Customization of the acoustic field produced by a piezoelectric array through interelement delays

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(Received 27 September 2007; revised 31 March 2008; accepted 31 March 2008)

A method for producing a prescribed acoustic pressure field from a piezoelectric array was investigated. The array consisted of 170 elements placed on the inner surface of a 15 cm radius spherical cap. Each element was independently driven by using individual pulsers each capable of generating 1.2 kV. Acoustic field customization was achieved by independently controlling the time when each element was excited. The set of time delays necessary to produce a particular acoustic field was determined by using an optimization scheme. The acoustic field at the focal plane was simulated by using the angular spectrum method, and the optimization searched for the time delays that minimized the least squared difference between the magnitudes of the simulated and desired pressure fields. The acoustic field was shaped in two different ways: the -6 dB focal width was increased to different desired widths and the ring-shaped pressure distributions of various prescribed diameters were produced. For both cases, the set of delays resulting from the respective optimization schemes were confirmed to yield the desired pressure distributions by using simulations and measurements. The simulations, however, predicted peak positive pressures roughly half those obtained from the measurements, which was attributed to the exclusion of nonlinearity in the simulations. © 2008 Acoustical Society of America. [DOI: 10.1121/1.2912448]

PACS number(s): 43.38.Hz, 43.60.Fg [AJZ]

Pages: 4174–4185

I. INTRODUCTION

This work was motivated by an application in shock wave lithotripsy (SWL). SWL has been clinically used to treat kidney stones since 1980,^{1,2} but there is still debate over the process by which the SWs comminute kidney stones. There are two main categories of mechanisms deemed plausible for stone breakage: direct stress effects, such as spallation,^{1,3} squeezing,⁴ shear,^{5,6} and cavitation effects consisting of microjets,^{7,8} and secondary SWs emitted from collapsing bubbles.^{9–12}

Studies have shown that altering the temporal wave profile can affect cavitation,^{13–19} thus influencing the cavitation related mechanisms of therapeutic ultrasound. Conceivably, altering the spatial distribution could influence the direct stress related mechanisms. For instance, spallation depends primarily on the portion of the SW that enters the stone.^{1,3} Similarly, mechanisms such as squeezing and shear depend more on the pressure at the periphery of the stone.^{4–6} Therefore, stress related mechanisms might be manipulated by altering the spatial pressure distribution.

To date, the primary focus of customizing the acoustic field has been to electronically steer the focus to track the kidney stone during treatment.^{20,21} Both studies employed a spherical transducer array with high voltage pulser circuits for each individual element and achieved efficient focusing in an ellipsoidal region of about 4 cm in diameter and 6 cm

in length. The present work employs a similar spherical transducer and pulse generator system to customize spatial pressure distribution.

The modification of spatial pressure distribution has not been widely implemented in lithotripsy. One notable example where the spatial pressure distribution was altered is the wide-focus lithotripter developed by Eisenmenger *et al.*²² It employs an electromagnetic source and generates a relatively low pressure amplitude of 20 MPa but a relatively wide focal width of 18 mm (two to four times the diameter of a typical stone). This “wide-focus” lithotripter was reported to trigger the squeezing mechanism⁴ leading to successful treatment while reducing patient discomfort. In another instance, Sapozhnikov *et al.*²³ treated cylindrical stones with an electrohydraulic lithotripter and modified the incident acoustic field by using baffles. They implemented various configurations of baffles to selectively block specific waves responsible for different stress related fracture mechanisms such as spallation, shear, or squeezing. They determined that neither spallation nor squeezing is solely responsible for fracture, and that shear stresses induced at the periphery of the proximal face of the stone play a dominant role in comminution.

The focus of the present work was to develop a system capable of producing a predetermined acoustic field by solving an inverse problem for a piezoelectric array. The method utilized in this study to customize the acoustic output of the piezoelectric lithotripter array is loosely based on the approach by Tanter *et al.*²⁴ There, it was demonstrated that an arbitrary pressure-time profile and spatial pressure distribution can be achieved by implementing a spatiotemporal inverse filter. Though they presented a viable solution of the

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inverse problem, the physical implementation of a spatiotemporal inverse filter is instrumentation intensive as it requires the ability to excite each element with an arbitrary signal. This is particularly challenging for the high amplitudes required in lithotripsy applications. The hardware available for this study (detailed description to follow) consisted of independent high voltage pulse discharge circuits for each element, where the only control parameter for each element was the time that the element was excited. The challenge was to find the appropriate timing for each element that best matches the desired acoustic field. We employed an inverse problem approach which resulted in a satisfactory approximation to the desired spatial pressure distribution.

II. THEORY

The theory is presented in three parts. First, the angular spectrum approach which is employed as the forward model is described. Second, the practical implementation of the angular spectrum method to the array source is defined, and, finally, the error function employed in the optimization routine is described.

