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A practical global distortion correction method for an image intensifier based x-ray fluoroscopy system

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Abstract

X-ray images acquired on systems with image intensifiers (II) exhibit characteristic distortion which is due to both external and internal factors. The distortion is dependent on the orientation of the II, a fact particularly relevant to II's mounted on C arms which have several degrees of freedom of motion. Previous descriptions of distortion correction strategies have relied on a dense sampling of the C-arm orientation space, and as such have been limited mostly to a single arc of the primary angle, α . We present a new method which smooths the trajectories of the segmented vertices of the grid phantom as a function of α prior to solving the two-dimensional warping problem. It also shows that the same residual errors of distortion correction could be achieved without fitting the trajectories of the grid vertices, but instead applying the previously described global method of distortion correction, followed by directly smoothing the values of the polynomial coefficients as functions of the C-arm orientation parameters. When this technique was applied to a series of test images at arbitrary α , the root-mean-square (RMS) residual error was 0.22 pixels. The new method was extended to three degrees of freedom of the C-arm motion: the primary angle, α ; the secondary angle, β ; and the source-to-intensifier distance, λ . Only 75 images were used to characterize the distortion for the following ranges: α , $\pm 45^\circ$ ($\Delta\alpha = 22.5^\circ$); β , $\pm 36^\circ$ ($\Delta\beta = 18^\circ$); λ , 98–118 cm ($\Delta\lambda = 10$ cm). When evaluated on a series of test images acquired at arbitrary (α, β, λ) , the RMS residual error was 0.33 pixels. This method is targeted at applications such as guidance of catheter-based interventions and treatment planning for brachytherapy, which require distortion-corrected images over a large range of C-arm orientations.

Keywords

x-ray image intensifier; x-ray fluoroscopy; angle dependent distortion correction

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I. INTRODUCTION

Distortions due to the use of image intensifiers (II) in x-ray fluoroscopy have long been known, and several approaches have been described^{1–13} to correct these distortions for applications where quantitatively accurate positional information is to be obtained from the images produced by these systems. The distortion arises from several sources but the three that are of primary concern at the scales relevant to medical imaging are the so-called “pincushion” distortion, an S-shaped sigmoidal distortion, and local distortions. The pincushion distortion is due to the mapping of the planar image onto the curved input phosphor of the II, resulting in a radial-dependent stretching of the images which is most apparent at the periphery of the field of view. The sigmoidal distortion arises as a result of the interaction of the electrons in the II with homogenous magnetic fields and is dependent on the orientation of the II within these magnetic fields. This is important for C-arm-based systems where the II has several degrees of freedom. Finally, local distortions are the result of imperfections in the hardware of the II or strong, localized magnetic fields.

In all cases, the general approach to correcting the distortion has been to image a rectilinear grid of known spacing, and model the observed grid vertex locations as some function of the ideal locations. The earliest methods^{1–4} focused primarily on the pincushion distortion, and they attempted to correct this by modeling the geometry of the II and determining the curvature of the input phosphor. Later approaches^{5–8} divided the II into several local regions defined on the basis of trilaterals or quadrilaterals of adjacent grid vertices and used interpolating functions defined within the local regions to map observed to ideal grid points. A global method of distortion correction was proposed by Fahrig *et al.*⁹ and Gronenschild¹⁰ in which high-order two-dimensional (2D) polynomials defined over the entire II were used for the warping.

Comparison studies between the local methods and global methods^{11–13} returned mixed results, suggesting that the choice of local versus global depended on the nature of the distortion. When significant local distortions are present, the global method, which fits a single function for the entire II, is unable to satisfactorily characterize the distortion. However, in most cases, local distortions are negligible and the global method performs robustly.

In their original description of the global method, Fahrig *et al.*⁹ characterized the distortion for an arc of 200° of the primary angle of the C-arm, α , which was appropriate for their application of CT-like three-dimensional (3D) reconstructions in which images would always be acquired on this same arc. As such, they acquired images with $\Delta\alpha = 2^\circ$, a sampling density they deemed necessary after observing rapid fluctuations in the values of the polynomial coefficients as a function of α . Liu *et al.*¹⁴ who also acquired images for a single arc of α , proposed a super-global method in which they applied the global method to images acquired every 5°, then fit the polynomial coefficients as functions of α with 6th-order polynomials to capture the fluctuations observed by Fahrig *et al.*

We have previously developed a system of registration and fusion of pre-procedural magnetic resonance (MR) images with live x-ray fluoroscopy to guide procedures where the soft tissue contrast of MR is coupled with the high temporal resolution and excellent visualization of interventional devices afforded by x-ray fluoroscopy.^{15,16} Such a system relies on distortion-free x-ray images, and we have used the global method with satisfactory results. Our previous approach has been to use the global method to model the distortion at a set of pre-defined C-arm orientations to which the operator is restricted during the fusion-guided procedure. It is desirable to remove this restriction by developing a method of characterizing the distortion at all possible C-arm orientations so that the operator can choose the optimal view for the task at hand.

