Published in final edited form as:

J Biomech. 2009 May 29; 42(8): 1069–1074. doi:10.1016/j.jbiomech.2009.02.021.

Determinants of friction in soft elastohydrodynamic lubrication

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Abstract

Elastohydrodynamic lubrication (EHL) protects soft tissues from damage and wear in many biological systems (e.g. synovial joints, cornea of the eye, and pleural surfaces of the lung and chest wall). Among studies of lubrication of deformable solids, few have examined the effects of external loads, geometry, and material properties on EHL of soft tissues. To examine these effects, we studied the tribology of soft tissues in a two dimensional finite element simulation of a thin layer of fluid separating a sliding rigid surface from a soft asperity or bump with an initial sinusoidal shape. We computed the frictional force, deformation of the solid, and change in fluid thickness as functions of independent variables: sliding velocity, normal load, material properties, and bump amplitude and length. Double logarithmic regression was used to determine the exponents of the scaling relationships of friction coefficient and minimum fluid thickness to the independent variables. The analysis showed that frictional shear force is strongly dependent on velocity, viscosity, and load, moderately dependent on bump length and elasticity, and only weakly dependent on the bump amplitude. The minimum fluid thickness is strongly dependent on velocity and viscosity, and changes moderately with load, elasticity, amplitude, and length. The shape of the bump has little effect. The results confirm that the shear-induced deformation of an initially symmetrical shape, including generalizations to other symmetrical geometries such as quadratic or piecewise linear bumps, leads to load-supporting behavior.

Keywords

Pleural space; Finite element analysis; Tribology

Introduction

The study of elastohydrodynamic lubrication (EHL) in biological systems has received considerable attention in the last two decades. Dowson and colleague investigated the role of tribology in natural synovial joint regions and metal joint replacements (Dowson and Zhong-Jin, 1986) and showed that a microelastohydrodynamic regime smoothes out the unevenness

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Conflict of interest statement

The authors have no financial or other interest that could affect the objectivity with which they approach the subject matter of this investigation.

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of cartilage surfaces under loading conditions in synovial joints and that EHL is a major regime associated with the tribology of synovial joints. Later, using an iterative numerical procedure, Dowson and Jin, (1992) studied the influence of geometrical variables of a single sinusoidal asperity on the minimum fluid thickness. They observed that fluid thickness increases with longer wavelengths as the sinusoidal bump becomes flattened. Later, they studied microelastohydrodynamic lubrication of a single asperity analytically (Jin and Dowson, 1997). They found that when the amplitude of roughness increases or the wavelength decreases the overall film thickness decreases, confirming their earlier numerical study. Skotheim and Mahadevan (2005) considered a combination of different geometries and material properties, and showed that in lubrication of soft materials, elastic deformation couples tangential and normal forces leading to load-supporting behavior.

For several years, we have been exploring mechanisms that may affect lubrication in serosal cavities (e.g. the pleural and pericardial spaces), in which soft tissues slide against each other without discernable wear. The nature of lubrication in these regions, whether mixed (implying points of contact) or elastohydrodynamic, is still in debate. Using computational methods, Lai et al. (2002) modeled the pleural space in two dimensions as a deformable flat membrane sliding against a rigid bump. Their study showed that motion between pleural surfaces reduces unevenness of the surfaces and prevents asperities from touching. Subsequently, Gouldstone et al. (2003) used finite element methods to model the pleural space in two dimensions. The results confirmed that the deformation due to sliding leads to smoothening of the surface and promotes uniformity of fluid thickness. They found that longer bumps undergo larger deformations resulting in more uniform fluid thickness. We later modeled the interaction of soft solid and fluid in 3D using a simple cylindrical geometry with an uneven solid-fluid interface in rotation (Moghani et al., 2007). Our result indicated that deformation of soft solids in EHL generates a positive net pressure profile which is load supporting. Frictional shear torque was less than proportional to normal load, as expected in soft material lubrication (Hamrock, 1994).

When an uneven soft surface is deformed by sliding to generate lift and reduce friction, how do material properties and morphologic features of the sliding surface affect frictional force? In this paper we modeled the interaction of an initially symmetrical deformable asperity with a lubricated sliding surface, using ADINA-FSI, a finite element software package for analysis of solid interactions with fluid. We examined the effects of material properties, geometry, velocity, and load on friction coefficient, minimum fluid thickness, and deformation of the asperity. We captured the essence of these relationships using scaling arguments, and determined the scaling exponents with double logarithmic regression.

