Borelli, 1680; Dyson, 1977). Recently, this basic idea has been developed into the concept of feasible stability region (Pai and Patton, 1997; Pai and Iqbal, 1999), which demonstrated that the COM state (i.e., its displacement and velocity) is important in considering the thresholds for triggering protective stepping. Yet it is unclear what is required for recovery once a person’s COM state has exceeded those predicted thresholds of stability (Pai and Patton, 1997) and, as a consequence, a fall is initiated.

Experimental results have shown that with the increase in the intensity of disturbances, an increase of step length is required for balance recovery (Hsiao and Robinovitch, 1999; Wojcik et al., 1999; Do et al., 1982). Thus it appears that step length is one of the critical factors influencing the balance recovery of human body when disturbed. If a person can achieve a greater step length, it is possible to produce more mechanical work at the joints and thus to absorb larger amounts of the perturbing energy. In contrast, if the mechanical work generated in a small step length is insufficient to absorb the amount of mechanical energy for balance recovery, the current reasoning would conclude that another step will be required to continue the braking process (Figure 1). We posit a minimal step length – as a function of the COM state – that in a single step can achieve balance recovery. Little is known about this minimal step length necessary for balance recovery.

The purpose of this paper was therefore to present a novel theoretical framework within which the minimal step length needed for movement termination and balance recovery can be estimated. This study focused on forward stepping associated with forward loss of balance, although we expect that a similar approach could also be applied to solve backward stepping. The work-energy principle of Newtonian mechanics was the essential tool employed in this simulation, which also offers a conceptual and methodological alternative to previous approaches with forward dynamics and optimization in the determination of the feasible stability region (Pai and Iqbal, 1999).

METHODS

A 4-segment model was developed in the sagittal plane to determine the minimal length of a forward step for recovery from forward loss of balance. Anatomical details of the human body and kinematic details of regular gait were not considered in this reductionist approach. Our approach assumed two feet formally defined as massless, rigid triangles; two rigid massless legs without knee joint flexion; and a concentration of body mass at the hip joint (Figure 2). The lower limbs were assumed to be bilaterally symmetrical and their movement occurred in the sagittal plane. Each foot was equal in length, l_f , with equal distance from the ankle to the heel, a . Each ankle joint, C and C' for the stance and swing leg, respectively, was reduced to a pin joint with plantar-flexor moments for braking forward momentum and balance recovery. The hip joint was represented as a frictionless pin joint. We assumed that no slip occurred between the plantar surface and the ground. Typical male proportions were used to determine anatomical dimensions (Winter, 1990; Table 1).

The simulation began with the COM posterior to the standing right limb, without any representation of the instantaneous position of the swing (left) limb, for the sake of clarity (Figure 2). The effect of the massless leg swing on braking is negligible. The body mass then rotated forward around the ankle of the right limb from this *initial* position, A^0 . The *final* position A'' was associated with a diminished forward velocity of the COM, achieved by stepping with the left limb, again without representation of the instantaneous position of the right limb in Figure 2. During the process of braking of forward momentum, the eccentric plantar-flexor moments generated first by the ankle of right limb and subsequently by the ankle of left limb were considered the primary contributors. We assumed that the torque generated by the ankle of right limb would *cease* to contribute to braking when the horizontal projection of the COM, x_{com} , is located in front of the extreme anterior edge of right foot, point B. Thus we could neglect the effect of the ankle moments of stance limb during later (push-off) part of the stance phase. Overall, it is reasonable to assume that protective stepping for braking and movement termination is a departure from the regular gait pattern for forward progression.

The process of braking was divided into two phases: first, the COM moved from initial position A^0 to A' , at which point the right limb ceased to produce any braking effect prior to its liftoff. Second, the COM moved from A' to final position A'' , during which phase the stepping left limb effected a similar braking of forward momentum. At the end of phase one, x_{com} approached B, and $D_x = 0$. There are two possible outcomes at that moment, depending on the velocity of the COM at that position. First, the forward velocity of the COM can be reduced to zero at the time when the COM reaches position A' . The movement termination will then be successful and no forward stepping will be needed to recover balance; i.e., phase two is unnecessary. This scenario has been studied (Pai and Patton, 1997; Pai and Iqbal, 1999). Second, if the forward velocity of COM at position A' is greater than zero at the end of the first period, the body must continue to move forward and a forward step must be initiated to avert a fall (Figure 1).