A. Forward model

The forward model employed the angular spectrum method which models diffractive wave propagation from one plane to any other parallel plane.²⁵⁻²⁷ The salient features of the angular spectrum theory will be presented in this section. Readers may refer to Ref. 27 for further details. The pressure can be described in the frequency domain by taking a temporal Fourier transform which is defined here as

$$P(x, y, z, \omega) = \int_{-\infty}^{\infty} p(x, y, z, t) e^{i\omega t} dt. \quad (1)$$

Given $P(x, y, z, \omega)$ on some $z=z_0$ plane, the angular spectrum is defined as the spatial Fourier transform of P with respect to x and y , defined here as

$$\tilde{P}(k_x, k_y, z_0, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x, y, z_0, \omega) e^{-i(k_x x + k_y y)} dx dy. \quad (2)$$

Since $P(x, y, z, \omega)$ obeys the Helmholtz equation, the angular spectrum at any other plane of interest $z=z_1$ can be obtained by multiplying the initial angular spectrum by a k -space propagation operator,

$$\tilde{P}(k_x, k_y, z_1, \omega) = \tilde{P}(k_x, k_y, z_0, \omega) e^{i(z_1 - z_0) \sqrt{k^2 - k_x^2 - k_y^2}}, \quad (3)$$

where k is the free-space wave number. The relationship described as a product in k -space [Eq. (3)] can be written as a convolution in the space domain,

$$P(x, y, z_1, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(\acute{x}, \acute{y}, z_0, \omega) G(x - \acute{x}, y - \acute{y}_1, z_1 - z_0) d\acute{x} d\acute{y}, \quad (4)$$

where $G(x, y, z)$ is the inverse spatial Fourier transform of the propagation operator and can be explicitly written²⁸ as

$$G(x, y, z) = \frac{e^{ikr}}{2\pi r} \left(\frac{1}{r} - ik \right) \frac{z}{r}, \quad (5)$$

where $r = \sqrt{x^2 + y^2 + z^2}$. For the purposes in this work, the angular spectrum method was symbolically represented in the form given in Eqs. (4) and (5). Numerically, it was implemented by using the MATLAB convolution algorithm which employs two dimensional (2D) Fourier transforms.

B. Wave propagation model for the piezoelectric lithotripter array

The array was considered as a source with L elements and focal axis along the z direction and the geometric focus taken to be the origin. The angular spectrum technique is not easily adapted to model a source condition prescribed on a curved surface. Furthermore, a large area needs to be discretized in order to obtain the source condition at the face of the array, thus making the model computationally cumbersome. Therefore, the approach employed in this study involved individually measuring the impulse response of each element on a prefocal plane ($z_0 = -20$ mm) and then propagating each measurement forward to the focal plane by using the angular spectrum method. The pressure at any point (x, y) on the $z=z_0$ plane can then be defined as

$$p(x, y, z_0, t) = \sum_{l=1}^L h_l(x, y, z_0, t - \Delta t_l), \quad (6)$$

where $h_l(x, y, z_0, t)$ is the impulse response of element l measured at (x, y, z_0) and Δt_l is the delay for the l th element that will be used to control the acoustic field. Equation (6) can be rewritten in the frequency domain as

$$P(x, y, z_0, \omega) = \sum_{l=1}^L H_l(x, y, z_0, \omega) e^{i\Delta t_l \omega}. \quad (7)$$

The pressure field described in Eq. (7) is then propagated to the focal plane by using the angular spectrum approach. The angular spectrum operator commutes with the summation and time delay factor yielding a field at $z=z_1$ which can be expressed as

$$P(x, y, z_1, \omega) = \sum_{l=1}^L e^{i\Delta t_l \omega} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H_l(\acute{x}, \acute{y}, z_0, \omega) G(x - \acute{x}, y - \acute{y}_1, z_1 - z_0) d\acute{x} d\acute{y}. \quad (8)$$

Numerically, the equivalent is achieved in three steps. First, the impulse response field for each element, $H_l(z_0, \omega)$ for l th element, is propagated to the plane of interest ($z=z_1$) by using Eq. (4). Second, the propagated impulse response field $H_l(z_1, \omega)$ is multiplied by $e^{i\Delta t_l \omega}$ which describes the time delay applied to each element. Finally, the cumulative pressure field resulting from the array is simulated by invoking the linear superposition of pressure and summing the products of the propagated impulse response and the delay term for all the elements. This procedure captures both the diffraction from each element and the interelement delay.

C. Optimization scheme

The optimization scheme was implemented with the goal of determining the delays of each element so that the difference between a prescribed pressure field $\hat{P}(x, y, z, \omega)$ and a simulated pressure field $P(x, y, z, \omega)$ was minimized. The prescribed pressure field was defined on a plane at $z = z_1$ over M control points $\hat{P}(x_m, y_m, z_1, \omega)$. The simulated pressure at the $z = z_1$ plane resulting from a set of delays Δt_l is given by Eq. (8), which can be rewritten for each point m in the control plane as the following sum over L elements:

$$P_m(\omega) = \sum_{l=1}^L \hat{H}_{ml}(\omega) e^{i\Delta t_l \omega}, \quad (9)$$

where $\hat{H}_{ml}(\omega) = H(x, y, z_0, \omega) *_{x,y} G(x, y, z_1 - z_0)|_{x_m, y_m}$ and H_{ml} represents the transfer function from the l th element to the m th control point. The simulated pressure at all M points can be represented by using M equations which can be written in matrix form as

$$\mathbf{P}(\omega) = \hat{\mathbf{H}}(\omega)\mathbf{D}(\omega), \quad (10)$$

where $D_l(\omega) = e^{i\Delta t_l \omega}$, $l = 1, \dots, L$. The goal is to choose each Δt_l , such that the simulated pressure $\mathbf{P}(\omega)$ best matches the prescribed pressure $\hat{\mathbf{P}}(\omega)$.