Problem formulation and methods

We consider a deformable 2D sine-shaped geometry separated by fluid from a sliding flat surface. We adopt Cartesian coordinates(x, z), where x and z are longitudinal and transverse axes relative to the fluid channel respectively, as shown in Fig 1. The top boundary of the solid is constrained to vertical displacements and is initially located at $z = h_0 + A + h_s$, where h_0 is the initial minimum fluid thickness, A is the amplitude of the asperity, and h_s is the wall thickness common to the entrance and exit boundaries.

The initial undeformed geometry of the bottom surface of the solid is given by,

$$h = h_0 + A(1 - \sin \pi x / L) \quad (1)$$

where h is the fluid thickness as a function of x in the interval $[0, L]$.

The solid is pressed downward by a uniform pressure P at the top boundary. The bottom plane of the fluid moves with uniform velocity V in the x direction. Zero fluid pressure is imposed at the fluid inlet and outlet ($x = 0, L$). The solid-fluid interface has a no-slip boundary condition.

We modeled the solid as linearly elastic, isotropic, and nearly incompressible with Poisson's ratio $\nu=0.49$. Hydrodynamic pressures remain sufficiently small to enable the lubricant to be treated as Newtonian and incompressible. We neglect gravitational, inertial, and thermal effects.

Special mixed-interpolated elements are used to improve the continuity of pressure within the soft solid. We used plane strain 9-node solid elements, the most accurate 2D elements available in ADINA for soft materials, and 9-node fluid elements suitable for Stokes flows. We used a direct method in which solid and fluid models are solved together in one matrix system, improving the stability and robustness. Because all pressures in biological spaces are atmospheric to within a few percent, no attempt was made to simulate cavitation.

We studied the effects of the independent variables: amplitude A , length L , velocity V , normal load (applied pressure) P , viscosity μ , and elastic Young's modulus E on the friction coefficient F_c (defined as the ratio of axial shear force on the bottom solid boundary to the normal force load), and minimum fluid thickness $h_{\min}$. To measure these effects, we introduced a reference model with parameters shown in Table 1 and then compared the lubrication behavior by varying each one of the parameters independently.

With six independent variables and three fundamental dimensions (mass, length, time), there are three dimensionless groups according to Buckingham's theorem (Shames, 1982). For convenience (and specifically to isolate the pressure to only one dimensionless variable), we choose A/L , P/E , and $\mu V/LE$ as the sufficient set of three. The friction coefficient is already nondimensional; the minimum fluid thickness is nondimensionalized by channel length. We write the scaling equations as

$$F_c = k_1 (A/L)^{\alpha_1} (P/E)^{\beta_1} (\mu V/LE)^{\gamma_1} \quad (2)$$

$$h_{\min}/L = k_2 (A/L)^{\alpha_2} (P/E)^{\beta_2} (\mu V/LE)^{\gamma_2} \quad (3)$$

The logarithms of Eq. 2 and 3 are linear functions of the exponents; these were determined by regression over variations in the dimensionless groups. This allowed us to characterize the dependence of friction coefficient F_c and minimum fluid thickness $h_{\min}$ over a range of geometries, loading conditions, material properties, and velocities. We used SI units for pressure and viscosity, and cm for length and velocity.

Results and Discussion

The steady state behavior of the model was examined for a combination of geometrical variables (A, L), material properties (E, μ), loading condition (P), and velocity (V), by performing a transient analysis and allowing sufficient time for the results to reach steady state. Fig. 2 shows the pressure distribution of the deformed solid and fluid models simulated at reference values (Table 1). Positive and negative hydrodynamic pressures upstream and downstream of the bump deformed it, elongating the converging channel and shortening the diverging channel to create an asymmetric wedge that supports the normal load.

Friction coefficient

The dependence of the friction coefficient F_c on the dimensionless groups A/L , P/E , and $\mu V/LE$ is shown in Fig. 3. Fig. 3a shows the very weak dependency of F_c on roughness. F_c decreases slightly and then increases as A/L increases under constant $\mu V/LE$ and P/E . The dependency of F_c on A/L diminishes as P/E increases and $\mu V/LE$ decreases. This behavior is consistent with the following argument: anything that causes the fluid layer to become thinner concentrates the hydrodynamic pressures in a smaller, flatter region of the bump, reducing the importance of the more peripheral bump shape in determining F_c . Thus, higher P/E results in greater normal and transverse deformation (Fig. 4) and causes greater flattening of the central region of the bump, which makes the fluid channel more uniform in thickness and reduces the influence of A/L . Similarly, lower values of $\mu V/LE$ (e.g. lower values of velocity) thin the fluid layer and restrict the hydrodynamic pressure distribution to a smaller region of the asperity beneath the flattened bump (see Fig. 2a, b), where the fluid is of nearly constant thickness. Thus, other things being equal, restricting the pressure distribution to a smaller region causes less of the bump's shape to affect hydrodynamics, thus decreasing the influence of A/L on F_c .