In this second scenario, the body took a forward step with a length l_{step} . A new base of support would be formed following the touchdown of the left leg. The body would rotate around the rotational axis of the ankle joint of new stance limb at C' and the ankle's plantar-flexor moments generated by the left limb after touchdown would be required to prevent the body's forward rotation. This process is quite similar to that of phase one. At the end of the phase two, the x_{com} reached the extreme anterior edge of the newly established BOS at B' . Similar to what was described in the first scenario earlier, we considered this the necessary condition of successful movement termination for balance recovery (Pai and Patton, 1997; Pai and Iqbal, 1999). Based on the same rationale, when the forward velocity of the COM is still greater than zero at position A'' , the body continues to move forward and another forward step is required. This scenario was not modeled in the present study, although a similar approach could be further extended to investigate the minimal length required in the following step (Figure 1).

The work-energy principle can be applied to predict that at a given joint work profile, the amount of kinetic energy (KE) associated with the initial condition can diminish while approaching the final condition of the simulation. As the COM moves from the initial position A^0 to A' , the change in potential energy (ΔPE_1) and the reduction in KE (ΔKE_1) will be equal to the negative work absorbed at the right ankle, W_1 .

$$W_1 = \Delta PE_1 + \Delta KE_1 \quad (1)$$

A similar analysis is applicable when the COM moves from the position A' to A'' for the work done at the ankle joint of the left limb following the step, W_2 . For successful movement termination, the total kinetic energy of body is of course equal to zero at the final position,

where standing balance can be established. When initial KE is greater (because initial COM velocity is greater), greater W_1 and W_2 are necessary to absorb the energy. The energy balance equation with one forward step can be expressed as the following:

$$W_1 + W_2 = \Delta PE_1 + \Delta KE_1 + \Delta PE_2 + \Delta KE_2 \quad (2)$$

Eq. (2) can be expressed by the following:

$$\int_{\cos^{-1}((D_x - l_f + a)/r)}^{\pi - \cos^{-1}((l_f - a)/r)} \tau_1 d\theta + \int_{\cos^{-1}((l_{step}/2)/r)}^{\pi - \cos^{-1}((l_f - a)/r)} \tau_1' d\theta = \frac{1}{2} I \left(\frac{v_x}{r \sin \theta_0} \right)^2 - mgr \left[\sin(\pi - \cos^{-1} \frac{l_f - a}{r}) - \sin(\cos^{-1} \frac{D_x - l_f + a}{r}) \right] \quad (3)$$

Where a was $0.19l_f$ based on parameters in Table 1, and r the distance between the COM and the ankle; v_x was the horizontal velocity of the COM at initial position (Appendix).

The predicted minimal step length can be obtained through the numerical solution of equation (3). Given the values of the respective parameters of body (Table 1), gravitational acceleration, and the torque of ankle joint, the variables undetermined in equation (3) are initial COM displacement, D_x , its velocity, v_x , and step length, l_{step} . Therefore, given the various magnitudes of D_x and v_x , the corresponding values of l_{step} can be determined through equation (3). The Riemann sum was used to calculate the work which generated by the ankle joints, and an optimization routine, *FMINSEARCH*, in Matlab platform (Math Works, Inc., Natick, MA, USA), was employed to calculate the corresponding numerical solution of the l_{step} in equation (3). These step lengths are the minimal step lengths needed for balance recovery with one forward step for various initial states of COM.

Additional analyses were also conducted to identify the effects of one essential parameter, the strength of the muscle of ankle joint, on the minimal step length. For this, the muscle strength was reduced to 80% and increased to 120% of normal values and the process of the numerical solution of equation (3) was repeated as described above. The minimal step lengths with reduced/increased muscle strength of ankle joint were then calculated. The predicted minimal step lengths (normalized to the foot length) were demonstrated for the conditions in which the initial velocity of the COM varied from 0 to 0.5 (normalized to $\sqrt{gH}$, where g is acceleration due to gravity and H the height of the person) and the initial displacement of the COM ran from 0 to 2.5 (normalized to the foot length).