One optimization approach is to employ a direct inverse filter, *à la* Tanter *et al.*,²⁴ in which Eq. (10) can be solved for $\mathbf{D}(\omega)=[\hat{\mathbf{H}}(\omega)]^{-1}\hat{\mathbf{P}}(\omega)$. Implementation of the inverse solution obtained in this manner results in a $\mathbf{D}(\omega)$ with varying amplitude and phase. The high voltage pulsers used to drive the transducer array, however, produce a pulse of fixed amplitude and shape; that is, the hardware imposes a constraint of $|D_f(\omega)|=1$. Further, because the only control available is over the time delay Δt_f of an element, the phase must be linear in frequency of the form $\arg[D_f(\omega)]=\Delta t_f\omega$.

Therefore, in this work an alternate method involving nonlinear regression of squared error was used to solve for optimal delays. Because our motivation is to control the pressure amplitude, we choose the objective function to be

$$|\hat{P}(\omega_n)| = |P(\omega_n)|. \quad (11)$$

That is, we do not specify any phase variation. The impact of this is considered in the Discussion. The error function to be minimized was the square error summed over all frequencies and all spatial control points,

$$\Pi(\Delta t_l) = \sum_{n=1}^N \left\{ \sum_{m=1}^M W_m [|\hat{\mathbf{P}}_m(\omega_n)| - |\mathbf{P}_m(\omega_n)|]^2 \right\}, \quad (12)$$

where W_m was a weighting function defined across the control points to allocate sensitivity of the optimization routine to the error at the m th point. The weighting function was independent of frequency and was dependent on space alone. The optimization was performed by using the optimization toolbox in MATLAB (Release 14, Mathworks, Natick, MA). The optimization function used was “lsqnonlin” which employs the Levenberg–Marquardt algorithm²⁹ for determining the set of delays that minimizes the error function [Eq. (12)].

FIG. 1. A diagram of transducer elements in the spherical bowl and their numerical identification.

The following parameters were used for all the optimization runs. The maximum number of functional evaluations was 32 600 and maximum number of iterations was 500. The tolerance for functional evaluation was 10^{-4} and the tolerances for Δt_l were 1 ns for the wide-focus optimization and 10 ns for the ring focus optimization. In all cases, the optimization was terminated when the magnitude of the search direction was less than the tolerance for Δt_l .

III. EXPERIMENTAL SETUP

The instrumentation and experimental methods will be presented in this section. The section addresses three topics: the description of the source, the instrumentation for acoustic pressure measurements, and methods for customizing the acoustic field.

A. Piezoelectric lithotripter array

The piezoelectric lithotripter array used in this study comprised of a 170 element focused array (Imasonic S. A., Besancon, France). The elements were placed on a spherical cap with a radius of curvature of 150 mm and an aperture diameter of 154 mm. The nominal center frequency of each element was 600 kHz ($Q=2$). The transducer was constructed from a “1–3” piezocomposite material which consists of thin ceramic rods encapsulated within a polymer matrix.

The layout of the 170 elements (referred to as e_1 through e_{170}) is shown in Fig. 1. The central element (e_1) had circular cross section, and the other elements were trapezoidal in shape placed on seven axisymmetric rings. Each element had a surface area of 88 mm². The maximum permissible driving voltage was 6 kV. The average electrical impedance of an element when the array was in water at the center frequency (600 kHz) was $Z=313-j504 \Omega$.

The piezoelectric array was driven by a 170 channel high voltage pulser (Gammell Applied Technologies, LLC, Exmore, VA). The circuit details are described in Ref. 30, and the version used here was the nondoubling circuit. Briefly, each channel of the pulser employed a 220 nF ca-

capacitor which can be charged to 1200 V. A high voltage insulated gate bipolar transistor (IGBT) was used to discharge the capacitor through the transducer element. Each channel was independently controlled by a TTL-level pulse generated by a 196 line pulse-pattern generator (81104A, Agilent, Palo Alto, CA). Both the duration and the timing of the TTL pulse for each channel could be independently adjusted with a 100 ns resolution. A single high voltage power supply (N02HP30, Acopian, Easton, PA, USA) was used to charge the capacitors. The 1200 V limit was governed by the breakdown voltage of the IGBT.

The transducer was mounted into the wall of a cast-acrylic test tank of dimensions of $0.5 \times 0.9 \times 0.5 \text{ m}^3$. The tank was fitted with a filtration and degassing system. A $5 \mu\text{m}$ subparticle filter (02913-30, Cole-Parmer, Vernon Hills, IL) along with a magnetic nonsubmersible pump was used for filtering the water. A pinhead degassing system³¹ was used for degassing the water. The dissolved gas content of the water in the test tank was maintained at approximately 40% of gas-saturated water. Positioning in the test tank was accomplished by using a computer controlled three axis positioner (Velmex, Inc., Bloomfield, NY) with a nominal spatial resolution of $5 \mu\text{m}$.