For interventional applications, it is not sufficient to limit C-arm orientations to a single arc of α ; indeed the whole range of achievable C-arm orientations is often necessary, depending on the type of procedure. For example, in cardiac catheterizations, images are acquired over a range of α of approximately 90° and secondary angle, β , approximately 70° ; if images were acquired every 5° , 285 images would be required for distortion correction. When one considers the range of source-to-intensifier distances and zoom factors available, this number grows larger still. For example, Rhode *et al.*¹⁷ applied the methods of Liu *et al.* to a characterize a range of α and β and used a total of 4611 images. The goal of this work was to determine if the distortion could be adequately characterized without the need to sample the space of C-arm orientations using the large number of images previously described.

The following sections will describe a method of smoothing the trajectories of grid vertices as a function of C-arm orientation and evaluate the effect of this smoothing on the global method of distortion correction. These results will be compared with previously described methods and the findings will be applied to a large range of C-arm orientations.

II. METHODS AND MATERIALS

II. A. Imaging

The imaging system used was a Siemens Artis Axiom cardiac single-plane, floor-mounted x-ray system. Images were acquired at the maximum field of view of 33 cm, and they were digitized at 512×512 pixels resolution. The phantom used for characterizing the distortion was a 34×34 cm sheet of plastic with a series of metal annular rings placed in a regular grid pattern. The rings had an outer diameter of 3 mm, an inner diameter of 1 mm, and a center-to-center spacing of 1 cm, with $R+1=426$ rings covering the useful part of the II. Images were acquired with the grid attached directly to the input window of the II, and the rings were segmented following the method described by Fahrig *et al.*⁹ The segmentation error was evaluated by acquiring 40 images at a constant C-arm orientation, performing the segmentation algorithm for each image, and measuring the standard deviation of the ring locations across all 40 images.

II. A.1. Images in a single arc—Images were acquired through an arc of $(A+1=21)$ primary angles, $\alpha = \{0^\circ, \pm 5^\circ, \dots, \pm 50^\circ\}$, which covered the range of α that is typically used during interventional applications (initially the secondary angle β and source-to-intensifier distance λ were kept constant at 0° and 98 cm, respectively). Ring segmentation generated ordered pairs $(u_{ra}, v_{ra}) \forall r \in \{0, \dots, R\}, a \in \alpha$. The high sampling density ($\Delta\alpha = 5^\circ$) of this dataset allowed evaluation of the necessary sampling density by serially leaving out some images and evaluating the resultant distortion correction models on all images, as described below in Sec. II C.

After acquiring the images used to model the distortion correction, the grid was rotated and translated on the input window of the II by an arbitrary amount, and a new series of ten images was acquired at arbitrary but known α . These test images were used to evaluate the distortion correction models described in Sec. II B by measuring their residual errors (see Appendix).

II. A.2. Beyond a single arc—Based on the results from the images on a single arc of α , another set of grid images was acquired covering a range of primary angle ($\alpha = \{0^\circ, \pm 22.5^\circ, \pm 45^\circ\}$), secondary angle ($\beta = \{0^\circ, \pm 18^\circ, \pm 36^\circ\}$), and source-to-intensifier distances ($\lambda = \{98 \text{ cm}, 108 \text{ cm}, 118 \text{ cm}\}$). These 75 images yielded a range suitable for the cardiovascular interventions in which we are interested. Again, a set of ten test images at arbitrary but known (α, β, λ) were also acquired.

II. B. Generation of distortion correction polynomials

The distorted ring locations in the images are modeled as a two-dimensional warping of the ideal ring locations $(\zeta_{ra}, \xi_{ra}) \forall r \in \{0, \dots, R\}, a \in \alpha$ (using the images on a single arc from Sec. II A 1). The ideal grid was inferred from the four rings closest to the center of the image acquired at $\alpha = 0^\circ$. These central rings were chosen because they experience almost no relative distortion, though they do experience some rotation, θ_a , and translation, $\vec{d}_a$. By correcting the ring locations in each image to the reference ideal grid at $\alpha = 0^\circ$, the corrected images will have no relative rotation or translation, though they will each still be rotated and translated by unknown θ_0 and $\vec{d}_0$, respectively. These residual rotation and translation parameters are routinely computed and corrected during the 3D calibration process of a C-arm system, and are not a topic of the current paper. The coefficients of the distortion correction polynomials were generated by three methods, which are described schematically in Fig. 1.

II. B.1. Method 1—This is a reimplementation of the method previously published by Fahrig *et al.*⁹ and Gronenschild.¹⁰ The segmented rings locations were modeled as (2D) 5th-order polynomials of the ideal ring locations, and the coefficients of the polynomials were calculated by solving (see Appendix)

$$[{}_1\vec{p}_a | {}_1\vec{q}_a] = (\vec{V}_R^{55}(\vec{\zeta}_{a=0}, \vec{\xi}_{a=0}))^+ [{}_1\vec{u}_a | {}_1\vec{v}_a] \quad \forall a \in \alpha, \quad (1)$$

where ${}_1\vec{p}_a$ is the vector of 36 coefficients for the polynomial fitting the horizontal component of the distortion in the a th image, and ${}_1\vec{q}_a$ the 36 coefficients for the vertical component. The coefficients ${}_1\vec{p}$ and ${}_1\vec{q}$ were then used to calculate the least-squares solution of the distorted ring locations, $({}_1\vec{u}, {}_1\vec{v})$, where the left subscript is used to denote Method 1.