Fig. 3b shows that F_c decreases with P/E for fixed A/L and $\mu V/LE$. This dependency weakens as P/E increases. The graph also shows little dependency on A/L , in agreement with Fig 3a.

Fig. 3c shows that F_c increases with $\mu V/LE$ for constant values of A/L and P/E . The rate of increase of F_c weakens at higher values of $\mu V/LE$. Note that since μV appears only as a product in one dimensionless group, V plays the same role as μ in EHL. The figure also shows little effect of roughness on the friction coefficient, in agreement with Fig 3a.

Fig. 4 shows normal (z) and longitudinal (x) deformations of the bump surface under reference conditions with loads $P = 20$ and 40 Pa. Normal and longitudinal deformations are of similar magnitude and increase slightly with increasing P .

Minimum fluid thickness

Fig. 5a shows changes in $h_{\min}$ with A/L for fixed P/E and $\mu V/LE$. As A/L increases $h_{\min}$ decreases, i.e. for a constant length L , rougher bumps lead to thinner $h_{\min}$. It is of interest that roughness has little effect on F_c , whereas it has a considerable influence on $h_{\min}$. Fig. 6 compares the fluid thickness of two bumps with amplitudes of 0.0005 and 0.001 cm under the same loading and velocity conditions. $h_{\min}$ is slightly less for the rougher bump contributing to an increase in shear stress while the thin fluid extends over a shorter flattened length contributing to a decrease in shear force. These opposing factors seem to be responsible for the friction coefficient being only weakly dependent on roughness.

Fig. 5b shows that under higher P/E , $h_{\min}$ gets thinner while the rate of thinning decreases as P/E increases.

Fig. 5c shows that $h_{\min}$ increases as $\mu V/LE$ increases for constant A/L and P/E (i.e. the fluid layer gets thicker for higher values of V or μ).

Scaling relations

For the friction coefficient, we used the data from 120 simulations using various geometrical, loading and material conditions. The independent variables were changed in the range provided in Table 2 for all the simulations. Regression analysis resulted in the following relationships:

$$F_c = 2.58(A/L)^{-0.018}(P/E)^{-0.66}(\mu V/LE)^{0.52} \quad (4)$$

$$h_{\min}/L=0.84(A/L)^{-0.21}(P/E)^{-0.30}(\mu V/LE)^{0.59} \quad (5)$$

Figs. 7 and 8 show the fit of F_c and $h_{\min}$ from Eqs. 4 and 5 compared to the data from finite element simulations.

Effect of different shapes

In the previous section we characterized lubrication properties for a sine-shaped geometry. Here we examine the role of shape in EHL. Consider a simple case of a piecewise linear wedge. We computed friction coefficients of linear and sine-shaped asperities under identical conditions. Fig. 9 shows a comparison of friction coefficient for the two geometries. It is of interest that the friction coefficients for the two geometries are almost identical while minimum fluid thickness is less for the linear wedge according to Fig. 10. This is consistent with the following argument: the effective fluid thickness (occurring under the flattened segment) happens over a longer length of the asperity in the sine-shaped geometry, increasing F_c , whereas the minimum thickness is thicker for this geometry and contributes to a decrease in F_c . We conjecture that these two factors together cause F_c to remain only weakly dependent on geometrical shapes, similar to the argument for bumps of different roughness.

Effect of multiple bumps

Our simulations included only a single bump, whereas biological tissues and other soft deformable surfaces have multiple bumps on their surface. The load support generated by a single bump in our 2-D model could also be simulated in a 3-D model with multiple bumps, each bump supplying load support to maintain separation of tissues. However, in a simple 2-D model configured with multiple bumps, there would be no mechanism whereby the fluid in the space between bumps could be replenished independent of the flow past neighboring bumps. This would cause pressure in the spaces between bumps to be determined by the imbalance of flow past the adjacent bumps, and each bump would thus strongly influence its neighbors. In a 3-D model, this artifactual interdependence would be less important because the fluid between bumps forms an interconnected reservoir of approximately constant pressure, allowing multiple bumps to provide relatively independent load support. Although we were limited by computational considerations to a 2D model with one bump, we believe our qualitative findings are valid for 3D situations with multiple bumps.