B. Acoustic measurements

A polyvinylidene fluoride (PVDF) membrane hydrophone (model 0200, Precision Acoustics, Dorchester, England) with a 0.2 mm diameter active area and a bandwidth of 30 MHz was used for acoustic measurements where only one element was excited at a time. The hydrophone, referred to as the PA hydrophone hereon, used in conjunction with a 20 dB preamplifier (50 Ω termination), had a sensitivity of 33.3 MPa/V (flat within 2 dB over a frequency range of 0.1–20 MHz). The PA hydrophone signal was passed through a tunable high pass filter (model 3940, Kronhite, Avon, MA) with a cutoff frequency of 10 kHz and gain of 20 dB with a nominal frequency range of 0–10 MHz to further improve the signal to noise ratio. The input impedance of the high pass filter was 1 M Ω and output impedance was 50 Ω . A 50 Ω termination was attached to the input of the high pass filter to match the output impedance of the hydrophone's preamplifier.

Shock wave measurements were performed by using a fiber optic probe hydrophone (FOPH) or a second PVDF membrane hydrophone. The FOPH (model 500, R. P. Acoustics, Leutenbach, Germany) had a bandwidth of 0.1–30 MHz and a receiving aperture of 100 μm . This hydrophone measures a change in refractive index of water which can be related to the pressure in the water.³² The PVDF hydrophone was developed at University of Washington,³³ referred to as UW hydrophone hereon, and had a bandwidth of 0.1–20 MHz, sensitivity of 35.8 MPa/V, and a receiving aperture size of 0.5 mm. Individual waveforms were sampled at a rate of 100 MHz on a digital oscilloscope (Waverunner 6000A, Lecroy, Chestnut Ridge, NY) with 8 bit resolution and 200 MHz low-pass filter and transferred to a computer for postprocessing by using MATLAB (Mathworks, Natick, MA).

C. Customization of acoustic output

The response $h(x, y, z_0, t)$ was obtained for each element at $z_0 = -20$, that is, 20 mm prefocal, on a $50 \times 50 \text{ mm}^2$ grid with internode distance of 0.5 mm (10 000 points) and then zero padded to $80 \times 80 \text{ mm}^2$ (25 600 points) to reduce wrap-around errors that may occur in evaluation of convolution by using fast Fourier transform. The grid size of $50 \times 50 \text{ mm}^2$ was the minimum size necessary to faithfully include all the features of the impulse response from all the elements. It will be shown in Sec. IV A that the majority of the energy (97%) of the pulse produced by each element lies in the 0–1.5 MHz band. Therefore, the angular spectrum approach was implemented within this band. The internode distance of 0.5 mm was the maximum possible distance that satisfied the Nyquist sampling criterion of $\lambda/2$.

The optimization scheme required an initial guess for the delays. For inversion problems that are not robust, careful selection of initial guess for the delays is crucial to converge to the global minimum. To further facilitate convergence of the optimization routine toward the desired solution, a weighting function was used to allocate a higher sensitivity of the error function [Eq. (12)] at key locations of interest. The choice of initial condition and weighting function varied depending on whether a broad focus or a ring-shaped focus was desired.

For the prescribed acoustic fields of increased -6 dB focal widths, a constant delay (1 or 3 μs) was assigned to all elements as the initial state for the optimization routine. This was done to prevent negative results for time delays. The weighting function used for this series of optimizations placed a higher emphasis on points along the circle describing the desired focal width as shown in Fig. 2(a).

The constant delay condition as the initial state for the optimization failed to produce the ring-shaped pressure field. Apparently, the optimization method used was unable to overcome the natural tendency of a spherically focused array to produce a peak pressure at the geometric focus. In order to better guide the solution, the initial delays were selected in order to focus groups of elements onto various points on desired ring. Eight points were chosen on the ring and the elements were divided into eight corresponding groups. Elements 1, 9, 17, ..., 169 comprised the first group, elements 2, 10, 17, ..., 170 were in the second group, and so on. The initial delays were set so that each group focused on its specified point on the desired ring. Since the ring-shaped pressure distribution critically depends on reducing the pressure at the geometric focus, its weighting function gave highest emphasis at the geometric focus extending out to the ring diameter in the form a 2D Gaussian function, as shown in Fig. 2(b).

IV. RESULTS

The results will be presented in three sections. First, the acoustic output of a single element of the piezoelectric lithotripter array is characterized. Second, the acoustic output of the entire piezoelectric lithotripter array without implement-

FIG. 2. The weight function employed to control the emphasis given to error at each node on the optimization plane for customizing the spatial distribution of pressure. (a) For the wide focus, a ringlike weighting function was used. (b) For the ring focus, the weighting function was highest on the axis.

ing any customization of the acoustic output is described. Third, results from customizing the spatial pressure distribution will be shown.