II. B.2. Method 2—The locations of the rings in the series of acquired images create a two-dimensional trajectory for each ring. Each component of the trajectory was fitted independently with an ℓ th-order polynomial of α , whose coefficients were calculated by solving

$$[\vec{\sigma}_r | \vec{\tau}_r] = (\vec{V}_A^\ell(\alpha))^+ [\vec{u}_r | \vec{v}_r] \quad \forall r \in \{0, \dots, R\}, \quad (2)$$

where $\vec{\sigma}_r$ is the vector of coefficients for the polynomial fitting the u component of the trajectory for the r th ring, and $\vec{\tau}_r$ the coefficients for the v component. The trajectory fitting was evaluated for $\ell = \{2, 3, 4, 5\}$, and $\ell = 3$ was chosen for subsequent experiments. The trajectory-smoothed grid locations, $(\vec{s}, \vec{t})$, were then used as the input to the global distortion correction problem [note below that $(\vec{s}, \vec{t})$ replace $(\vec{u}, \vec{v})$, the original distorted grid locations in Eq. (1)]:

$$[{}_2\vec{p}_a | {}_2\vec{q}_a] = (\vec{V}_R^{55}(\vec{\zeta}_{a=0}, \vec{\xi}_{a=0}))^+ [{}_2\vec{s}_a | {}_2\vec{t}_a] \quad \forall a \in \alpha, \quad (3)$$

yielding the least-squares solution $({}_2\vec{u}, {}_2\vec{v})$.

II. B.3. Method 3—As a point of comparison for Method 2, which relies on trajectory-smoothed ring locations as inputs to the global warping problem, we considered the distortion models that result from fitting ${}_1\vec{p}$ and ${}_1\vec{q}$ with polynomials of the form of Eq. (2):

$$[\vec{\pi}_{ij}|\vec{\varphi}_{ij}] = (\vec{V}_A^3(\alpha))^+ [{}_1\vec{p}_{ij}|{}_1\vec{q}_{ij}] \quad \forall i \in \{0, \dots, 5\}, j \in \{0, \dots, 5\}, \quad (4)$$

where $\pi \rightarrow_{ij}$ is the vector of coefficients for the polynomial fitting ${}_1\vec{p}_{ij}$, and $\varphi \rightarrow_{ij}$ the coefficients for ${}_1\vec{q}_{ij}$ [note that in Eq. (4), the value of A depends on the sampling density used]. The least-squares solutions of the polynomial coefficient fitting were denoted ${}_3\vec{p}$ and ${}_3\vec{q}$, and these were used to calculate the least-squares solution of the distorted ring locations $({}_3\hat{u}, {}_3\hat{v})$. This is similar to the work described by Liu *et al.*,¹⁴ except that where they used high (6th) order polynomials to fit the distortion correction coefficients, we deliberately chose the same low (3rd) order fit as used in Method 2, and as such the fitted distortion correction coefficients did not capture the fluctuations of the original coefficients. In this case, the residual errors of the distortion correction were evaluated relative to $(\vec{s}, \vec{t})$, as for Method 2.

II. C. Generation of sparsely sampled distortion correction polynomials

In order to determine the sampling density necessary to characterize the distortion over the range of C-arm orientations, serially smaller subsets of the acquired single arc images were used. The fully sampled data had $\Delta\alpha=5^\circ$; subsets with $\Delta\alpha=\{10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ\}$ were generated by keeping every 2nd, 3rd, 4th, 5th, and 6th images, respectively, always ensuring that the subset of α was centered about 0° (note that for $\Delta\alpha=\{15^\circ, 30^\circ\}$ the range of sampled α was $+45^\circ$, rather than $\pm 50^\circ$). These subsampled data were then used as the starting point for Methods 2 and 3 of the distortion correction polynomial fitting.

In examining the accuracy of the sparsely sampled distortion models from Methods 2 and 3, the residuals errors were calculated not only for the images used to determine the polynomial coefficients but also for those left out of the sampling. By the nature of Eq. (4), ${}_3\vec{p}$ and ${}_3\vec{q}$ could be evaluated at arbitrary α , allowing calculation of $({}_3\hat{u}, {}_3\hat{v})$ at these arbitrary α and subsequent calculation of the residual errors; it was desired to do the same for Method 2. Examination of ${}_2\vec{p}$ and ${}_2\vec{q}$ showed these to be smooth functions which could be very well characterized by low-order polynomials as a function of α , as in Eq. (4). In this way, the residual errors of Method 2 could also be calculated for all images including those not used in modeling the distortion.