RESULTS

We determined that the minimal step length is a function of the various initial states of COM (Figure 3a). In general, at any given COM location, a higher forward COM velocity requires greater work to be generated at the ankle joint, which would be associated with a greater joint displacement and a greater step length. Likewise, a more anteriorly positioned COM at a given COM velocity will also require a greater step length. At region NS, the minimal step length needed for balance recovery equals zero; i.e., no forward stepping is needed when the initial states of the COM are located at this region. In contrast, at region ST, the minimal step lengths are greater than zero. Thus a forward step is needed. Furthermore, the magnitude of step length should be larger than the minimal step length predicted in this model for balance recovery under these initial states of the COM. The feasible stable region associated with stepping was extended (Figure 3b).

The muscle strength at the ankle joint, it appears, influences the magnitude of minimal step lengths. We found that the minimal step length required for balance recovery increased in proportion to a decrease of muscle strength and decreased in proportion to an increase of muscle strength. The extended stable region was reduced with a decrease of the muscle strength and extended with an increase of muscle strength (Figure 4). Thus associated with the decrease of the muscle strength, a larger step length is needed for balance recovery.

DISCUSSION

The simulation presented in this study has demonstrated that a simple mathematical model based on the work-energy principle can be used to estimate the minimal step length needed for balance recovery. The three dimensional surface indicates the relationship between the initial states of COM and the minimal requirements of step length, which is necessary for balance recovery with one forward step after perturbation. Using this principle, we were able to estimate not only under *which* conditions a forward step is needed, but also *how long* the recovery step length necessary for balance recovery must be if an additional step is to be avoided. The feasible stable region can be substantially enlarged following the stepping, though the size of this region depends on the length of the step. These results highlighted the importance of the stepping in the balance recovery.

Stepping is a unique and irreplaceable strategy during balance recovery, whereby excess forward momentum and kinetic energy can be absorbed while eccentric muscular work is produced at the joints of the stepping limb. The feasible stable region has been previously computed through solution of the equations of motion with forward dynamics combined with iterative optimization processes (Pai and Patton, 1997; Pai and Iqbal, 1999). The present study has advanced this line of work in two respects. It has provided a framework for calculating the estimated braking effect and the necessary conditions for executing a single protective step, or even multiple steps for balance recovery. Furthermore, the present approach, which is characterized by the application of work-energy principle, made possible to reduce considerable computational complexity and computing time.

Such distinguished advantage is achieved at a cost, however, whereby this approach relying on the work-energy principle cannot account for the amount of time required during the stepping process. This limitation may impact its application in some dynamic tasks when the available time became an important factor that determines the success of a fall arresting strategy (Bogert et al., 2002; Cao et al., 1997). Older adults tend to have a longer reaction time and a slower rate in generating ankle joint torque (Thelen et al., 1996). In this regard, time-dependent formulation of Newton's 2nd law can be applied to predict the temporal aspect of the stability limits (Patton, et al., 2000).

Further, the model employed to derive these results is based on several simplifying assumptions, which therefore might limit its predictive capacity. For example, the human body was assumed to be composed by four rigid segments. The influence of arm swing, leg swing, and push-off on the balance recovery was not considered in the model. Movement of body recovery was assumed to occur in the sagittal plane. The plantar-flexor moment of ankle is assumed dependent on joint position alone. Overall, the effects of three-dimensional movements, of joint velocity on strength limits, of rate limits on muscle activation (Zajac, 1989), and of other neurological constraints (Nashner et al., 1989) may influence the precision of predicted minimal step length.

In spite of the simplifications made in the present study, the model predictions are still consistent with the previous published work. Our results are consistent as well with the previously-established feasible stable region that is equivalent to our zero step length (Pai and

Patton, 1997; Pai and Iqbal, 1999). Our results are also consistent with experimental results from perturbed stepping (McIlroy and Maki, 1996). It is important to note that subjects recovered balance using a single step when their step lengths were larger than the predicted minimal step length, and failed, i.e., used multiple steps, when their first step lengths were less than model predictions (Figure 5). The general trend of minimal step length we predicted was also similar to the results derived from human leaning-and-release experiments (Wojcik et al., 1999). In these experiments, human subjects took larger step lengths after release at the maximum lean angles; that is, the initial displacements of COM were small (Thelen et al., 1997). Their results indicated that minimal step length required for balance recovery was greater when the initial displacement of COM was more anteriorly positioned.