A. Single element characterization

A set of ten waveforms obtained from a single element (e_1) driven at 1.2 kV pulse repetition frequency (PRF) of 1 Hz is shown in Fig. 3(a). The received signal was highly reproducible with a temporal jitter of less than 30 ns and is characteristic of waveforms produced by all of the elements. The maximum temporal jitter between any two elements was determined to be less than 50 ns. The peak positive pressure produced from a single element was observed to be around 0.2 MPa at the focus of the piezoelectric array. Figure 3(b) shows the frequency spectrum of the pressure measurement obtained from e_1 at the focus. The emitted signal from the transducer element is a wide-band signal with a peak close to the specified center frequency of 0.6 MHz. The majority of the energy (97%) lies below 1.5 MHz and so the inversion only used signal content in this band. The discrepancy in the peak pressures due to this was less than 15%.

The map of the acoustic response of each element at a prefocal plane ($z_1 = -20$ mm) is necessary for implementing

the angular spectrum method in the wave propagation model. Formally, one should measure the field for each element in the array, however, for each element this requires measuring 10 000 waveforms, for a total of 1.7×10^6 measurements. We note that because each element has the same area and measurements were obtained in the far field, one could theoretically determine all the data from measuring one element. However, we found variations in field maps from ring to ring in excess of 20%. Therefore, measurements from one element per ring were used for initialization. The response of any other element on that ring was obtained by a rotation based on the difference in angle of the two elements. For instance, the map of P_+ of e_{170} on the axisymmetric plane $z = z_0$ can be obtained by rotating the response of e_{135} measured on the same plane by 60° , as shown in Fig. 4. The rotation of the field was performed by using the MATLAB function “imrotate” with the linear interpolation scheme. Measurements from multiple elements on multiple rings were conducted to confirm that the geometric transformation described above was sufficient to accurately predict the response of any element on a ring. Thus, the response of all 170 elements could be determined from eight measurements.

FIG. 3. (Color online) (a) Overlay of ten consecutive waveforms obtained from e_1 driven with 1.2 kV pulse lasting $0.5 \mu\text{s}$ at a PRF of 1 Hz. (b) Amplitude spectrum of the pressure waveform shown in (a).

FIG. 4. (Color online) (a) Normalized impulse response of e_{135} measured 20 mm prefocal and rotated by 60° . (b) Normalized impulse response of e_{170} measured 20 mm prefocal.

B. Piezoelectric lithotripter array characterization

The piezoelectric lithotripter array was first characterized while all elements were synchronously driven, that is, without manipulating the acoustic output. Figure 5 shows the waveform produced by driving the array with a 1.2 kV pulse. The waveform shown was obtained by first measuring ten waveforms with the PVDF hydrophone, aligning their main shock fronts, and then averaging the waveforms. Unlike the more typical lithotripsy waveforms generated by electrohydraulic and electromagnetic devices,³⁴ the waveform has four distinct features. First, a negative precursor of $0.5 \mu\text{s}$ long and a negative pressure of $P_1 = -10 \text{ MPa}$ followed by the main shock front with a positive peak $P_{1+} = 68 \text{ MPa}$. The rise time based on time difference between 10% of P_{1+} and 90% of P_{1+} was 30 ns and was limited by the 20 MHz bandwidth of the UW hydrophone. The duration of the main positive phase was $t_{1+} = 0.4 \mu\text{s}$. The negative phase following the main positive peak was $1 \mu\text{s}$ long with peak negative pressure $P_2 = -15 \text{ MPa}$. The second shock front had a peak positive pressure $P_{2+} = 36 \text{ MPa}$. The entire shock wave pulse had a duration $t = 5 \mu\text{s}$ measured as the time between when the pressure first exceeds 10% of P_{1+} and when the pressure last exceeds 10% of P_{1+} . Figure 6 shows the mean P_{1+} and P_{2-} as a function of charging voltage. The error bars

represent the standard deviation of ten measurements. Measurements indicated a monotonic increase in pressure with increase in the driving voltage.

The spatial pressure distribution of the acoustic field produced by the array was also mapped. Since the maximum operating voltage for the IGBTs used in the high voltage drivers was 1.2 kV, pressure maps were acquired while driving the piezoelectric array lithotripter at 1 kV to prevent overload from any voltage fluctuations. Pressure measurements were acquired with the FOPH as its $100 \mu\text{m}$ fiber diameter provided better spatial resolution than the UW hydrophone, albeit with a modest decrease in signal to noise ratio. The equivalency between the measurements acquired via both the hydrophones was verified by acquiring a waveform measurements at the focus. Figure 7 shows the average of ten waveforms acquired by using both hydrophones at the focus when the array was driven by using a 1.2 kV pulse. All prominent features of the waveform and P_{1+} are in good agreement with each other. The pressure measurements along all three axes are shown in Fig. 8. The top curve in each figure shows P_{1+} and the lower curve P_{2-} with respect to displacement from the focus. The -6 dB isobar of the peak positive pressure of the main shock front P_{1+} was used to describe the focal region which was thus determined to be $1.4 \times 1.9 \times 15 \text{ mm}^3$.