II. D. Evaluation on test images

After acquiring the images used to model the distortion correction, the grid was rotated and translated on the input window of the II by an arbitrary amount, and a new series of ten images was acquired at arbitrary but known α . These images were corrected using the polynomials determined by Methods 2 and 3 for all $\Delta\alpha$, and residuals were measured with respect to the ideal grid for this new grid orientation. The rotation and translation of the ideal grid was again inferred from the central beads of an image acquired with $\alpha=0^\circ$, but the pixel spacing of the ideal grid was forced to be the same as that used in the previous sections. The accuracy of the distortion correction was assessed by measuring the residual error relative to this new ideal grid.

II. E. Extension to multiple dimensions

Based on the results from the images on a single arc of α , another set of 75 grid images was acquired over a range of primary angles, secondary angles, and source-to-intensifier distances to encompass the full range of C-arm orientations necessary for the cardiovascular interventions in which we are interested. Sparsely sampled distortion correction polynomials were generated from these images, with $\Delta\alpha=22.5^\circ$, $\Delta\beta=18^\circ$, and $\Delta\lambda=10$ cm. These sampling

frequencies were determined by examining the results of Sec. II D. Again, a set of ten test images at arbitrary but known (α, β, λ) were also acquired, and residuals were measured.

Using Method 2, the ring trajectories were fit with polynomials of the form

$$[\vec{\sigma}_r | \vec{\tau}_r] = (\vec{V}_{74}^{332}(\alpha, \beta, \lambda))^+ [\vec{u}_r | \vec{v}_r] \quad \forall r \in \{0, \dots, R\}, \quad (5)$$

resulting in the least-squares solution $(\vec{s}, \vec{t})$. The distortion of these trajectory-smoothed rings was then modeled by

$$[{}_2\vec{p}_a | {}_2\vec{q}_a] = (\vec{V}_R^{55}(\vec{\zeta}_{\alpha=0}, \vec{\xi}_{\alpha=0}))^+ [\vec{s}_{abl} | \vec{t}_{abl}] \quad \forall a \in \alpha, b \in \beta, l \in \lambda. \quad (6)$$

As described in Sec. II C, ${}_2\vec{p}$ and ${}_2\vec{q}$ were smooth functions of the C-arm orientation parameters, and they were interpolated at arbitrary (α, β, λ) using polynomials of the form of Eq. (5). Method 3 was also evaluated for this dataset by fitting the distortion according to Method 1, then smoothing the resultant, rapidly fluctuating polynomial coefficients by the same manner as the trajectories [Eq. (5)].

The order of the polynomials for the β dimension was chosen to be the same as for α due to the similar nature of the primary and secondary angles. For the same reason, the sampling frequency for β was chosen to be similar to that for α ; however, the achievable set of C-arm configurations when all three parameters were varied limited the range of angles. This combined with the five samples of α used in Sec. II C resulted in the sampling frequency $\Delta\beta = 18^\circ$. For λ , our previous experience was that distortion was not significantly dependent on this parameter, and we made an *a priori* decision to use linear fitting. Choosing to have one more sample than the minimum for the linear fit resulted in the three samples with $\Delta\lambda = 10$ cm.

III. RESULTS

III. A. Imaging

In order to compare our results to those of other authors, we first ensured that our ring location data were determined in a similar fashion and with similar results. From the 40 images acquired at a constant C-arm orientation, the standard deviation of the ring segmentation was measured as 0.011 pixels. This is similar to the segmentation errors reported previously.⁹

Figure 2 is a minimum intensity projection of the 21 images acquired of the grid phantom over the $\pm 50^\circ$ arc of α . The trajectories of the rings are apparent and appear to be quite smooth. This smooth appearance suggests the possibility to characterize the distortion with a sparser sampling than previously described.

III. B. Trajectory fitting of segmented rings (Method 2)

Figure 3(A) highlights a region in the upper left of Fig. 2 and shows the segmented ring locations $(\vec{u}, \vec{v})$ as circles ($^\circ$) and a 3rd-order polynomial fit $(\vec{s}, \vec{t})$ of each of the four rings' trajectories (lines). Note from Fig. 2 that these four rings exhibit some of the most exaggerated trajectories on the II. From Fig. 3(A), it can be seen that three of the four highlighted rings have segmented locations that follow the polynomial fit very closely, while one of them (upper left) has segmented ring locations for some α that do not follow the fit as well.

Figure 3(B) shows the RMS and maximum residual errors as a function of the polynomial order ℓ used in the trajectory fitting. Here the RMS and maximum were taken from all 426 rings on all 21 acquired images. Panel C shows the RMS error as a function of primary angle α , where the RMS operation was performed over all 426 rings for a given α . Though the RMS of the 2nd-order fits can be seen to be quite low, there is a clear improvement with the 3rd-order fit. Also, the 2nd-order fit shows a distinct pattern as a function of α , whereas the higher order fits are nearly equal and seem to have no α dependence. For these, the RMS error is less than 0.05 pixels; the 3rd-order trajectory fitting was selected for subsequent experiments.

