On the application of the lattice Boltzmann method to the investigation of glottal flow

Bogdan R. Kucinski^{a)} and Abdollah A. Afjeh

Mechanical, Industrial and Manufacturing Engineering Department, University of Toledo, Ohio 43606

Ronald C. Scherer^{b)}

Department of Communication Disorders, Bowling Green State University, Ohio 43403

(Received 7 September 2007; revised 8 April 2008; accepted 17 April 2008)

The production of voice is directly related to the vibration of the vocal folds, which is generated by the interaction between the glottal flow and the tissue of the vocal folds. In the current study, the aerodynamics of the symmetric glottis is investigated numerically for a number of static configurations. The numerical investigation is based on the lattice Boltzmann method (LBM), which is an alternative approach within computational fluid dynamics. Compared to the traditional Navier–Stokes computational fluid dynamics methods, the LBM is relatively easy to implement and can deal with complex geometries without requiring a dedicated grid generator. The multiple relaxation time model was used to improve the numerical stability. The results obtained with LBM were compared to the results provided by a traditional Navier–Stokes solver and experimental data. It was shown that LBM results are satisfactory for all the investigated cases.

© 2008 Acoustical Society of America. [DOI: 10.1121/1.2924137]

PACS number(s): 43.70.Bk, 43.70.Aj [BHS]

Pages: 523–534

I. INTRODUCTION

The production of voice is related to the quasiperiodic motion of the vocal folds, which is primarily a fluid-structure interaction process. This process is determined by both the biomechanical characteristics of the vocal fold tissue¹ and aerodynamic pressures.² It is accepted that the normal phonation involves a symmetrical motion of the vocal folds, where the motion of one vocal fold mirrors the motion of the other.^{3,4} The symmetry of the glottal motion permits simplifications of the theoretical and empirical models used for the study of phonation. There are, however, situations when the glottis is asymmetric (oblique), associated with either normal or pathological conditions.^{5–8} The study of the glottal flow and the resulting glottal wall pressures is important since it constitutes the driving force of the glottal vibration. Although the motion of the vocal folds is complex, a number of simplifications can be used to reduce the complexity of the models, as will be shown in Sec. II.

The present work deals with the numerical prediction of the flow through different static symmetrical configurations of the glottis. The approach proposed here is to use a relatively new computational method, namely, the lattice Boltzmann method (LBM). In the last two decades, LBM has emerged as a viable computational fluid dynamics technique. Compared to the Navier–Stokes solvers, the LBM presents the advantage of simplicity of computational algorithms, which are also suitable for parallelization. Also, it can handle efficiently complex geometric configurations, a necessary condition for the analysis of the glottal flow. Traditionally,

the numerical simulations of glottal flow are based on solving the Navier–Stokes equations for both static^{8–10} and moving-wall^{11–13} models.

Recently, the LBM solvers have become serious competitors to conventional Navier–Stokes solvers¹⁴ for a wide diversity of flows, including compressible and thermal flows.^{15,16} Also, progress was recently made toward aeroacoustic simulations based on LBM.^{17,18}

According to the authors' knowledge, the LBM method has not been used for the study of the glottal flow. The unsteady compressible nature of LBM makes this method potentially attractive for the investigation of phonatory acoustics. Although in the present work only static configurations were considered, the LBM method can be applied to moving boundaries.^{19–21} The LBM appears particularly convenient for the flow within a moving glottis since it does not use a deforming mesh, while the Navier–Stokes solvers require the deformation or regeneration of the computational cells (a challenging process for glottal geometries¹³).

II. LARYNGEAL GEOMETRY

The actual human glottis is a three-dimensional airway with two primary portions. The membranous, or anterior, glottis extends from the vocal processes of the arytenoid cartilages to the anterior commissure, while the posterior, or cartilaginous, glottis extends from the vocal processes to the posterior mucosal wall between the two arytenoid cartilages.²² The present study focuses on the membranous glottis, where the normal coronal configuration during phonation alternates between converging and diverging shapes, during the glottal opening and closing.³ This change of shape is generated by the wall pressures, the morphological structure of the vocal folds, and the dynamics of the fluid-structure interaction process. The anterior-posterior shape of

^{a)}Electronic mail: bkucinsc@eng.utoledo.edu

^{b)}Electronic mail: ronalds@bgsu.edu

FIG. 1. The geometrical configuration of the larynx model.

the membranous glottis is not uniform during phonation, such that the glottis opens wider near the middle and tapers toward the anterior and posterior ends. Although phonation involves three-dimensional moving configurations, much simpler, two-dimensional static empirical models are commonly used for pressure measurements. The two-dimensional character of the flow is obtained by typically allowing for a uniform anterior-posterior dimension. The use of static models is justified by the so-called *quasisteady approximation*, which states that the flow physics for a certain static configuration satisfactorily approximates the flow physics in an identical configuration when the vocal folds are moving with the relatively low speeds typical for phonation. A number of studies have reasonably shown the validity of this assumption.^{13,23}

The static, symmetric laryngeal configurations analyzed in the present study are based on the two-dimensional geometry of the experimental model M5, reported by Scherer and co-workers.^{8,10} This is a simplified Plexiglas model of the larynx with linear dimensions 7.5 times larger than an average normal male larynx. The larger dimensions of the model have permitted the installation of pressure taps in the mid-section of model folds for accurate wall pressure measurements. The vocal folds are replaceable, such that various glottal configurations (i.e., convergent, uniform, or divergent, with different minimum glottal openings) can be obtained.

Three representative configurations have been investigated: convergent ($\alpha = -10^\circ$), uniform ($\alpha = 0^\circ$), and diver-

gent ($\alpha = +10^\circ$) for a glottal opening (“diameter”) $d_{\min}$ of 0.04 cm. Figure 1 presents the glottal profile corresponding to the convergent geometry. The details for each configuration are shown in Table I. The x axis coincides with the glottal axis, and its origin corresponds to the abscissa of point D. The length of the inlet region L_i and exit region L_e are the same for all cases, having values of 0.2 and 0.1 cm, respectively. They have been chosen in order to decrease the computational effort, while providing a uniform flow at the inlet and a satisfactory pressure recovery downstream of the vocal folds. The use of inlet and exit regions shorter than the actual length of trachea and vocal tract, respectively, is a simplification which does not affect the predicted pressures.^{8,9,13}

There is a small difference between the configurations studied in the present paper and the M5 model. For reasons related to the application of the lattice Boltzmann method, the rectangular region in M5 around point A' in Fig. 1 was trimmed with the 45° linear segment A–M. The purpose was to avoid singular nodes in the lattice, as will be explained in Sec. III B 1. Since the experiments showed that the pressures in this region are approximately uniform and equal to the downstream pressure, this profile alteration does not introduce significant differences in the pressure distribution.

III. METHODS

The flow through the glottal configurations presented in Sec. II was investigated computationally, using both the lattice Boltzmann method and a Navier–Stokes solver (FLUENT 6.3). The numerical results were compared against the empirical data obtained with the experimental model M5. Some of these results have been reported.^{8,10,24}

A. The lattice Boltzmann method

Traditional computational fluid dynamics (CFD) methods solve the Navier–Stokes equations in order to directly determine the pressure and velocity (i.e., macroscopic variables) in the fluid. In contrast, the Boltzmann equation governs a particle distribution function $f(\mathbf{x}, \mathbf{v}, t)$, which depends on space $\mathbf{x}$, particle velocity $\mathbf{v}$, and time t such that

TABLE I. Geometric characteristics of the laryngeal model (all values are given in centimeters). The minimum glottal diameter is $d_{\min} = 0.04$ cm for all cases.