It is also noteworthy that regardless of the simplicity of the inverted pendulum model employed in these studies, the results were remarkably robust and appropriate for a range of task conditions with similar symmetric bipedal standing postures. These task conditions include bi-manual pull (Patton et al., 2000; Patton et al., 1999), resisting floor perturbation (Pai et al., 2000) and waist pull (Pai et al., 1998), or chair-rise (Pai et al., 2003). Precisely because of the success of these and other reductionist approaches, the present study was undertaken.

Finally, our study has highlighted the mechanical basis of a strategy one might adopt to compensate for reduction in muscle strength. For example, older adults, whose muscle strength is thought to be weaker than that of the young (Brooks and Faulkner 1994; Doherty et al., 1993; Binda et al., 2003), preferred a longer step or multiple steps for balance recovery than did the young after perturbations (McIlroy and Maki, 1996; Pavol et al., 2001). A decrease in muscle strength may indeed be the cause of taking a longer step or even multiple steps at a given perturbation intensity, although other factors, such as neurological, cognitive, or psychological processes may be involved as well. These results are consistent with the findings from the simulation, namely that the muscle strength at the ankle had significant influence on the minimal step length with the minimal step length increase associated with the decrease of the muscle strength. The subject enlarged the range of motion of ankle joint through increasing the step length to generate a given work that was required for balance recovery.

In summary, the present study has bridged a gap in our knowledge by providing theoretical estimates on the braking effect that can be produced by taking a protective step. The work-energy principle was employed as a part of the theoretical framework upon which the minimal step length needed for balance recovery was derived. This basic framework has the potential to solve many of the complex problems that remain and to provide guidance for the evaluation of balance dysfunction – guidance urgently needed in the field of physical rehabilitation (O'Sullivan, 1994).

Acknowledgments

This study was supported by a grant (30170242) from the National Science Foundation of China, and in part by NIH R01-AG16727 (YCP). We thank Dr. James Girsch for his careful editorial work.

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APPENDIX

A1. The ankle angle at initial and final position

In this study, the angle of the initial stance limb θ_0 was defined as a function of the initial COM position and its displacement D_x .

$$\theta_0 = \cos^{-1} \frac{D_x - l_f + a}{r} \quad (\text{A1})$$

Where a was $0.19l_f$ based on parameters in Table 1, and r the distance between the COM and the ankle. When x_{com} approached the anterior edge of stance foot at point B in Figure 2, the angle of the initial stance limb θ_{end} can be expressed as:

$$\theta_{end} = \pi - \cos^{-1} \frac{l_f - a}{r} \quad (\text{A2})$$

Similarly, when x_{com} approaches the most anterior edges of the left foot after its touchdown at point B' in Figure 2, the angle of this limb θ'_{end} will be of the same magnitude as θ_{end} .

Therefore, we could use θ_{end} for both legs. Furthermore, θ'_0 can be expressed as a function of the step length l_{step} .

$$\theta'_0 = \cos^{-1} \frac{l_{step}/2}{r} \quad (\text{A3})$$

A2. The energy and work during balance recovery

The magnitude of potential energy at its initial and final positions A^0 and A'' can be expressed as the follows.

$$PE_1 = mgr \sin \theta_0 \quad (\text{A4})$$

$$PE_2 = mgr \sin \theta_{end} \quad (\text{A5})$$

Where m is the mass of the body and g the acceleration due to gravity.

The kinetic energy of the body is:

$$KE = \frac{1}{2} I \omega^2 \quad (A6)$$

Where I is the moment of inertia of the body with respect to ankle joint; ω is the angular velocity of the support limb at initial position. Its magnitude is calculated by the following:

$$\omega = \frac{v_x}{r \sin \theta_0} \quad (A7)$$

Where v_x is the horizontal velocity of COM at initial position.

The energy absorbed by the ankle joints depended on the plantar-flexor moments within range of motion that the limb went through during balance recovery.

$$W_1 = \int_{\theta_0}^{\theta_{end}} \tau_1 d\theta \quad (A8)$$

Where τ_1 was the maximal plantar-flexor moment at the ankle joint of stance limb, which was assumed to be a function of joint position, and was taken from a musculoskeletal model (Delp et al., 1990). In this Hill-based model, the force-length properties of muscles crossing the ankle and the lines of action of all musculotendonous elements were considered to estimate the maximum plantar-flexion torque. A polynomial curve fitting was employed to obtain the relationship of the torque and angular displacement of ankle joint, i.e.,

$$\tau_1 = -0.0024\theta^3 - 0.091\theta^2 + 3.4\theta + 160 \text{ (Nm)} \quad (A9)$$

Where θ is the ankle flexion angle with its range from 20° to -40° (plantar-flexion).