III. C. Comparison of Methods 1 and 2 with full sampling

Figures 4(A) and 4(B) show the 36 coefficients of the distortion correction polynomials in the u and v directions determined by both Method 1 and Method 2 for the fully sampled data. As previously described by Fahrig *et al.*,⁹ the coefficients determined by Method 1 (thin line) can be seen to vary greatly as a function of α . On the other hand, in Method 2, by smoothing the trajectories of the segmented rings, the coefficients (thick line) behave much better with α for all 36×2 coefficients of the distortion model. As is seen in Fig. 4(C), the residual errors measured for Method 2 are also smoother as a function of α , and they are slightly improved from Method 1. Note, however, that the scale of this graph is very small, and the errors of Method 1 are already very low (<0.1 pixels).

III. D. Comparison of Methods 1, 2 and 3 with sparse sampling

Figure 5 compares the segmented, distorted ring locations with the modeled, distorted ring locations from the three methods. In order to clearly visualize the results, the trajectory of one ring is highlighted [this is the same ring seen in the upper left corner of Fig. 2(B)]. In panel A, the segmented ring locations ($\vec{u}, \vec{v}$) in the 21 images of the fully sampled data are shown by the circles($^\circ$). The lines in panel A are the fitted trajectories of the rings ($\vec{s}, \vec{t}$) determined for the fully sampled dataset and the five subsampled datasets. All the trajectories are virtually indistinguishable at this resolution, and they all follow the segmented ring locations quite faithfully (with the exception of the trajectory fit with the $\Delta\alpha=30^\circ$ dataset, which starts to fail at one end, though this last point on the line is an extrapolated point). It can be seen, however, (zoomed inset) that the location of the ring for one α seems to deviate from the underlying trend. The effect of the trajectory fitting in this case could be interpreted as smoothing out this data point.

Figure 5(B) shows the distortion correction results of Method 1, ($_1\hat{u}, _1\hat{v}$), for the fully sampled dataset and Method 2, ($_2\hat{u}, _2\hat{v}$), for all six sampling densities. The modeled trajectory of the ring can be seen to be nearly identical for all seven traces. Examination of the zoomed inset shows the one clear difference between ($_1\hat{u}, _1\hat{v}$) and ($_2\hat{u}, _2\hat{v}$): where the modeled trajectory of Method 1 attempts to capture the deviation of the highlighted ring, the results of Method 2 do not. Figure 5(C) compares ($_1\hat{u}, _1\hat{v}$) for the fully sampled dataset with ($_3\hat{u}, _3\hat{v}$) for the six sampling densities. The results here are nearly identical as those described Fig. 5(B).

Figure 6 displays the residual errors of the distortion correction using all three Methods, including the six sampling densities used for Methods 2 and 3. Note that at this scale, the results for Methods 2 and 3 are identical, and they are slightly better than the results for Method 1 for all sampling densities except $\Delta\alpha=30^\circ$.

III. E. Evaluation on test images

The ten test images were acquired with $\alpha=\{-44^\circ, -15^\circ, 31^\circ, -49^\circ, -36^\circ, -30^\circ, 27^\circ, 10^\circ, -23^\circ, 37^\circ\}$. Figure 7(A) shows the minimum intensity projection of the test images after they were corrected using the polynomials modeled with the sampling density $\Delta\alpha=30^\circ$. Comparing this figure with Fig. 2(A) shows that the gross angular dependence of distortion has been removed.

Also, as is highlighted by the superimposed lines, the rings now fall on a regular grid. Figure 7(B) displays the residual errors of correction of about 0.22 pixels for the test images, after correction with the polynomials of all six sampling densities. Also shown on this graph is the number of images used to characterize the distortion for each sampling density.

III. F. Sparse sampling in multiple dimensions

Table I shows the α , β and λ parameters of the ten test images used to evaluate the sparsely sampled distortion correction in multiple dimensions. Figure 8 displays the minimum intensity projection of the test images before (A) and after (B) distortion correction by Method 2. Panel A shows the larger trajectory of the uncorrected rings compared to the images acquired over a single arc of α [see Fig. 2(A)]. After distortion correction, the gross angular dependence is removed and the rings fall on a regular grid. The RMS residual error after distortion correction was 0.33 pixels for both Method 2 and Method 3.