FIG. 5. A characteristic waveform produced by the piezoelectric lithotripter array when all elements were synchronously driven. The displayed waveform is an average of ten waveforms recorded consecutively at a PRF of 1 Hz.

FIG. 6. Peak positive pressure (P_{1+}) and peak negative pressure (P_{2-}) as a function of driving voltage. Peak pressures monotonically increased with increase in driving voltage.

FIG. 7. (Color online) Comparison of waveforms measured by the FOPH and UW hydrophone at the focus while synchronously driving the array with a 1.2 kV pulse. Waveforms are the average of ten measurements.

C. Customization of spatial pressure distribution

The optimization routine was employed by using prescribed fields of different focal widths defined by -6 dB of P_{1+} . The optimal set of delays for the array, for each focal width, was first investigated by using the forward propagation model discussed in the previous section and then programmed into the pulse pattern generator and the resulting pressure field measured.

Figure 9 shows the simulated pressure distribution (based on a driving voltage of 1.2 kV) of P_{1+} along the lateral axis X obtained from the optimization routine for varying focal widths. The displayed results were obtained from the linear forward wave propagation simulations by using the set of optimal delays determined for each of the desired -6 dB focal widths. As the only control parameter used for optimization is the time delay for each element and the optimization is based on minimization of least squares errors, the resulting simulated pressure fields do not exactly match the prescribed pressure field. However, an increase in the desired acoustic focal width resulted in an increase in the focal width of the simulated pressure obtained by using the optimal time delays. Prescribed focal widths of 3, 4, and 5 mm resulted in simulated focal widths of 3.4, 3.6, and 5.1 mm. As one might expect the “broadening” of the focal width results in a decrease of the peak pressure. The simulated focal width obtained by setting the delays to zero was 2.9 mm, which is different from the measured focal width reported in the previous section. The discrepancy can be attributed to the fact that the simulation is linear and therefore

FIG. 9. (Color online) Simulated distribution of P_{1+} along the lateral axis X produced from the optimization routine for prescribed focal widths of 3, 4, and 5 mm. The optimal delays were predicted to produce focal widths of 3.4, 3.6, and 5.1 mm. The predicted focal width when all elements were synchronously fired was 2.9 mm.

does not include the higher harmonics generated due to the nonlinear effects which subsequently lead to tighter focusing of the acoustic field.

The effectiveness of the lsqnonlin function in obtaining the optimal delays for achieving a pressure field of predetermined focal width was investigated via behavior of the error residual and the magnitude of the directional derivative obtained for each iteration during the optimization routine shown in Fig. 10. For the prescribed focal width of 5 mm, the optimization routine effectively reduced the residual and the magnitude of directional derivative, which indicates that the optimization routine approached a primary minimum. The derivative exhibited a significant reduction in oscillations through the course of optimization, indicating that the optimization surface was smooth and that the optimization might be converging toward a unique solution. Similar performance was achieved for the other focal widths.

The lateral pressure distributions measured upon implementing the delay sets corresponding to each prescribed -6 dB focal width are shown in Fig. 11. The driving voltage for each run was adjusted so that magnitude of P_{1+} at $X=0$ was roughly 40 MPa for all focal widths with the long term goal of investigating the dependence of stone fragmentation

FIG. 8. Peak pressure measurements along the X , Y , and Z (acoustic) axes. The -6 dB focal region based on the primary peak positive pressure was 15 mm long and between 1.4 and 1.9 mm wide.

FIG. 10. (Color online) Parameters for monitoring the optimization routine for the prescribed -6 dB focal width of 5 mm. (a) Error residual. (b) Magnitude of the directional derivative.

on focal width alone. The driving voltages for 5, 4, and 3 mm focal widths were 1200, 850, and 800 V, respectively. For the case where the elements were synchronously driven, the field with a maximum $P_{1+}=40$ MPa was obtained by driving the elements at 575 V.

Similar to the wave propagation simulations, the measured focal widths increased with increase in prescribed focal width. Prescribed focal widths of 3, 4, and 5 mm resulted in simulated focal widths of 2.8, 3.1, and 3.9 mm. The focal width measured when all elements were synchronously driven was 1.7 mm. The focal width of 3.9 mm was the maximum possible focal width we were able to obtain while maintaining $P_{1+}=40$ MPa. As shown in the previous section, the focal width when all elements are driven at 1 kV with zero delay was 1.4 mm along the X axis. The optimization routine scaled the focal width by a factor of over 2 with a 33% reduction in the magnitude of P_{1+} .

Both the predictions and measurements show that the increase in focal width was achieved with a loss of peak pressure. This was investigated further by scanning the hydrophone in the axial direction. Figure 12 shows that peak pressure has been shifted from the geometrical focus to a point of 8 mm closer to the transducer. That is, the optimization routine's solution was to shift the focus closer to the transducer and exploit the postfocal divergence to create a broad focus.

The optimization routine was also implemented for obtaining a ring-shaped pressure distribution with a diameter of 4 or 7 mm. Figure 13 shows simulated and measured spatial pressure distribution based on P_{1+} and P_{2-} for both cases.