	Convergent 10°		Uniform		Divergent 10°	
	X	Y	X	Y	X	Y
A	0.3000	0.7800	0.3000	0.7800	0.3000	0.7800
B	−0.7592	0.8600	−0.7750	0.8600	−0.7750	0.8600
C	0.2092	0.1108	0.2013	0.1187	0.1920	0.1446
D	0.0000	0.1885	0.0000	0.1700	0.0000	0.1700
E	0.3000	0.1108	0.3000	0.1187	0.3000	0.1446
F	0.2013	0.0203	0.2013	0.0200	0.2014	0.0370
G	0.0131	0.0391	0.0000	0.0200	0.0131	0.0206
I	−0.1059	0.0823	−0.1149	0.0736	−0.1149	0.0736
R_1	0.15		0.15		0.15	
R_2	0.0908		0.0987		0.108

FIG. 2. The two-dimensional, nine-velocity lattice.

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = Q(f, f), \quad (1)$$

where Q is a collision operator. It has been shown that in the limit of a small Knudsen number, the Navier–Stokes equations can be recovered from the Boltzmann equation if the Chapman–Enskog expansion is used.²⁵ It is assumed that the solution of the Boltzmann equation can be expressed in the form of an infinite series,

$$f = f^{(0)} + f^{(1)} + f^{(2)} + \dots, \quad (2)$$

where the lowest order term, $f^{(0)}$, is the equilibrium distribution function, corresponding to the Maxwell–Boltzmann distribution. If only the first two terms, $f^{(0)}$ and $f^{(1)}$, are taken into account in Eq. (2), the Navier–Stokes equations are obtained. The collision operator Q in Eq. (1) has a complex form, but it can be simplified by using the Bhatnagar–Gross–Krook (BGK) approximation,²⁶

$$Q(f, f) = -\frac{1}{\lambda}(f - f^{(0)}), \quad (3)$$

where λ is the relaxation time due to collision. The relationship between the macroscopic quantities (density ρ and momentum density $\rho \mathbf{u}$) and the distribution function f is given by²⁷

$$\rho = \int f(\mathbf{x}, \mathbf{v}, t) d\mathbf{v}, \quad \rho \mathbf{u} = \int \mathbf{v} f(\mathbf{x}, \mathbf{v}, t) d\mathbf{v}. \quad (4)$$

The velocity space can be discretized into a finite set of velocities, $\mathbf{v}_i$, whose directions define a lattice (e.g., Wolf–Gladrow²⁸). The corresponding distribution functions become $f_i(\mathbf{x}, t) \equiv f(\mathbf{x}, \mathbf{v}_i, t)$. The corresponding discrete Boltzmann equation can be written as

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla f_i = -\frac{1}{\lambda}(f_i - f_i^{(0)}). \quad (5)$$

In the present paper, a two-dimensional square lattice with nine-velocity directions (Fig. 2) is used. The discrete velocities are expressed in terms of the lattice directions, $\mathbf{v}_i = c\mathbf{e}_i$, where c is the lattice speed, $c = \delta_x / \delta_t$. Here, δ_x and δ_t are the lattice space and time steps, respectively, and the lattice directions are indicated by

$$\mathbf{e}_i = \begin{cases} (0, 0), & i = 0 \\ \left(\cos\left(\frac{\pi}{2}(i-1)\right), \sin\left(\frac{\pi}{2}(i-1)\right) \right), & i = 1, 2, 3, 4 \\ \sqrt{2} \left(\cos\left(\frac{\pi}{2}\left(i - \frac{9}{2}\right)\right), \sin\left(\frac{\pi}{2}\left(i - \frac{9}{2}\right)\right) \right), & i = 5, 6, 7, 8. \end{cases} \quad (6)$$

This model²⁹ is commonly referred to as D2Q9. There are only three distinct values of speed in the lattice: 0 (“rest particle”) and c and $c\sqrt{2}$ on the rectangular and oblique directions, respectively. The method deals with individualized particles that travel between two lattice nodes during one time step, such that it has a Lagrangian character.

1. The lattice Boltzmann–BGK (LBGK) equation

The discrete Boltzmann equation can be discretized such that the LBGK equation is obtained,^{27,28}

$$f_i(\mathbf{x} + c\mathbf{e}_i\delta_t, t + \delta_t) - f_i(\mathbf{x}, t) = -\omega[f_i(\mathbf{x}, t) - f_i^{(0)}(\mathbf{x}, t)], \quad (7)$$

where $\omega = 1/\tau$ is a relaxation parameter corresponding to the dimensionless collision time, $\tau = \lambda / \delta_t$. Although this discretization is of first order, it has been shown³⁰ that at low Mach numbers, the method is second-order accurate in both space and time due to its Lagrangian nature if a suitable relationship between the dimensionless collision time and kinematic viscosity [Eq. (15)] is used. The equilibrium part of the distribution function for the nine-velocity model is given in terms of macroscopic quantities (i.e., density and velocity),

$$f_i^{(0)} = w_i \rho \left[1 + 3 \frac{\mathbf{e}_i \cdot \mathbf{u}}{c} + \frac{9}{2} \frac{(\mathbf{e}_i \cdot \mathbf{u})^2}{c^2} - \frac{3}{2} \left(\frac{\mathbf{u}}{c} \right)^2 \right]. \quad (8)$$

The weights w_i depend on the lattice direction,

$$w_i = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{1}{9}, & i = 1, 2, 3, 4 \\ \frac{1}{36}, & i = 5, 6, 7, 8. \end{cases} \quad (9)$$

The LBGK equation can be regarded as containing two steps: a local collision relaxation step, followed by a propagation on the direction i of the lattice (also referred to as “streaming step”). Equation (7) can thus be written as

$$f_i^+(\mathbf{x}, t) = (1 - \omega)f_i(\mathbf{x}, t) + \omega f_i^{(0)}(\mathbf{x}, t),$$

$$f_i(\mathbf{x} + c\mathbf{e}_i\delta_t, t + \delta_t) = f_i^+(\mathbf{x}, t), \quad (10)$$

where the first equation constitutes the collision step (the “+” subscript refer to the postcollision value of the distribution function), while the second equation describes the propagation. The collision process is strictly local since it involves only f_i at the current site. The relationship between the lattice speed c and the speed of sound c_s is particular for each type of lattice. For the square lattice used in the present work, it has been shown²⁸ that $c_s^2 = c^2/3$.

It is customary in the literature dedicated to LBM to use dimensionless notations, such that the (dimensionless) space

and time step, as well as the lattice speed c , become equal to unity. The propagation part of Eq. (10) may thus be written as

$$f_i(\mathbf{x} + \mathbf{e}_i, t + 1) = f_i^+(\mathbf{x}, t). \quad (11)$$

The fluid density and velocity are obtained from the distribution function by applying the discrete form of Eq. (4),

$$\begin{aligned} \rho &= \sum_i f_i, \\ \rho \mathbf{u} &= \sum_i c \mathbf{e}_i f_i. \end{aligned} \quad (12)$$

For low velocity flows, characterized by a low Mach number, the density variations are small. If the density is expressed as $\rho = \rho_0 + \delta\rho$, the density fluctuations should be of the order $O(M^2)$ as the $M \ll 1$. By neglecting the terms of the equilibrium density functions that involve higher order dependency on M , He and Luo³¹ have proposed an *incompressible* LBM model [sometimes referred to as ID2Q9 (Ref. 32)] characterized by

$$f_i^{(0)} = w_i \left\{ \rho + \rho_0 \left[3 \frac{\mathbf{e}_i \cdot \mathbf{u}}{c} + \frac{9}{2} \frac{(\mathbf{e}_i \cdot \mathbf{u})^2}{c^2} - \frac{3}{2} \left(\frac{\mathbf{u}}{c} \right)^2 \right] \right\}. \quad (13)$$