IV. DISCUSSION

X-ray fluoroscopy systems with image intensifiers have many clinical uses, including treatment planning for brachytherapy,^{18,19} generation of 3D volumes by rotational angiography,^{20,21} and guidance of interventions such as cardiac catheterizations.²² When accurate positional information is necessary, the distortion present in the images has been corrected by a variety of techniques discussed above.^{1–10}

Our experience¹⁶ with the global method of distortion correction has yielded satisfactory results. Indeed, observation of images acquired at several C-arm orientations (Fig. 2) led us to believe that not only was the distortion global for a given image, but that it was also a smooth, slowly varying function of the C-arm orientation. We aimed to exploit this fact to characterize the distortion for the wide range of C-arm orientations used during interventional procedures without the need to sample the distortion as densely as had been previously published. Cosby and Leszczynski¹⁹ had a similar goal, since their application of computer-aided radiation therapy simulation also requires the flexibility to position the II in arbitrary orientations. In their method, correction coefficients were interpolated for arbitrary C-arm orientations as a weighted mean of coefficients at the nearest neighbors of acquired calibration images. Although they too used a global approach to distortion correction, they employed a polynomial with significantly fewer coefficients (10) than the method described by Fahrig *et al.* and Gronenschild, and as such their published results were not as low, leading us to follow the work of the latter authors. It should also be noted that two groups^{23,24} have designed devices that attach to the II for online correction at arbitrary C-arm orientations with near real-time modeling of the distortion. These approaches are robust and general, although they may add time to procedures, and the devices may not always be compatible if they present space problems that interfere with the patient or operator.

The observation by Fahrig *et al.* that the polynomial coefficients fluctuated greatly with α , which seemed contrary to the smooth appearance of the composite image in their Fig. 1(B) [as well as our Fig. 2(A)] led us to believe that the source of the fluctuations was an attempt to capture the types of deviations highlighted in the zoomed inset of Fig. 5. This is because the distortion correction polynomials have a large number of degrees of freedom which can characterize not only the true distortion but also noise or non-reproducible deviations (such as might arise from vibrations in the C arm). This same idea was described by Gronenschild (see Ref. 11, Sec. IV A), who argued that the optimal degree of the correction polynomial can therefore not be known beforehand, but that it must rather be determined by statistical evaluation. Figures 5(A) and 5(B) show that the effect of inputting the smoothed trajectories of Method 2 to the global method results in distortion that ignores the small, possibly non-reproducible deviations. The smooth nature of the correction coefficients from Method 2

suggests the possibility of sparse sampling of the C-arm orientations, since the now smooth coefficients can be very well characterized with a few number of points fit to a low-order polynomial.

A more interesting result was the fact that directly smoothing the original polynomial coefficients, as done in Method 3, yielded practically the same results as Method 2 (Figs. 5(B), 5(C), and 6), even though this smoothing returns distortion correction polynomial coefficients ($_{3p}, _{3q}$) which do not capture the fluctuations in the original coefficients. However, closer inspection of the coefficient traces (Fig. 4, thin lines) reveals underlying trends that are not immediately obvious. Although the values of the coefficients fluctuate widely, it can be seen that the traces of coefficients on a diagonal have very similar shapes (their magnitudes differ due to the different powers of the terms the coefficients scale). Neighboring (non-diagonal) coefficients have traces that run nearly opposite. Given these observations, it is understandable that smoothing of these coefficients could yield an acceptable result.

Another approach of comparing Methods 2 and 3 is to consider the fitting of the ring trajectories ($\vec{s}, \vec{t}$) as a low-pass filter applied in the image domain to the input data of the distortion model. The smoothing of the correction coefficients in Method 3 takes the same form as the trajectory fitting (see Fig. 1, dashed boxes). In other words, the low-pass filter which was applied in the image domain in Method 2 was the same one applied to the correction coefficients in Method 3, returning practically identical results in both cases.

Considering the similarity of the results from Methods 2 and 3, the preferred approach is Method 3, since this does not require identifying corresponding rings in the images acquired at different α . Instead the correction polynomials are determined by the same method described by Fahrig *et al.* and Gronenschild, then the values of the coefficients as functions of the C-arm orientation are smoothed by low-order polynomials. These low-order polynomials allow for sparse sampling of the range of C-arm orientations with satisfactory results at arbitrary views (Figs. 7 and 8). Finally, considering both these results and those of Cosby and Leszczynski suggests that an alternate approach is simply to employ polynomials of an intermediate order such that all the components of the distortion are characterized without being influenced by small variations in the input data.

The recent emergence of flat panel detectors offers the possibility of distortion-free x-ray fluoroscopy.²⁵ Where available, it seems that these systems should clearly be used for applications in which accurate positional information is important; however, there exists a large base of installed II-based systems, and the adoption of these new systems is costly and will likely take some time before they are the predominant type. In the meantime, novel applications will continue to be developed for II-based systems, explaining the continued need for distortion correction schemes.

V. CONCLUSIONS

We have shown that image intensifier distortion correction for a large range of C-arm orientations can be determined by acquiring grid images at a sparse sampling of these angles and fitting their resulting distortion correction polynomial coefficients to a lower order polynomial as a function of the C-arm orientation parameters. We characterized the distortion over a range of primary angles, α , of $\pm 45^\circ$, secondary angles, β , $\pm 36^\circ$, and source-to-intensifier distance, λ , of 98–118 cm with only 75 images ($\Delta\alpha=22.5^\circ$, $\Delta\beta=18^\circ$, $\Delta\lambda=10$ cm). This resulted in residuals, measured on test images acquired at arbitrary (α, β, λ) , of 0.33 pixels, and corrected grid images showed no gross angular-dependent rotation or translation and had grid vertices that fell on a regular grid. This may allow for use of distortion-corrected images at the large

range of C-arm orientations necessary for applications such as guidance of interventions or treatment planning for brachytherapy.