FIG. 12. Axial pressure maps for -6 dB focal width of 3.9 mm from measurements acquired by using the PVDF hydrophone. (a) Map of P_{1+} . (b) Map of P_{2-} . The rectangle denotes the spatial extent of a typical stone. Gray scale represents pressure in MPa.

FIG. 11. (Color online) Measured distribution of P_{1+} along the lateral axis X produced from the optimization routine for desired focal widths of 3, 4, and 5 mm. The optimal delays produced focal widths of 2.8, 3.1, and 3.9 mm. The focal width when all elements were synchronously fired was 1.7 mm.

The optimization routine was successful in producing a ring-shaped focal field. The simulated field, obtained by using the linear wave propagation model, and the measured field, when the array was driven by using the set of optimal delays determined for each ring diameter, were in good agreement. The discrepancy in the magnitude of P_{1+} can be attributed to the neglect of nonlinear steepening in the angular spectrum approach.

The error residual and magnitude of the directional derivative for both the 4 and 7 mm rings are shown in Fig. 14. The residual exhibited a modest decrease unlike the case for the optimization results for wider focal width shown in Fig. 10. Even so, the set of delays obtained produced an acceptable ring-shaped pressure distribution. The derivative, on the other hand, shows high oscillations around a large magnitude. This indicates that optimization surface is quite rough, and that it is likely that many different sets of delays could produce tolerably good ring-shaped pressure distributions.

Figure 15 shows the measured distribution of P_{1+} along the lateral axis X for ring diameters of 4 and 7 mm (driving voltage of 1.2 kV) compared to that obtained when elements are synchronously driven at 1 kV. The “no delay” waveform

FIG. 13. The results from optimization routine aimed at obtaining a ring-shaped pressure distribution. Left frames are field maps obtained by using the linear propagation model. Right frames are field maps from measurements acquired by using the PVDF hydrophone. (a) Prescribed ring diameter of 4 mm. (b) Prescribed ring diameter of 7 mm. Gray scale represents pressure in MPa.

had a maximum pressure in excess of 60 MPa. In contrast, both ring pressure distributions had a pressure minimum on axis with a peak less than 5 MPa. The ring focal field peaked off axis at an amplitude higher than the no delay field. However, the peak pressures did not exceed 20 MPa.

The relevant parameters from the ring optimization are shown in Table I. Both the simulations and the measurements were obtained at a spatial resolution of 0.5 mm. The ring diameter was determined along both lateral axes based on the maximum value of P_{1+} in the focal plane. Both the simulated and measured ring diameters resulting from the optimization routine were consistent with the prescribed ring diameter. The ring thickness based on -6 dB of P_{1+} (local peak) was between 2 and 4 mm in all the cases. The magnitude of P_{1+} at the focus obtained by using the angular spectrum simulation was around 1 MPa and that observed through measurements was 4 MPa.

The motivation of the ring focus was to produce a wavefront that traveled around the periphery of a kidney stone to efficiently couple into shear waves in the stone. This requires that the distribution remain ring shaped over an axial extent of about 10 mm. The optimization routine invoked to obtain the delays necessary to produce a ring-shaped pressure field prescribed the pressure only at one plane. Figure 16 shows the distribution of P_{1+} and P_{2-} over the axial plane for

acoustic field with a ring of 4 and 7 mm in diameter. The measurements indicated that the optimization routine for producing ring acoustic fields does indeed reduce the pressure along the acoustic axis and the ring shape is maintained over typical length of a stone.

Sample waveforms at four points that lie on the peak of the ring pressure field are displayed in Figs. 17(a). Since the ring pressure distribution is not exactly symmetrical, four sampling locations were chosen from the points that constitute the maximum ring pressure. The measured waveforms indicate that different portions of the ring consist of different waveform shapes. The measurements indicate that the time of arrival of the waveforms is not uniform at all locations on the focal plane. This can be attributed to the optimization routine with only time delays as the control parameter in which the pressure distribution was the only constraint and no constraint was placed on phase. The difference in arrival times of pulses at different locations on the ring presents a need for including an arrival time constraint in the optimization routine for the ring pressure distribution (which was beyond the scope of this work). The variation in the arrival

FIG. 14. Optimization results for ring-shaped pressure fields of 4 and 7 mm in diameter. (a) Error residual. (b) Directional derivative. During the course of the optimization, the residue, and the derivative exhibited a modest decrease, indicating that optimization runs aimed at producing a ring focus might have encountered a local minimum.

FIG. 15. Comparison between pressure distribution (P_{1+}) along a lateral axis (X) obtained from measurements across the focal plane for acoustic field obtained by synchronously driving elements and the ring pressure distribution of diameters of 4 and 7 mm.

TABLE I. Parameters from ring field optimization.