Note that θ_0 and θ_{end} are the angles of the right limb at the beginning and the end of the first period, respectively. To focus on the braking effect achieved by *protective* stepping, the complex effect of double stance of *regular* gait was ignored, and the work generated at the ankle joint during the second period after the touchdown of left limb is as follows:

$$W_2 = \int_{\theta'_0}^{\theta'_{end}} \tau'_1 d\theta \quad (A10)$$

Where τ'_1 was the maximal plantar-flexor moments of ankle joint of left leg; θ'_0 and θ'_{end} represented the angle of the left limb at its touchdown and at the end of this phase, respectively.

Movement Termination

Figure 1.

Scheme of balance recovery process with stepping after perturbation, which can be characterized by the amount of mechanical work that can be generated at the joints during braking of excessive mechanical energy. When the former is smaller than the latter, another step must be executed and the braking cycle to be repeated, until this relationship is reversed and balance for upright standing is established.

Figure 2.

A 4-segment human model used to study the braking effect. A^0 and A'' indicate respective initial and final positions for the simulation. The phase one and two are defined as the COM moves from A^0 to A' and from A' to A'' , respectively. During phase one, only the right foot provides the support, therefore the left limb is not simultaneously shown here. Similarly, in phase two when the left limb provides support, the right limb is not shown. Effect of the swing limb on the body dynamics is therefore neglected. The D_x and v_x are the displacement and velocity of COM at initial position; l_f and l_{step} , the foot length and step length; r , the distance between the COM and ankle; a , the horizontal distance between heel and ankle; and θ , the angular displacement of ankle.

(A)

(B)

Figure 3.

The minimal step length (normalized with the length of foot) needed for balance recovery of various initial displacements (normalized by the length of foot) and velocities (normalized by the $\sqrt{gH}$, where H is height of body and g , acceleration due to gravity) of COM. (a) The height of the three-dimensional curve indicates the minimal step length. The region NS indicates that the minimal step lengths are equal to zero, or no forward step is needed for balance recovery following perturbations, while region ST indicates that the minimal step lengths are greater than zero. (b) Stable region without/with one forward step. The region F indicates that no stepping is needed when velocities and displacements of the COM are located in this region,

i.e., feasible stable region. The region E indicates a stable region following one forward step, i.e., extended feasible stable region. The numbers present on curves indicate the step length needed for balance recovery.

Figure 4.

The minimal step length (normalized by the foot length) needed for balance recovery while the strength of muscle equaled 100%, reduced to 80% or increased to 120% of normal values at a given velocity and a given displacement. The initial displacements of the COM are normalized by the length of foot and the velocities of the COM are normalized by $\sqrt{gH}$. The minimal step length is normalized by the length of foot.

Figure 5.

The comparison of model predicted minimal step lengths and previously published experimental results from perturbed stepping (McIlroy and Maki, 1996). The measured step lengths from 14 subjects and predicted minimal step lengths were normalized by the length of foot. The average body weight (69 Kg) and height (1.71 m) from experimental results were used for model predictions. The average displacement (normalized by the length of foot) and velocity (normalized by the $\sqrt{gH}$) of the COM at the foot contact derived from their experimental results were used to deduce their initial states assuming upright posture using our model. Then, these data at initial states (i.e., $D_x = 0.81$ and $v_x = 0.23$ and $v_x = 0.28$ for single step and multiple steps, respectively) were applied in our model to predict minimal step lengths. In situations where the subject failed to recover balance with a single step and multiple steps occurred, only the first step length was used for calculation and comparison. For the condition when the subject recovered balance with a single step, the measured step length was used directly for comparison. It appears that the average step length of those who recovered with a single step was greater than our predicted minimal step length. In contrast, those who took more than one step had step lengths less than the predicted value.

Table 1

Summary of the anthropometric scaling scheme used in simulation.

Item	Formula
Height	$H=1.78m$
Mass	$m=80\text{ Kg}$
Distance between COM and ankle	$r=0.575H$
Moment of inertia	$I=m* r^2$
Foot length	$l_f=0.152H$
Horizontal ankle-to-heel distance	$a=0.19l_f$