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APPENDIX: POLYNOMIAL NOTATION

A n th-order polynomial of one variable, x , can be written as

$$\mathbf{P}(x) = c_0 + c_1 x^1 + \dots + c_n x^n = \sum_{i=0}^n c_i x^i, \quad (\text{A1})$$

where n is the degree of the polynomial $\mathbf{P}$ and the constants $c_i \forall i \in \{0, \dots, n\}$ are the coefficients of $\mathbf{P}$.

Given a set of $m+1$ points (x_i, z_i) , with unique $x_i \forall i \in \{0, \dots, m\}$, the polynomial $\mathbf{P}$ can be said to interpolate the data points z_i if

$$\mathbf{P}(x_i) = z_i \quad \forall i \in \{0, \dots, m\}. \quad (\text{A2})$$

Combining Eq. (A1) and Eq. (A2) yields the system of equations, written in matrix form as

$$\begin{bmatrix} x_0^0 & x_0^1 & \dots & x_0^n \\ x_1^0 & x_1^1 & \dots & x_1^n \\ \vdots & \vdots & \ddots & \vdots \\ x_m^0 & x_m^1 & \dots & x_m^n \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} z_0 \\ z_1 \\ \vdots \\ z_m \end{bmatrix}, \quad (\text{A3})$$

or more compactly as

$$\vec{V}_m^n(\vec{x}) \vec{c} = \vec{z}, \quad (\text{A4})$$

where $\vec{V}_m^n(\vec{x})$ is the Vandermonde matrix of $x \rightarrow$, $c \rightarrow$ is the vector of polynomial coefficients, and $z \rightarrow$ is the vector of data points to interpolate. When $m=n$, this system of equation is uniquely determined, and the coefficients $c \rightarrow$ can be calculated by solving

$$\vec{c} = (\vec{V}_m^n(\vec{x}))^{-1} \vec{z}, \quad (\text{A5})$$

where the superscript -1 refers to the inverse. However, in most experimental cases, the measured $z \rightarrow$ are noisy and extra data are acquired, making $m > n$ such that the system is over determined, and the coefficients of the polynomial modeling the data can be calculated by obtaining the least-squares solution $\hat{z}$, using the pseudoinverse:

$$\vec{c} = (\vec{V}_m^n(\vec{x}))^+ \vec{z} \quad (\text{A6})$$

$$\hat{z} = \vec{V}_m^n(\vec{x}) \vec{c}, \quad (\text{A7})$$

where the superscript $+$ refers to the pseudoinverse and $\hat{z}$ is the vector of interpolated data points. The interpolation errors (sometimes referred to as the residuals) are

$$\vec{e} = \hat{z} - \vec{z}, \quad (\text{A8})$$

and a commonly used scalar measure of this error is the root-mean-square (RMS) of the error,

$$|\vec{e}|_{RMS} \equiv \sqrt{\frac{1}{m} \sum_{i=0}^m \|e_i\|^2}. \quad (\text{A9})$$

This type of problem can be extended to two dimensions by introducing the two-dimensional polynomial

$$\mathbf{P}(x,y) = \sum_{j=0}^{n_y} \sum_{i=0}^{n_x} c_{ij} x^i y^j \quad (\text{A10})$$

and the equivalent of the Vandermonde matrix for the two-dimensional polynomial system of equations is

$$\vec{V}_m^{n_x n_y}(\vec{x}, \vec{y}) = \begin{bmatrix} x_0^0 y_0^0 & x_0^1 y_0^0 & \dots & x_0^{n_x} y_0^0 & x_0^0 y_0^1 & x_0^1 y_0^1 & \dots & x_0^{n_x} y_0^1 & \dots & x_0^0 y_0^{n_y} & x_0^1 y_0^{n_y} & \dots & x_0^{n_x} y_0^{n_y} \\ x_1^0 y_1^0 & x_1^1 y_1^0 & \dots & x_1^{n_x} y_1^0 & x_1^0 y_1^1 & x_1^1 y_1^1 & \dots & x_1^{n_x} y_1^1 & \dots & x_1^0 y_1^{n_y} & x_1^1 y_1^{n_y} & \dots & x_1^{n_x} y_1^{n_y} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_m^0 y_m^0 & x_m^1 y_m^0 & \dots & x_m^{n_x} y_m^0 & x_m^0 y_m^1 & x_m^1 y_m^1 & \dots & x_m^{n_x} y_m^1 & \dots & x_m^0 y_m^{n_y} & x_m^1 y_m^{n_y} & \dots & x_m^{n_x} y_m^{n_y} \end{bmatrix}, \quad (\text{A11})$$

which operates on the coefficient vector

$$\vec{c} = \left[\begin{array}{cccc|cccc|ccc} c_{00} & c_{10} & \dots & c_{n_x 0} & c_{01} & c_{11} & \dots & c_{n_x 1} & \dots & c_{0n_y} & c_1^{n_y} & \dots & c_{n_x n_y} \end{array} \right] \quad (\text{A12})$$

Fig. 1.