He and Luo recommend that $M < 0.15$ in numerical simulations. The model is not truly incompressible, and the pressure is related to the density by $\delta p = c_s^2 \delta \rho$. The incompressible Navier–Stokes equations are recovered from the incompressible lattice Boltzmann model by using the Chapman–Enskog procedure,

$$\begin{aligned} \frac{1}{c_s^2} \frac{\partial P}{\partial t} + \nabla \cdot \mathbf{u} &= 0, \\ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\nabla P + \nu \nabla^2 \mathbf{u}, \end{aligned} \quad (14)$$

where $P = c_s^2 \rho / \rho_0$ is the normalized pressure. The kinematic viscosity ν depends on the dimensionless collision time, as well as on the space and time steps,

$$\nu = \frac{(2\tau - 1)}{6} \frac{\delta_x^2}{\delta_t}. \quad (15)$$

In the case of the incompressible model, the fluid density and velocity are calculated as

$$\begin{aligned} \rho &= \sum_i f_i, \\ \rho_0 \mathbf{u} &= \sum_i c \mathbf{e}_i f_i. \end{aligned} \quad (16)$$

2. The multiple relaxation time (MRT) model

The dimensionless collision time τ is always greater than 0.5, but numerical instabilities may occur for the classic LBGK method for values very close to 0.5. Small values of τ are typically obtained for practical grids if the fluid presents a small kinematic viscosity (as in the case of air). Two approaches can be used to address this problem. One ap-

proach (proposed by Guo *et al.*³³) consists of improving the discretization of the Boltzmann equation, regarded as a partial differential equation. This has the effect of introducing an artificial viscosity at the cost of significantly increasing the complexity of the implementation (a three-node stencil is required for the propagation step, instead of the classic two-node stencil). The second approach recognizes that the limitations of LBGK stem from the single-relaxation time. A generalized lattice Boltzmann equation based on multiple relaxation times has been proposed by d’Humières,³⁴ where the collision is performed in moment space rather than in discrete velocity space. Lallemand and Luo³⁵ have investigated the stability of the modified method (as well as its dispersion and dissipation) and found it superior to the classic method due to the separation of the relaxation of the kinetic modes. This MRT model will be briefly outlined as follows, by contrast with the classic model which is sometimes referred to as the single-relaxation time (SRT) model.^{15,36} The distribution functions can be written as a vector in the $\mathbb{R}^9$ vector space,

$$\{f\} = \{f_i | i = 0, 1, \dots, 8\}. \quad (17)$$

A new set of variables corresponding to the moments of $\{f\}$ can be defined, and a vector $\{\Xi\}$ can be constructed in the $\mathbb{R}^9$ vector space,

$$\{\Xi\} = (\rho, e, \varepsilon, j_x, q_x, j_y, q_y, p_{xx}, p_{xy})^T, \quad (18)$$

where e is the energy, ε is related to the square of the energy, j_x and j_y are momentum densities, q_x and q_y correspond to the energy flux, while p_{xx} and p_{xy} correspond to the components of the viscous stress tensor. A transformation matrix,

$$[M] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \end{bmatrix}, \quad (19)$$

exists such that

$$\{\Xi\} = [M]\{f\}. \quad (20)$$

The collision in the MRT model is similar in form with that for SRT [see Eq. (10)], but with different relaxation parameters, s_k ,

$$\Xi_k^+ = \Xi_k - s_k(\Xi_k - \Xi_k^{(0)}), \quad (21)$$

with $k=0, \dots, 8$. Indeed, if the distribution function is taken as a vector in the $\mathbb{R}^9$ vector space, then Eq. (7) becomes

$$\{f(\mathbf{x} + \mathbf{e}_i, t + 1)\} = \{f(\mathbf{x}, t)\} - [\Omega](\{f(\mathbf{x}, t)\} - \{f^{(0)}(\mathbf{x}, t)\}), \quad (22)$$

where $[\Omega]$ is a 9×9 diagonal relaxation matrix for the D2Q9 lattice,

$$[\Omega] = \text{diag}(\omega, \omega, \dots, \omega).$$

In the case of the MRT model, the collision is executed in moment space while the propagation is still executed in discrete velocity space. The transformation between the velocity space and the moment space is contained in the relationship

$$\{f(\mathbf{x} + \mathbf{e}_i, t + 1)\} = \{f(\mathbf{x}, t)\} - [\Omega_{\text{MRT}}](\{f(\mathbf{x}, t)\} - \{f^{(0)}(\mathbf{x}, t)\}), \quad (23)$$

where $[\Omega_{\text{MRT}}] = [M]^{-1}[S][M]$ and $[S]$ is the diagonal relaxation matrix in the moment space,

$$[S] = \text{diag}(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8). \quad (24)$$

The conserved quantities (i.e., the quantities that are not changed by the collision process) in the athermal MRT model are the density ρ and the linear momentum $\mathbf{j} = (j_x, j_y)$, corresponding to the parameters with indices 0, 3, and 5, respectively, in Eq. (24). Thus, $s_0 = s_3 = s_5 = 0$. It can be shown that $s_7 = s_8 = \omega = 1/\tau$. The rest of the parameters can be determined based on an analysis of the modes of the linearized lattice Boltzmann equation, as indicated by Lallemand and Luo.³⁵ In the computational code used for the present work, the values indicated by Yu³⁶ were used, i.e., $s_1 = s_2 = s_4 = s_6 = 1.2$. The evolution equation of the MRT model can be written as

$$\{f(\mathbf{x} + \mathbf{e}_i, t + 1)\} = \{f(\mathbf{x}, t)\} - [M]^{-1}[S](\{\Xi\} - \{\Xi^{(0)}\}). \quad (25)$$

For computational efficiency,³⁷ the equilibrium values $\{\Xi^{(0)}\}$ are calculated explicitly in the moment space based on Eq. (20). Thus, $\Xi_0^{(0)} = -2\rho + 3\rho_0(u^2 + v^2)$, $\Xi_2^{(0)} = \rho - 3\rho_0(u^2 + v^2)$, $\Xi_4^{(0)} = -\rho u$, $\Xi_6^{(0)} = -\rho v$, $\Xi_7^{(0)} = -\rho(u^2 - v^2)$, and $\Xi_8^{(0)} = -\rho uv$, where ρ_0 is taken as unity. The computational speed is substantially improved by replacing the matrix multiplications with explicit programming. The MRT model increases the computational effort^{35,37} by about 10%–20%. The collision locality is not altered, while the propagation requires only the closest neighboring node.

B. Boundary and initial conditions

1. Wall boundary conditions

When solving the Navier–Stokes equations, a solid wall is specified by imposing no-slip boundary conditions for velocity $\mathbf{u}$ at the macroscopic level. For LBM, there is no corresponding physical boundary condition³⁸ for f . Traditionally, the so-called “bounce-back” condition is used at the walls, which was inherited from the lattice-gas cellular automata.²⁸ It can be intuitively interpreted as the back-scattering of a particle initially traveling along the direction $\mathbf{e}_i$ on the opposite direction, $\mathbf{e}_{-i} = -\mathbf{e}_i$, after encountering a wall. It was shown that the bounce-back boundary conditions may yield second-order accurate results³⁸ for straight walls. Several methods have been proposed for curved walls.^{19,38,39}

FIG. 3. Illustration of the linear BFL wall boundary conditions.