Conclusion

We studied the tribology of soft tissues in a two dimensional finite element simulation of a thin layer of fluid separating a sliding rigid surface from a soft asperity initially sine-shaped. We computed the frictional force and minimum fluid thickness as functions of sliding velocity, normal load, material properties, and bump amplitude and length.

Double logarithmic regression was used to determine the exponents of the scaling relationships of friction coefficient and minimum fluid thickness to the independent variables. Results showed that frictional shear force is strongly dependent on velocity, viscosity, and load, moderately dependent on bump length and elasticity, and only weakly dependent on the bump amplitude. The minimum fluid thickness is strongly dependent on velocity and viscosity, and changes moderately with load, elasticity, bump amplitude, and length.

We conclude that softer material and longer bump wavelength lead to a decrease in the friction coefficient and an increase in the minimum fluid thickness, improving lubrication performance. However, variations in velocity, viscosity, or load lead to changes in friction coefficient and

minimum fluid thickness of the same sign. For example, increasing load P decreases F_c by improving the load-bearing qualities of the shape but simultaneously decreases $h_{\min}$ which degrades lubrication performance. The effect of bump amplitude is negligible on the friction coefficient; however, bumps of lower amplitude result in greater minimum fluid thickness.

Note that in *vivo* not all these parameters enjoy a large range. For example, velocity and load are usually dictated by environment. However, in artificial joints one can improve the unevenness on the sliding surfaces or the elasticity of material to protect the surfaces from wear (e.g. modifying to longer bump wavelength or using softer materials).

We studied the influence of shape on lubrication behavior of soft asperities by comparing piecewise linear to sine-shaped wedge. The results showed little dependence of friction coefficient on shape in EHL, while fluid thickness was different depending on the geometrical shape. Importantly, the morphology of the asperity has little effect on friction. The results confirm that shear induced deformation of an initially symmetrical shape leads to load-supporting behavior, and suggest that an uneven soft surface sliding in lubricant will be deformed so as to generate lift and maintain separation between the surfaces.

Acknowledgements

This work was supported by a grant HL-63737 from the National Institutes of Health.

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Figure 1.
The undeformed geometry of solid and fluid models.

Figure 2.

Upper panel: Pressure distribution for the deformed model under reference conditions (Table 1) with $V = 0.6$ cm/s. Note the asymmetry of the pressure distribution and preponderance of positive pressure under the asymmetrically deformed bump. Lower panel: Pressure distribution as above but with $V = 0.02$ cm/s. Fluid is thinner, and the pressure variations are more concentrated at the central, flattened region of the bump. (Pressures on the scale are in $\text{Pa} \times 10$.)

Figure 3.
Friction coefficient as functions of dimensionless groups.

Figure 4. Normal and longitudinal deformation under 2 normal pressures. Normal and longitudinal displacements are of similar magnitude.

Figure 5.
Minimum fluid thickness as a function of dimensionless groups.

Figure 6. Fluid thickness as a function of position under bumps of amplitudes $A = 0.0005$ and $A = 0.001$ cm at $V = 0.6$ cm/s and $F_N = 40\mu\text{N}$.

Figure 7.
Friction coefficient from Eq. 4 compared to simulation data.

Figure 8.
Minimum fluid thickness from Eq. 5 compared to simulation data.

Figure 9. Comparison of friction coefficients between two geometries: sine-shaped wedge and piecewise linear wedge.

Figure 10.

Comparison of fluid thickness for sine-shaped wedge and linear wedge at $V = 0.6$ cm/sec, $A = 0.001$ cm, $P = 20$ Pa, $E = 500$ Pa, $\mu = 0.001$ Pa·s.

Table 1

The reference values of the variables and values representative of the pleural space.

Variable	Reference for value model	Values likely in human pleural space
Amplitude, A (cm)	0.0005	(Depends on L)
Length, L (cm)	0.02	0.001–30
Velocity, V (cm/sec)	0.6	0.001–10
Viscosity, μ (Pa·s)	0.001	0.0015
Pressure, P (Pa)	20	0–200 ? (Depends on L)
Young's modulus, E (Pa)	500	50–2000 (Gouldstone et al., 2003)

Table 2

Range of independent variables.

Variable	Min	Max	No. of Values tested
Amplitude (cm)	0.0005	0.006	5
Length (cm)	0.02	0.08	3
Velocity (cm/sec)	0.02	1.2	5
Pressure (Pa)	5	40	4
Elasticity (Pa)	250	1000	3
Viscosity (Pa·s)	0.001	0.003	3