Schematic diagram of the steps of the three methods of distortion correction. Method 1, which is the previously published method of global distortion correction, solves the coefficients of the distortion model by inverting the Vandermonde matrices of Eq. (1); the model returns the modeled ring locations, $(\vec{u}_1, \vec{v}_1)$. Method 2 modifies this by smoothing the input ring locations with 3rd-order polynomials of α . Method 3 applies the smoothing as a function of α directly to the coefficients determined in Method 1. Note that the smoothing functions in Methods 2 and 3 are of the same form (dashed box), though one acts on the grid coordinates in the image domain, while the other acts on polynomial coefficients.

Fig. 2.

The angular dependence of II distortion can be visualized by taking a minimum intensity projection of images acquired every 5° along an arc of primary angles from -50° to 50° . On any individual image, the rings of the grid phantom do not fall on a regular grid; additionally, images at different C-arm orientations have different amounts of in-plane rotation and translation, resulting in the pattern seen here.

Fig. 3.

(A) A closer look at the highlighted part of the image in Fig. 2 shows the smooth shape of the angular-dependent distortion pattern. The circles ($\circ$) show the ring locations ($\vec{u}$, $\vec{v}$) as they were segmented in each image, and the line is a two-dimensional 3rd-order polynomial ($\vec{s}$, $\vec{t}$) fitted to these points. (B) The RMS (solid line) and maximum (dashed line) errors of the ring locations as a function of the order ℓ of the trajectory-fitting polynomials $\vec{\sigma}$ and $\vec{\tau}$. (C) The RMS error of the trajectory-fitting polynomials as a function of primary angle. Note that the 2nd-order polynomial shows a distinct pattern, suggesting that it does not fully characterize the ring

trajectories, while the higher order polynomials are virtually equivalent, with RMS errors <0.05 pixels.

Fig. 4.

The 36 coefficients of the distortion correction polynomials in the u direction (A) and v direction (B) determined using Method 1 (thin line) and Method 2 (thick line). In all cases, Method 2 results in coefficients that exhibit dramatically decreased variability as a function of primary angle (the abscissa of each axis represents the primary angle, from -50° to 50°). (C) The residual errors of the modeled ring locations using the distortion correction polynomials shown in panels A–B.

Fig. 5.

(A) The 3rd-order polynomial fits $(\vec{s}, \vec{t})$ of the ring trajectories are shown for all six sampling frequencies, $\Delta\alpha = \{5^\circ, 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ\}$. With the exception of the $\Delta\alpha = 30^\circ$ curve, the trajectories are nearly indistinguishable. The circles ($\circ$) represent the segmented ring locations $(\vec{u}, \vec{v})$. The inset highlights 1 pixel containing a ring location which shows an occasional deviation from the underlying trend, but this is partly corrected by the trajectory-fitting polynomials. (B) The modeled ring locations $({}_2\hat{u}, {}_2\hat{v})$ determined with Method 2 were calculated for the six sampling frequencies (solid lines). Again, the curves are very similar. This panel also shows the modeled ring locations $({}_1\hat{u}, {}_1\hat{v})$ determined by Method 1 (dashed

line). This curve matches the others in most places, except that it follows the deviation (inset).
(C) The curves of $({}_3\hat{u}, {}_3\hat{v})$ are shown as for panel B.

Fig. 6.

The residual errors of the modeled ring locations for all three methods of distortion correction are presented, including all six sampling densities evaluated for Methods 2 and 3. Note that at this scale, the results for Methods 2 and 3 are identical.

Fig. 7. (A) The minimum intensity projection of distortion-corrected test images is shown for the $\Delta\alpha=30^\circ$ sampling density. The relative rotation and translation of the images due to the angular dependent distortion can be seen to be corrected. Also, the superimposed lines show that the rings now fall on a regular grid. (B) The RMS residual errors of the distortion-corrected test images are shown for all six sampling densities, and the curve is nearly flat, with the RMS errors about 0.22 pixels.

Fig. 8. The minimum intensity projection of the ten test images acquired at arbitrary (α, β, λ) , before (A) and after (B) distortion correction. Panel A shows the larger trajectory of the uncorrected rings compared to the images acquired over a single arc of α [see Fig. 2(A)]. After distortion correction based on only 75 images, the angular dependence is removed and the rings fall on a regular grid. The RMS residual error of distortion correction was 0.33 pixels.

Table I

Parameters of the test images used to evaluate the sparsely sampled distortion correction in multiple dimensions.

α (°)	β (°)	λ (cm)
-28	0	115
16	29	98
-18	23	112
4	10	106
-31	23	115
18	12	108
-11	-11	112
32	-15	107
32	-11	104
8	2	102