In the present work, the interpolation method due to Bouzidi *et al.*¹⁹ was applied. This method (referred to henceforth as “BFL”) assumes that the wall passes in between two lattice nodes. It is superior in terms of stability to other methods based on extrapolation and can be applied for both fixed and moving boundaries.²⁰ The BFL method has been recently used for the investigation of flow through constricted vascular tubes.⁴⁰ The concept behind BFL is that during a time step, the distribution is propagated over one lattice space step even when it is backscattered by the presence of a wall. The method will be summarized below for the case of linear interpolation. The location of the wall in between lattice nodes is specified as the fraction of the link in the fluid,

$$0 \leq q = |\mathbf{x}_C - \mathbf{x}_A|/|\mathbf{x}_B - \mathbf{x}_A| \leq 1,$$

where B is a solid node. For the case $q \leq \frac{1}{2}$ [see Fig. 3(a)], the distribution f_{-i} at node A at time $t+1$ is equal (due to propagation) to the postcollision distribution f_i at the virtual node D at time t ,

$$f_{-i}(\mathbf{x}_A, t + 1) = f_i^+(\mathbf{x}_D, t).$$

The latter can be calculated by interpolation based on the values of f_i at nodes A and E ,

$$f_{-i}(\mathbf{x}_D, t) = (1 - 2q)f_i^+(\mathbf{x}_E, t) + 2qf_i^+(\mathbf{x}_A, t).$$

Thus, one can write in lattice coordinates,

$$f_{-i}(\mathbf{x}, t + 1) = (1 - 2q)f_i^+(\mathbf{x} - \mathbf{e}_i, t) + 2qf_i^+(\mathbf{x}, t), \quad q \leq \frac{1}{2}. \quad (26)$$

It can be observed that a classic bounce-back condition is recovered if $q = \frac{1}{2}$,

$$f_{-i}(\mathbf{x}, t + 1) = f_i^+(\mathbf{x}, t). \quad (27)$$

For $q > \frac{1}{2}$, the value of f_{-i} at node A at time $t+1$ is obtained by linear interpolation between the values at nodes E and D [see Fig. 3(b)],

$$f_{-i}(\mathbf{x}_A, t + 1) = \frac{(2q - 1)f_{-i}(\mathbf{x}_E, t + 1) + f_{-i}(\mathbf{x}_D, t + 1)}{2q}. \quad (28)$$

Due to the propagation process,

$$f_{-i}(\mathbf{x}_E, t + 1) = f_{-i}^+(\mathbf{x}_A, t)$$

and

$$f_{-i}(\mathbf{x}_D, t+1) = f_i^+(\mathbf{x}_A, t).$$

By using the lattice coordinates, one can write

$$f_{-i}(\mathbf{x}, t+1) = \frac{2q-1}{2q} f_{-i}^+(\mathbf{x}, t) + \frac{1}{2q} f_i^+(\mathbf{x}, t), \quad q > 1/2. \quad (29)$$

It can be noted that two nodes adjacent to the wall are necessary for BFL with linear interpolation. A more accurate, quadratic interpolation can be used.¹⁹ In the present study, the BFL scheme with quadratic interpolation was observed to be less stable than its linear counterpart. A generalization of the BFL boundary conditions was found by Ginzburg and d’Humières based on rigorous kinetic considerations. These conditions, referred to as “multireflection” (MR) boundary conditions, provide third order kinetically accurate results.²¹ Although the MR conditions were not systematically tested in the present work, some preliminary simulations resulted in numerical oscillations.

It is assumed that a wall is bordered by a sufficiently large number of fluid nodes, which signifies that the fluid is a continuous medium. In case a fluid node is located in a “corner” (such as point A' in Fig. 1), there is a single node in between two solid boundaries in one lattice direction. In order to avoid this situation, the linear segment $A-M$ was defined. For the particular investigated problem, this alteration introduces negligible modifications in the solution.

2. Pressure boundary conditions

In the case of the laryngeal flow, it is convenient to use pressure boundary conditions for both inlet and outlet since the transglottal pressure drop is typically known. It is desirable to use numerical schemes for the boundary conditions that provide both accuracy and stability. Chen *et al.*³⁰ have proposed a second-order extrapolation scheme for the distribution functions at the boundaries. The extrapolation can have a negative impact on the numerical stability. Guo *et al.*⁴¹ proposed a method based on the extrapolation of only the nonequilibrium part of the distribution functions. This method is second-order accurate, while presenting superior numerical stability. Henceforth, the nonequilibrium extrapolation method for pressure boundary conditions will be outlined for completeness. A distribution function f_i can be decomposed into its equilibrium and nonequilibrium parts, consistent with Eq. (2),

$$f = f^{(0)} + f^{(\text{NE})}. \quad (30)$$

Hence, the collision equation from Eq. (10) can be rewritten for a boundary node O as

$$f_i^+(O, t) = f_i^{(0)}(O, t) + (1 - \omega) f_i^{(\text{NE})}(O, t), \quad (31)$$

with $\omega = 1/\tau$. Let B be the neighbor node of O in the first layer of fluid immediately adjacent to the boundary. The nonequilibrium part of f can be approximated as

$$f_i^{(\text{NE})}(O, t) \approx f_i^{(\text{NE})}(B, t) = f_i(B, t) - f_i^{(0)}(B, t). \quad (32)$$

The equilibrium part in O is approximated as

$$f_i^{(0)}(O, t) \approx \tilde{f}_i^{(0)}(O, t) = \text{function}(p(O, t), \mathbf{u}(B, t)), \quad (33)$$

where $p(O, t) = p_{\text{BC}}$ is the pressure on the boundary. The boundary pressure p_{BC} is related to the density on the boundary, such that the following expression is obtained:

$$\tilde{f}_i^{(0)} = w_i \left(\frac{p_{\text{BC}}}{c_s^2} + \rho_0 \mathcal{F}_i \right), \quad (34)$$

where

$$\mathcal{F}_i = 3 \frac{\mathbf{e}_i \cdot \mathbf{u}(B, t)}{c} + \frac{9}{2} \frac{[\mathbf{e}_i \cdot \mathbf{u}(B, t)]^2}{c^2} - \frac{3}{2} \left[\frac{\mathbf{u}(B, t)}{c} \right]^2.$$

Here, the incompressible model of He and Luo³¹ was used for Eq. (33). The same approach was reported by Huang *et al.*⁴⁰ Guo *et al.*⁴¹ have used a modified form of the incompressible model developed by the same team.⁴²

In the context of the MRT model, the pressure boundary conditions can be cast in a particular form. The relationship stated in Eq. (20) applies to the corresponding nonequilibrium parts,

$$\{\Xi^{(\text{NE})}(B, t)\} = [M] \{f^{(\text{NE})}(B, t)\}, \quad (35)$$

with

$$\{\Xi^{(\text{NE})}(B, t)\} = \{\Xi(B, t)\} - \{\Xi^{(0)}(B, t)\}.$$

A relation similar to Eq. (31) can also be written for the moment vector on the boundary,

$$\Xi_k^+(O, t) = \Xi_k^{(0)}(O, t) + (1 - s_k) \Xi_k^{(\text{NE})}(O, t), \quad (36)$$

where it was assumed that $\Xi_k^{(\text{NE})}(O, t) \approx \Xi_k^{(\text{NE})}(B, t)$. It can now be seen that the following relation in the discrete velocity space can be written:

$$\{f^+(O, t)\} = \{\tilde{f}^{(0)}(O, t)\} + [M]^{-1} [D] [M] \{f^{(\text{NE})}(B, t)\}, \quad (37)$$

where $[D] = [I] - [S]$, $[I]$ is the unit matrix in $\mathbb{R}^9$, and $[S]$ is the diagonal relaxation matrix [see Eq. (24)]. Although similar in form with the boundary condition due to Guo *et al.*,⁴¹ this modified boundary condition incorporates the MRT concept and was observed to lead to an improved numerical stability.

3. Initial condition

An initial condition is necessary because the LBM is intrinsically an unsteady method. In the case of glottal flow, the steady-state solution is sought, and a convenient initial condition corresponds to the no-flow state. The distribution functions f_i are initialized with their corresponding equilibrium parts, $f_i^{(0)}$, calculated with Eq. (8) or (13) for the null velocity distribution and average density,

$$f_i^{(0)}(\mathbf{x}, t = t_0) = w_i \rho_0. \quad (38)$$

This choice is also convenient because it is incorrect to use the equilibrium values as initial values for f_i when the initial velocity field is not zero.⁴³

TABLE II. Characteristics of the computational lattices used for the three configurations (convergent 10°, uniform and divergent 10°, and minimum diameter $d_{\min}=0.04$ cm) for a speed of sound $c_s=330$ m/s.