Prescribed diameter (mm)	Simulated		Measured	
	4	7	4	7
X diameter (mm)	3.5	6	3.5	6.5
Y diameter (mm)	3.5	6	3.5	6
Mean ring thickness (mm)	≈ 3	≈ 3	≈ 2	≈ 3
P_{1+} at focus (MPa)	1	1	4	4

time in space is illustrated in maps of the arrival time of waveform based on the largest peak (typically P_{1+}) in Fig. 17(b). The map indicates that the arrival times varied by up to $2 \mu\text{s}$ along the region that corresponded to the ring of high pressure. The discrete edges correspond to locations where peak pressure changes from P_{1+} to P_{2+} .

V. DISCUSSION

The work presented describes an approach with which a piezoelectric array could generate a prescribed acoustic field with limited control over the drive signal. The algorithm was able to successfully broaden the focus and produce ring-shaped focii. The acoustic fields produced by implementing the delays determined by the optimization routine were not an exact match to the prescribed acoustic fields. This was, in part, because the delays for the elements of the array were the only control parameter in our optimization and, in part, because the numerical propagation model was linear and

could not capture the nonlinear effects. Also, in both scenarios, the resulting pressure distributions were achieved at the price of reduced peak pressure.

In the case of the wide focal region, the optimization function electronically shifted the focus toward the source to achieve the objective. This is equivalent to placing a regular acoustic lens that shifts the focus and satisfies the requirement for broader pressure region in the focal plane but clearly results in large acoustic pressures in off-target locations which may not be desirable, particularly in clinical applications. To prevent this from happening, further constraints along the acoustic axis would need to be included in the optimization, e.g., a penalty if the pressure at other axial locations is greater than the pressure at the focus. In the broadest case, one could specify the pressure field in an entire volume around the geometrical focus, rather than just one plane, and optimize for that case. The broadening of the focal region can also be achieved in a more *ad hoc* manner by adjusting the delays to emulate a slightly heterogeneous phase screen to broaden the focus without shifting it. However, the motivation for this work was to develop a methodology that would enable automatic evaluation of the optimal set of delays to best match a prescribed spatial pressure distribution, and therefore the *ad hoc* methods were not pursued.

The optimization function for obtaining the ring acoustic field was not as well behaved as that for obtaining a wider focal field, and convergence to a ring shape depended more heavily on the initial guess for the delays and the weighting

FIG. 16. Axial pressure maps for ring-shaped acoustic fields from measurements acquired by using the PVDF hydrophone. (a) Map of P_{1+} for the 4 mm ring. (b) Map of P_{2-} for the 4 mm ring. (c) Map of P_{1+} for the 7 mm ring. (d) Map of P_{2-} for the 7 mm ring. Gray scale represents pressure in MPa.

FIG. 17. (a) Sample waveforms measured at the focal plane when the array was driven by using the optimal delays for the 7 mm ring. The measurements indicate spatial variation in the time of arrival of the waveforms. (b) Map of arrival time of waveforms based on the largest peak for the 7 mm ring. The gray scale represents time in μs . The region between the two circles denotes the ring of high pressure and differences in delay of up to $2 \mu\text{s}$ can be seen. The discrete edges correspond to locations where peak pressure changes from P_1 to P_2 .

function for the error. Further, the arrival times were not uniform around the ring (recall Fig. 17). Additional constraints that force the arrival times for the pulses along the ring to be uniform might also lead to a more robust convergence to the optimal set of delays for producing the ring acoustic field. We modified the error function by incorporating an additional term of the form $|\hat{P} - P|^2$ which incorporates phase difference. However, despite efforts with different weighting strategies, this modified error function resulted in a very modest reduction of phase error for the case of ring-shaped pressure field. Furthermore, this strategy did not lead to a significant reduction in the amplitude error. It appears that a more sophisticated optimization algorithm in conjunction with a modified error function is necessary for approaching a global minimum which might then lead to a ring-shaped pressure field where wavefronts on the ring simultaneously arrive. On the other hand, it may be impossible to obtain certain spatial distributions within the constraints of this device. For a system with expanded capabilities, such as independent control over both time delay and amplitude for each element, an inverse filter approach to manipulate acoustic vortices³⁵ might provide better means for producing a ring-shaped pressure distribution. A system that allows control over the waveform shape of the driving signal for each element can be used for implementing a time reversal approach³⁶ which could be exploited for customizing the acoustic field and automatically focusing the acoustic energy at the stone.

The model accurately predicted the relative spatial pressure distribution for both the wide-focus and ring-focus customizations. The main discrepancy was in the positive peaks and was attributed to the fact that the forward propagation model neglected nonlinear effects and thus failed to include the enhancement of the magnitude due to nonlinear distortion inherent in measurements. The fact that this discrepancy was present in the acoustic field maps derived from the peak positive pressure but not from that derived from the peak negative pressure supports this conjecture. Given the large number of iterations required by the optimization routine, incorporating a fully nonlinear propagation model is compu-

tationally infeasible at present. However, the use of a linear model provided an accurate enough model to modify the spatial distribution of the pressure field.

ACKNOWLEDGMENTS

This work was supported by the National Institutes of Health through Grant No. DK43881. We thank Dr. A. P. Evan and Dr. J. A. McAteer for providing access to the fiber optic probe hydrophone, Dr. M. R. Bailey for the PVDF hydrophone, and Dr. R. A. Roy for helpful discussions and guidance.

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