Case	Grid N_d	Space step δ_x (10^{-5} m)	Time step δ_t (10^{-8} s)	Collision time τ (—)	Envelope grid			Fluid grid	
					N_X	N_Y	Total nodes	Fluid nodes	Wall nodes
Convergent 10°	12	3.077	5.383	0.502 559	467	561	261 987	136 047	2026
	24	1.600	2.799	0.504 921	899	1077	968 223	504 075	3898
Uniform	12	3.077	5.383	0.502 559	472	561	264 792	135 526	2026
	24	1.600	2.799	0.504 921	909	1077	978 993	503 103	3918
Divergent 10°	12	3.077	5.383	0.502 559	472	561	264 792	136 294	2036
	24	1.600	2.799	0.504 921	909	1077	978 993	505 969	3918

C. Particularities of the LBM implementation

The kinematic viscosity ν is connected to the dimensionless collision time τ , the speed of sound c_s , and the spatial and time steps δ_x and δ_t through Eq. (15) and the relationship between the speed of sound and the lattice speed c . If the space step is chosen, the time step and the dimensionless collision time are determined as

$$\delta_t = \frac{1}{\sqrt{3}} \frac{\delta_x}{c_s},$$

$$\tau = \frac{1}{2} \left(1 + 6\nu \frac{\delta_t}{\delta_x^2} \right). \quad (39)$$

As mentioned above, the numerical stability of the method is affected for values of the dimensionless collision time τ very close to 0.5. It can be observed that for a given value of ν , the space step has to be decreased in order to increase τ . For a fluid with a small kinematic viscosity (as air), the grid may become very fine, while the time step becomes very small. It can be verified that for a low kinematic viscosity, the grid size may become prohibitive if sufficiently large values of the dimensionless collision time are to be maintained. The MRT model is more stable such that reasonably sized grids can be used. However, numerical instabilities may occur during the start-up phase of the simulations if the actual (small) value of τ is used. It was found convenient to start calculations with a larger value, τ_s , of the dimensionless collision time (e.g., 0.75) and to decrease it progressively. Physically, this is equivalent to using a more viscous fluid during the start up. A parabolic variation of τ for the start-up phase was implemented,

$$\tau = \begin{cases} (\tau_s - \tau)(t/T_p)[(t/T_p) - 2] + \tau_s, & t \leq T_p \\ \tau, & t > T_p, \end{cases} \quad (40)$$

where T_p is an arbitrary amount of time, sufficient to avoid instabilities. Since the steady-state results are of interest, the actual value of T_p does not affect the computational outcome. However, it has a significant influence on the total computational effort, such that it is desirable to use a value as small as possible, which is found by trial and error. A steady-state simulation was considered converged when the maximum error over the fluid nodes satisfies the criterion

$$\max_j \frac{\|\mathbf{u}(\mathbf{x}_j, t+1) - \mathbf{u}(\mathbf{x}_j, t)\|}{\|\mathbf{u}(\mathbf{x}_j, t+1)\|} \leq \epsilon, \quad (41)$$

where ϵ was set to 10^{-6} .

A LBM code traditionally uses a rectangular grid composed of $N_X \times N_Y$ nodes on the two Cartesian axes. In the case of the glottis geometry, such a rectangular “envelope” grid contains a large number of nodes that are located outside the flow domain. The amount of unused memory is quite large because nine double-precision variables (i.e., f_i , $i=0, \dots, 8$) are stored at each node. A different approach was taken in the current implementation: only one Boolean variable is stored at each node of the envelope grid, indicating whether the node is located within the flow domain. A data structure was created such that memory is allocated for the functions f_i only at the fluid nodes. Another data structure was subsequently created for the fluid nodes adjacent to a wall (“wall nodes,” whose links intersect a wall in at least one direction). The economy in terms of memory and computational time for the investigated geometries is significant because the number of nodes in the envelope grid is much larger than that actually needed for the computational domain (see Table II).

D. Finite volume Navier–Stokes solver

The general-purpose commercial CFD package FLUENT was chosen in order to provide a basis of comparison for the LBM results. FLUENT is a finite volume Navier–Stokes solver and was previously utilized for the study of the glottal flow in both fixed^{8,10} and moving¹³ glottal configurations. It has been shown to provide reliable results for glottal wall pressures and flow rates.

The two-dimensional, unsteady implicit version of the solver with simple pressure-velocity coupling was used for the present study. Second-order schemes were selected in FLUENT for both momentum and pressure. The unsteady solver was necessary since the convergence could not be obtained with the steady solver due to numerical oscillations at the outlet pressure boundary condition. A no-flow initial condition was considered. The simulations were carried out with a time step of 10^{-5} s until a steady incompressible laminar solution was obtained for each investigated case. The solutions were considered converged when all the reported

residuals were less than 10^{-5} . Based on symmetry, only half of the flow domain was considered in the simulations. The computational domain was discretized with quadratic cells. Unstructured mesh grids were used, with higher density in the minimum opening region. The minimum grid spacing δ_L , corresponds to the minimum glottal diameter. For the reported results, δ_L was of the order of 5×10^{-6} m (see Table VI). Different meshes were tested in order to verify that grid-independent results were obtained. A comparison between various meshes used during the tests is not presented here in order to avoid lengthening the presentation.

IV. RESULTS AND DISCUSSIONS

The values of fluid (air) properties considered in the computational simulations are as follows: density ρ_0 , 1.2 kg/m^3 , dynamic viscosity μ , $1.8 \times 10^{-5} \text{ kg/m/s}$ (with the resulting kinematic viscosity ν , $1.5 \times 10^{-5} \text{ m}^2/\text{s}$), and speed of sound c_s , 330 m/s . Three transglottal pressures have been used in this study, 3, 5, and 10 cm H_2O , corresponding to 294.3, 490.5, and 981 Pa, respectively. For the LBM simulations, it was found convenient to define the grid as a function of the number of lattice units N_d necessary to cover the minimum glottal diameter $d_{\min}$, such that $\delta_x \approx d_{\min}/N_d$. The space step is slightly modified by the code such that the straight inlet and outlet parts of the model glottal geometry are allowed to pass at half-distance in between two lattice nodes (leading to a link fraction $q = \frac{1}{2}$).

Grids of different densities were used for performing the LBM simulations. For each geometry, the coarser grid was characterized by a number of intervals across the minimum diameter $N_d = 12$, while for the finer grid, 24 intervals were used. The properties of the two grids are shown in Table II. Preliminary tests of the LBM code using different grid densities ($N_d = 12, 24$, and 36) have shown that $N_d = 24$ provide a good accuracy, together with a reasonable computational time. This also helped confirm the grid independence for the LBM simulations. The LBM grid encompasses the whole flow domain (not just half-domain, as for the FLUENT grids). One can observe in Table II that the values of the dimensionless collision time τ are close to the stability limit of 0.5 for the LBGK method. The test simulations performed with the LBGK method have lead to numerical oscillations, such that no useful set of results could be obtained. However, the MRT model presented in Sec. III A 2, together with the pressure boundary conditions of Sec. III B 2 and appropriate wall boundary conditions, has lead to stable simulations. It was observed that the wall boundary conditions have a significant influence on the stability of the numerical simulations. The linear BFL conditions lead to stable solutions for all cases. When the quadratic BFL conditions were used, bounded numerical oscillations were observed for kinematic viscosities corresponding to values of the dimensionless collision time τ generally less than 0.55 (as shown in Sec. III C, the simulations have been started with a larger value of τ , which was decreased progressively). The actual value of the dimensionless collision time for which oscillations start to occur appears to be particular to a specific geometry and transglottal pressure. It was observed that the instabilities first occur at

those nodes adjacent to boundaries where the link fractions q (in any direction) present values very close to either 0 or 1. Such values are difficult to avoid for a relatively complex geometry (e.g., when arcs are intersected by the lattice links). More investigations on the applicability of the quadratic BFL conditions and the multireflection conditions²¹ for the glottal flow are in progress.

The bounce-back wall boundary conditions can be enforced by imposing $q = 0.5$ at all nodes adjacent to a wall (even when it does not reflect the correct wall configuration). This condition was initially implemented in the LBM code (mainly for its simplicity) but was shown to provide unsatisfactory results in terms of accuracy and even stability. This is not surprising, as the asymptotic analysis of Junk and Yang⁴⁴ has shown that for any actual value of the parameter q other than 0.5, the simple bounce-back conditions lead to a solution of the lattice Boltzmann equation, which is not a correct solution of the incompressible Navier–Stokes equations.

For the reasons stated above, the results presented in this section were obtained by using the linear BFL wall boundary conditions. Since low values of the Mach number were expected, the incompressible form of the equilibrium distribution functions [see Eq. (13)] has been implemented in the computational code. As will be shown below, the results have confirmed this assumption.

The LBM simulations predict a symmetric flow through the glottis, which is due to the symmetry of geometry and boundary conditions. The assumption of the flow symmetry was made *a priori* in the CFD code FLUENT by choosing to solve only half of the flow domain. During the numerical experimentations with FLUENT, it was observed that if the

FIG. 4. Glottal wall pressures for the convergent 10° , $d_{\min} = 0.04 \text{ cm}$ case. The LBM results are represented with solid line, while the FLUENT results are shown with dashed line. The experimental data are represented with empty symbols for the FW and filled symbols for the NFW.

FIG. 5. Glottal wall pressures for the uniform, $d_{\min}=0.04$ cm case. The LBM results are represented with solid line, while FLUENT results are shown with dashed line. The experimental data are represented with empty symbols for FW and filled symbols for NFW.

whole flow domain is considered, the jet emerging from the glottis has a tendency to deviate (skew) from the direction of the glottal axis because of numerical reasons. Experimental observations^{8,10} have shown that the glottal jet does indeed skew because of the flow conditions downstream of the vocal folds (e.g., the length of the downstream tunnel in M5). The asymmetry of the jet allows the definition of a “flow wall” (FW), toward which the flow is deviated, and a “nonflow wall” (NFW) on the other side. The measured wall pressures are slightly different, such that the NFW pressures are higher than those on the FW.

Figure 4 shows the glottal wall pressures for the convergent 10° geometry. It was convenient to represent the wall pressures as the pressure drop from the trachea (modeled by the inlet of the computational domain). The LBM results correspond to a grid density $N_d=24$ and are represented by solid lines. The FLUENT results are shown as dashed lines. The pressures measured on the NFW are represented by filled squares, circles, and triangles for transglottal pressures of 3, 5, and 10 cm H₂O, respectively. The FW are represented by similar but unfilled symbols. The glottal entrance is conventionally defined as the axial location corresponding

FIG. 6. Glottal wall pressures for the divergent 10° , $d_{\min}=0.04$ cm case. The LBM results are represented with solid line, while FLUENT results are shown with dashed line. The experimental data are represented with empty symbols for FW and filled symbols for NFW.

to point *D* (i.e., $X=0$), while the glottal exit corresponds to the vertical segment *A–E* in Fig. 1. The decrease in the cross-sectional area upstream of the glottal entrance accelerates the flow and causes the pressure to drop, and the minimum pressure (equivalent to the largest pressure drop) is obtained at the location of the minimum glottal diameter. The LBM pressures are generally in satisfactory agreement with the experimental data, as well as with the FLUENT results. Compared to the FLUENT results, the LBM has slightly over-predicted the value of the maximum pressure drop for 10 cm H₂O. The concept of grid independence has been separately verified for FLUENT, such that quasi-identical results are obtained (at the expense of computational time) if finer grids are used.

The wall pressures for the uniform configurations are presented in Fig. 5. The largest pressure drop is obtained for a location slightly downstream to point *F* (defined in Fig. 1 and Table I), where the glottal cross-sectional area starts to increase. The pressure recovery takes place up to the glottal exit. For all transglottal pressures, the largest pressure drop is less than that for the convergent configurations because the uniform geometry provides less flow acceleration and the viscous effects are more pronounced because of the longer

TABLE III. Glottal flow rate (in cm³/s) for trans glottal pressures of 3, 5, and 10 cm H₂O: comparison between LBM results (for a grid density $N_d=24$), FLUENT solver results (NS), and experimental data (E) for a minimum glottal diameter $d_{\min}=0.04$ cm.

	Convergent, 10°			Uniform			Divergent, 10°		
	LBM	NS	E	LBM	NS	E	LBM	NS	E
3	93.1	92.7	87.1	77.8	78.8	80.7	107.7	107.6	97.1
5	123.1	122.1	113.6	104.3	104.1	106.4	141.9	141.5	126.4
10	178.7	176.5	166.8	154.5	153.8	161.3	205.1	203.4	185.2

TABLE IV. Reynolds and Mach numbers based on LBM data for transglottal pressures of 3, 5, and 10 cm H₂O (minimum glottal diameter $d_{\min}=0.04$ cm).

	Convergent 10°		Uniform		Divergent 10°	
	Re	M	Re	M	Re	M
3	620	0.071	604	0.069	693	0.079
5	800	0.091	787	0.089	899	0.102
10	1134	0.129	1115	0.127	1280	0.146

throat zone of small diameter. A good match between the numerical and experimental wall pressures can be observed for all three pressures due to the small asymmetry of the exit jet. The asymmetry increases with the transglottal pressure and is visible for 10 cm H₂O toward the glottal exit. The LBM appears to provide a slightly better approximation than FLUENT with regard to the experiment data. The LBM results were reported for a grid density $N_d=24$.

For the divergent 10° configurations, the wall pressures are shown in Fig. 6. As for the other cases, a grid density $N_d=24$ was used to obtain the plotted LBM results. The flow asymmetry in the experimental data is visible starting from 5 cm H₂O. For each transglottal pressure, the largest pressure drop is obtained at the glottal entrance (i.e., $X=0$). The maximum pressure drop for a divergent case is larger than the one obtained for the corresponding convergent or uniform case. The numerical results are in good agreement with the experimental data for 3 cm H₂O when the flow is symmetric. The separation points are located axially in the vicinity of $X=0$ (immediately upstream of point G), where the glottal diameter reaches its minimum for a divergent geometry. For larger pressures, the separation points on the two sides are not symmetric due to flow instability¹⁰ and unequal pressures will occur on the two glottal walls. Since in the simulations a symmetric flow was assumed, larger differences between the numerical results and empirical data occur once the flow asymmetry becomes significant. Given the imprecise method of establishing the flow asymmetry, the comparisons between the two sets of computational predictions and the experimental data were generally reasonable. The LBM and FLUENT predictions agree well with each other for the lower pressure cases, while LBM tends to yield slightly

larger values of the maximum pressure drop (possibly due to the linear BFL conditions).

The plots for all transglottal pressures show that in some instances, the LBM curves are less smooth than those provided by FLUENT. The explanation is that the wall pressures were reported at the fluid nodes closest to the wall (wall nodes, for which at least one link intersects the wall). This approximation leads to a small inaccuracy for the value and location of the reported pressure.

A good agreement between the LBM and FLUENT was observed for the glottal flow rate, as shown in Table III. In the calculation of the flow rate, a value of 1.2 cm was used for the anterior-posterior dimension, as in the experimental model M5. The difference between the predicted and experimental flow rates is explained by the flow asymmetry in M5. For instance, the asymmetry is evident for a divergent 10° geometry and a transglottal pressure of 5 cm H₂O, while the flow rate differs by 15.3 cm³/s, corresponding to a relative error $\varepsilon_r=12\%$. However, for the uniform configuration at 5 cm H₂O, presenting a good flow symmetry, the difference is 2.2 cm³/s (i.e., $\varepsilon_r=2\%$).

The values of both Reynolds and Mach number (calculated as $M=u_{\max}/c_s$, where $u_{\max}$ is the maximum speed) for the investigated cases are listed in Table IV. One can observe that the highest value of the Mach number is 0.146, which justifies the use of the incompressible model of He and Luo.³¹ Simulations at larger values of Mach number are possible for higher values of the transglottal pressures if the unaltered form [Eq. (8)] of the equilibrium distribution functions is used. The Reynolds number was calculated based on

TABLE V. Computational time for lattice Boltzmann simulations. The code was compiled with the GNU C compiler.

	Grid		CPU time (h) Δp (cm H ₂ O)		
	N_d	Fluid nodes	3	5	10
Convergent 10°	12	136 047	2.9	5.2	11.5
	24	504 075	27.9	29.1	36.7
Uniform	12	135 526	2.6	6.8	6.9
	24	503 103	28.5	29.0	34.8
Divergent 10°	12	136 294	5.3	5.8	6.9
	24	505 969	34.8	39.1	46.4

TABLE VI. Minimum grid spacing δ_L , local Courant number, and CPU time for FLUENT simulations.

	Grid		CPU time (h) Δp (cm H ₂ O)			Courant Δp (cm H ₂ O)		
	Cells	δ_L (10 ⁻⁶ m)	3	5	10	3	5	10
Convergent 10°	194 212	4.8	3.0	2.6	2.2	106	141	205
Uniform	209 736	5.1	3.7	3.4	2.8	69	93	136
Divergent 10°	178 088	5.0	3.8	3.4	3.2	101	132	188

the minimum glottal diameter, $Re = u_{\max} d_{\min} / \nu$. The values of Reynolds number are also low for all cases, being less than 1300.

The computational time was evaluated by using a 3.2 GHz Pentium 4 machine. The current LBM implementation is costlier when compared to FLUENT. Table V presents the computational time for the LBM code, which requires a regularly spaced lattice characterized by a typically small space step, δ_x (see Table II). In contrast, FLUENT simulations are performed for only half of the flow domain and make use of grids designed such that coarser grids are sufficient in the regions with smaller gradients. One important computational aspect is that for LBM, the Courant number is limited to 1 because $\delta_x = \delta_t$ (in dimensionless notation) and the lattice speed is $c = \delta_x / \delta_t$. This involves a slow convergence for steady-state problems.¹⁵ The no-flow initial solution also contributed to a relatively long simulation time (see Sec. III C). The FLUENT simulations (see Table VI) were performed by using an unsteady implicit solver, which allowed for large values of the local Courant number. The speed of the LBM code can be improved by using locally refined grids^{39,45} and multigrid techniques.⁴⁶ There also exist algorithms for implementing LBM on arbitrary, nonuniform grids, where the nodal values of the distribution functions are estimated by interpolation.^{47,48}

The current study was limited to one value of the minimal glottal diameter (i.e., 0.04 cm). However, the investigated configurations are representative of the actual geometry of the glottis, which assumed successively convergent, uniform, and divergent postures.^{49,50} Although not presented here, numerical experiments for a glottal diameter of 0.08 cm suggest that LBM can be used for a wider range of glottal configurations.

V. CONCLUSIONS

A numerical method based on the solution of the lattice Boltzmann equation was proposed for the investigation of glottal flow. In the present work, the glottal flow was investigated by using LBM with the MRT model, for which suitable pressure boundary conditions were defined. Three representative static glottal configurations were studied by using this method, corresponding to different shapes of the vocal folds during phonation. The results were validated by comparison with experimental data, as well as with the results obtained with a Navier–Stokes CFD package (FLUENT). Both the glottal wall pressures and flow rates were generally in good agreement with the experimental data and the FLUENT results.

The present work was motivated by the fact that the LBM presents a number of advantages for the investigation of glottal flow. The local character of the interparticle collisions implies that most of the computational work takes place in the grid nodes, while the propagation involves only data shift to the neighbor node. For this reason, the LBM is easy to implement and is well suited for parallel computing.^{14,48,51} The simple topology of the LBM lattices ensures the convenient handling of large grids. Another aspect of particular interest for glottal flows is the treatment of moving boundaries,^{19,20} which from an implementation viewpoint requires only modifications in the formulation of f at the wall. Also, the inherent transient and weakly compressible character of the LBM shows potential for approaching the laryngeal acoustics.

The in-house LBM implementation used for the present work was less efficient than the Navier–Stokes solver, primarily because of the classical uniform lattice that leads to a dense computational grid. Further work envisages the implementation of algorithms that would allow to use nonuniform grids. Also, the case of moving vocal folds will be investigated, which will also permit the study of the fluid-structure interaction in the glottis.

ACKNOWLEDGMENTS

This research was supported in part by the NIH Grant No. 2R56DC003577-10A1.

¹K. Ishizaka and J. L. Flanagan, “Synthesis of voiced sounds from a two-mass model of the vocal cords,” *Bell Syst. Tech. J.* **51**, 1233–1268 (1972).

²K. Ishizaka and M. Matsudaira, “Fluid mechanical considerations of vocal cord vibration,” in *SCRL Monograph* (Speech Communications Research Laboratory, Santa Barbara, 1972), Vol. 8.

³M. Hirano, *Clinical Examination of Voice* (Springer-Verlag, New York, 1981).

⁴I. R. Titze, *Principles of Voice Production* (Prentice-Hall, Englewood Cliffs, NJ, 1994).

⁵J. G. Svec and H. K. Schutte, “Videokymography: High-speed line scanning of vocal fold vibration,” *J. Voice* **10**, 201–205 (1996).

⁶J. G. Svec, H. K. Schutte, and F. Sram, “Variability of vibration of normal vocal folds as seen in videokymography,” in *Communication and Its Disorders: A Science In Progress*, Proceedings 24th Congress International Association of Logopedics and Phoniatrics, edited by P. H. Dejonckere and H. F. M. Peters (Nijmegen University Press, Amsterdam, 1999), Vol. 1, pp. 122–125.

⁷J. G. Svec, F. Sram, and H. K. Schutte, “Videokymography: A new high-speed method for the examination of vocal-fold vibrations,” *Otorinolaryngologie a foniatrie* **48**(3), 155–162 (1999).

⁸R. C. Scherer, D. Shinwari, K. J. DeWitt, C. Zhang, B. R. Kucinski, and A. A. Afjeh, “Intraglottal pressure profiles for a symmetric and oblique glottis with a divergence angle of 10 degrees,” *J. Acoust. Soc. Am.* **109**, 1616–1630 (2001).

⁹C. G. Guo and R. C. Scherer, “Finite element simulation of the glottal flow

- and pressure," J. Acoust. Soc. Am. **94**, 688–700 (1993).
- ¹⁰D. Shinwari, R. C. Scherer, K. J. DeWitt, and A. A. Afjeh, "Flow visualization and pressure distributions in a model of the glottis with a symmetric and oblique divergent angle of 10 degrees," J. Acoust. Soc. Am. **113**, 487–497 (2003).
 - ¹¹F. Alipour, C. Fan, and R. C. Scherer, "A numerical simulation of laryngeal flow in a forced-oscillation glottal model," Comput. Speech Lang. **10**, 75–93 (1996).
 - ¹²S. L. Thomson, L. Mongeau, and S. H. Frankel, "Aerodynamic transfer of energy to the vocal folds," J. Acoust. Soc. Am. **118**, 1689–1700 (2005).
 - ¹³B. Kucinski, R. C. Scherer, K. J. DeWitt, and T. Ng, "An experimental analysis of the pressures and flows within a driven mechanical model of phonation," J. Acoust. Soc. Am. **119**, 3011–3021 (2006).
 - ¹⁴S. Succi, *The Lattice Boltzmann Equation for Fluid Dynamics and Beyond* (Oxford University Press, New York, 2001).
 - ¹⁵D. Yu, R. Mei, L.-S. Luo, and W. Shyy, "Viscous flow computations with the method of lattice Boltzmann equation," Prog. Aerosp. Sci. **38**, 329–367 (2003).
 - ¹⁶G. Yan, Y. Chen, and S. Hu, "Simple lattice Boltzmann model for simulating flows with shock wave," Phys. Rev. E **59**, 454–459 (1999).
 - ¹⁷X. M. Li, R. C. K. Leung, and R. M. C. So, "One-step aeroacoustics simulation using lattice Boltzmann method," AIAA J. **44**, 78–89 (2006).
 - ¹⁸E. W. S. Kam, R. M. C. So, and R. C. K. Leung, "Lattice Boltzmann method simulation of aeroacoustics and nonreflecting boundary conditions," AIAA J. **45**, 1703–1712 (2007).
 - ¹⁹M. Bouzidi, M. Firdaouss, and P. Lallemand, "Momentum transfer of a Boltzmann-lattice fluid with boundaries," Phys. Fluids **13**, 3452–3459 (2001).
 - ²⁰P. Lallemand and L.-S. Luo, "Lattice Boltzmann method for moving boundaries," J. Comput. Phys. **184**, 406–421 (2003).
 - ²¹I. Ginzburg and D. d'Humières, "Multireflection boundary conditions for lattice Boltzmann models," Phys. Rev. E **68**, 066614 (2003).
 - ²²M. Hirano, K. Kiyokawa, and S. Kuria, "Laryngeal muscles and glottis shaping," in *Vocal Physiology: Voice Production, Mechanism and Functions*, edited by O. Fujimura (Raven, New York, 1988), Vol. **49–65**.
 - ²³Z. Zhang, L. Mongeau, and S. H. Frankel, "Experimental verification of the quasi-steady approximation for aerodynamic sound generation by pulsating jets in tubes," J. Acoust. Soc. Am. **112**, 1652–1663 (2002).
 - ²⁴R. C. Scherer, D. Shinwari, K. J. DeWitt, C. Zhang, B. R. Kucinski, and A. A. Afjeh, "Intraglottal pressure distributions for a symmetric and oblique glottis with a uniform duct," J. Acoust. Soc. Am. **112**, 1253–1256 (2002).
 - ²⁵S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, Cambridge, 1960).
 - ²⁶P. Bhatnagar, E. P. Gross, and M. K. Krook, "A model for collision processes in gases. I. Small amplitude processes in charged and neutral one-component systems," Phys. Rev. **94**, 511–525 (1954).
 - ²⁷X. He and L. S. Luo, "Theory of the lattice Boltzmann method: From the Boltzmann equation to the lattice Boltzmann equation," Phys. Rev. E **56**, 6811–6817 (1997).
 - ²⁸D. Wolf-Gladrow, *Lattice-Gas Cellular Automata and Lattice Boltzmann Models* (Springer-Verlag, Berlin, 2000).
 - ²⁹Q. H. Qian, D. d'Humières, and P. Lallemand, "Lattice BGK models for Navier-Stokes equation," Europhys. Lett. **17**, 479–484 (1992).
 - ³⁰S. Chen, D. Martinez, and R. Mei, "On boundary conditions in lattice Boltzmann methods," Phys. Fluids **8**, 2527–2536 (1996).
 - ³¹X. He and L. S. Luo, "Lattice Boltzmann model for the incompressible Navier-Stokes equation," J. Stat. Phys. **88**, 927–944 (1997).
 - ³²Z. Guo, C. Zheng, and B. Shi, "An extrapolation method for boundary conditions in lattice Boltzmann method," Phys. Fluids **14**, 2007–2010 (2002).
 - ³³Z. Guo, C. Zheng, and T. S. Zhao, "A Lattice BGK scheme with general propagation," J. Sci. Comput. **16**, 569–585 (2001).
 - ³⁴D. d'Humières, "Generalized lattice-Boltzmann equations," in *Rarefied Gas Dynamics: Theory and Simulations, Progress in Astronautics and Aeronautics*, edited by B. D. Shizgal and D. P. Weaver (AIAA, Washington, DC, 1992), Vol. **159**, pp. 450–458.
 - ³⁵P. Lallemand and L.-S. Luo, "Theory of the lattice Boltzmann method: Dispersion, dissipation, isotropy, Galilean invariance, and stability," Phys. Rev. E **61**, 6546–6582 (2000).
 - ³⁶D. Yu, Ph.D. thesis, University of Florida, 2002.
 - ³⁷D. d'Humières, I. Ginzburg, M. Krafczyk, P. Lallemand, and L.-S. Luo, "Multiple-relaxation-time lattice Boltzmann models in three dimensions," Philos. Trans. R. Soc. London, Ser. A **360**, 437–451 (2002).
 - ³⁸R. Mei, L. S. Luo, and W. Shyy, "An accurate curved boundary treatment in the lattice Boltzmann method," J. Comput. Phys. **155**, 307–330 (1999).
 - ³⁹O. Filippova and D. Hänel, "Grid refinement for lattice-BGK models," J. Comput. Phys. **147**, 219–228 (1998).
 - ⁴⁰H. Huang, T. S. Lee, and C. Shu, "Lattice-BGK simulation of steady flow through vascular tubes with double constrictions," Int. J. Numer. Methods Heat Fluid Flow **16**, 185–203 (2006).
 - ⁴¹Z. Guo, C. Zheng, and B. Shi, "Non-equilibrium extrapolation method for velocity and pressure boundary conditions in the lattice Boltzmann method," Chin. Phys. **11**, 366–374 (2002).
 - ⁴²Z. Guo, B. Shi, and N. Wang, "Lattice BGK model for incompressible Navier-Stokes equation," J. Comput. Phys. **165**, 288–306 (2000).
 - ⁴³P. A. Skordos, "Initial and boundary conditions for the lattice Boltzmann method," Phys. Rev. E **48**, 4823–4842 (1993).
 - ⁴⁴M. Junk and Z. Yang, "Asymptotic analysis of Lattice Boltzmann boundary conditions," J. Stat. Phys. **121**, 3–35 (2005).
 - ⁴⁵J. Tölke, S. Freudiger, and M. Krafczyk, "An adaptive scheme using hierarchical grids for lattice Boltzmann multi-phase flow simulations," Comput. Fluids **35**, 820–830 (2006).
 - ⁴⁶J. Tölke, M. Krafczyk, and E. Rank, "A multigrid-solver for the discrete Boltzmann equation," J. Stat. Phys. **107**, 573–591 (2002).
 - ⁴⁷X. He, L. S. Luo, and M. Dembo, "Some progress in lattice Boltzmann method. Part I. Nonuniform mesh grids," J. Comput. Phys. **129**, 357–363 (1996).
 - ⁴⁸C. S. Sunder, G. Baskar, V. Babu, and D. Strenski, "A detailed performance analysis of the interpolation supplemented lattice Boltzmann method on the cray T3E and cray X1," Int. J. High Perform. Comput. Appl. **20**, 557–570 (2006).
 - ⁴⁹M. Hirano, "Structure and vibratory behavior of the vocal folds," in *Dynamic Aspects of Speech Production*, edited by M. Sawashima and F. Cooper (University of Tokyo, Tokyo, 1977), pp. 13–27.
 - ⁵⁰K. N. Stevens and D. H. Klatt, "Current models of sound sources for speech," in *Ventilatory and Phonatory Control Systems*, edited by B. Wyke (Oxford University Press, New York, 1974), pp. 279–292.
 - ⁵¹M. Krafczyk, "Lattice-Boltzmann Methods: From basics to fluid-structure interaction," Course on Advanced Computational Methods for Fluid-Structure Interaction, Ibiza, Spain, 3–7 May 2006.