Negative dispersion in bone: The role of interference in measurements of the apparent phase velocity of two temporally overlapping signals

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In this study the attenuation coefficient and dispersion (frequency dependence of phase velocity) are measured using a phase sensitive (piezoelectric) receiver in a phantom in which two temporally overlapping signals are detected, analogous to the fast and slow waves typically found in measurements of cancellous bone. The phantom consisted of a flat and parallel Plexiglas™ plate into which a step discontinuity was milled. The phase velocity and attenuation coefficient of the plate were measured using both broadband and narrowband data and were calculated using standard magnitude and phase spectroscopy techniques. The observed frequency dependence of the phase velocity and attenuation coefficient exhibit significant changes in their frequency dependences as the interrogating ultrasonic field is translated across the step discontinuity of the plate. Negative dispersion is observed at specific spatial locations of the plate at which the attenuation coefficient rises linearly with frequency, a behavior analogous to that of bone measurements reported in the literature. For all sites investigated, broadband and narrowband data (3–7 MHz) demonstrate excellent consistency. Evidence suggests that the interference between the two signals simultaneously reaching the phase sensitive piezoelectric receiver is responsible for this negative dispersion. © 2008 Acoustical Society of America. [DOI: 10.1121/1.2839893]

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I. INTRODUCTION

A number of studies have addressed the anomalous negative dispersion (decreasing phase velocity with increasing frequency) widely observed in ultrasonic studies of bone *in vitro*^{1–5} and *in vivo*.⁶ In those studies, for the sites that display negative dispersion, the bone specimens investigated typically exhibit an attenuation coefficient that rises approximately linearly with frequency. A positive dispersion might be expected in these cases based on consideration of the causality-induced Kramers–Kronig relations.^{7–9}

In previous work we proposed an alternative explanation of the observed negative dispersion and presented the results of numerical simulations demonstrating that negative dispersion can result from the interference of two propagating modes, each of which exhibits a positive dispersion, consistent with the Kramers–Kronig predictions.^{10,11} The goal of the present study is to demonstrate this negative dispersion experimentally using the simplest example of a phantom that is capable of producing two such interfering waves.

Most ultrasonic experimental data are acquired using phase sensitive piezoelectric devices. Effects associated with phase cancellation at a piezoelectric detector on ultrasonic measurements have consistently been investigated over the past fifty years.^{12–21} Artifacts resulting from phase aberrations can manifest themselves as acoustic speckle in *B*-mode

images,²² or as gross overestimations of the (apparent) attenuation coefficient.^{12,14,23} In a previous paper,²¹ we addressed the issue of phase cancellation at a phase sensitive piezoelectric receiving aperture and its effect on the validity of measurements made with an experimental setup sensitive to phase cancellation errors. Other authors have addressed these issues in bone, and have demonstrated the artifacts inherent in phase sensitive measurements.^{20,24,25}

II. METHODS

A. Sample preparation

The phantom used in this investigation was constructed from a single sheet of polymethyl methacrylate (Plexiglas™) initially machined into a flat and parallel plate 2 in. (50.8 mm) long, 1 in. (25.4 mm) wide, and 0.5 in. (12.7 mm) thick. The resulting plate then had a small step discontinuity of 0.3 mm milled into one of its surfaces, as seen in Fig. 1. This step size was chosen so that phase cancellation artifacts would occur at approximately the mid-bandwidth frequency of the insonifying ultrasonic field, which for our experiment was approximately 5 MHz. Additional details can be found in Bauer *et al.*²¹

B. Equipment and experimental setup

Through-transmission measurements were performed in a water tank ($T=21.50\pm0.02$ °C) using separate transmitting and receiving 5 MHz center frequency, 0.25 in. diam-

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FIG. 1. Measured attenuation coefficient (second column) and phase velocity (third column) as a function of insonification location. Panels (a) and (f) show the measured ultrasonic properties for the thin and thick regions, respectively. Panels (b)–(e) show the measured apparent ultrasonic properties as the interrogating field traverses the step from the thin side to the thick side. The percentages shown serve as a qualitative description of the amount of the thin and thick side of the plate interrogated by the ultrasonic field.

eter, single element, unfocused, immersion transducers (Panametrics V310; Waltham, MA). The propagation path between the transmitting and receiving transducers was 3 in.

Broadband measurements were collected by exciting the transmitting transducer with a broadband radio frequency (rf)

pulse from a pulser/receiver (Panametrics 5800, Waltham, MA) in through-transmission mode. The output of the receiving transducer was fed into the 50 Ω input of an 8-bit digitizing oscilloscope (Tektronix 5052b, Beaverton, OR). The resulting time-domain broadband signals were digitized

at a 2.5 GS/s digitization rate (400 ps/pt) and temporally averaged 256 times.

Narrowband measurements were performed using a pulse/function generator (Hewlett-Packard HP8116A; Palo Alto, CA) to create a sinusoidal tone burst at selected frequencies from 3 to 7 MHz in steps of 500 kHz. The sinusoidal tone bursts were used to drive the transmitting transducer and the output from the receiving transducer was fed into the 50 Ω input of the Tektronix 5052b oscilloscope. The received time-domain narrowband signals were digitized at a rate of 1.25 GS/s (800 ps/pt) and averaged 256 times. The number of cycles in each tone burst was varied so that each frequency component exhibited the same absolute bandwidth. The length of the tone burst was chosen to be sufficiently long (4 μ s) to minimize correlation errors associated with the transients occurring at the beginning and end of each tone burst.

The phase velocity and attenuation properties of the plastic were measured by utilizing a substitution method.²⁶ Systematic measurements of different locations of the stepped plastic plate were achieved by motion control (Aerotech Unidex 511; Pittsburgh, PA). Values for the phase velocity and attenuation of the Plexiglas plate uncomplicated with details of the stepped region were obtained by averaging the results 25 individual sites for the thin and thick regions. The stepped region of the plate was investigated by translating the region of insonification in increments of 0.050 mm (0.002 in.) from the thin region of the stepped plastic plate to the thick region, as seen in Fig. 1. One hundred and fifty one (151) sites were insonified across the step, from the thin side of the plate to the thick side of the plate. (Only selected regions are displayed in Fig. 1 for illustrative purposes.)

C. Data

1. Broadband analysis

The data collected were analyzed using a phase spectroscopy broadband ultrasonic technique described elsewhere.²¹ The frequency resolution of the acquired signals was enhanced by zero padding the digitized time-domain wave forms prior to data reduction in order to achieve an interpolated frequency spacing of $\Delta\omega/2\pi=25$ kHz between successive data points. The frequency dependent phase velocity, $c(\omega)$, of the sample was calculated by comparing the difference in the phase spectra, $\Delta\phi$, of the sample trace to that of the reference trace,

$$c(\omega) = c_w \left[\frac{\omega d}{\omega d + c_w \Delta\phi(\omega)} \right], \quad (1)$$

where c_w is the known speed of sound in the water²⁷ and d is the effective sample thickness. The effective sample thickness in the region of the step was calculated by linearly interpolating the sample thickness from the thin side, d_{thin} , of the plate to the thick side

$$d = d_{\text{thin}} + \frac{m}{n} \Delta x, \quad (2)$$

where $m=1, 2, 3, \dots, 151$, Δx is the thickness of the step, and $n=151$, the number of locations investigated across the step.

The amplitude attenuation coefficient of the sample (dB/cm) is determined from a log-spectral subtraction technique,²⁸

$$\alpha(\omega) = \frac{20 \log(|\tilde{U}_w(\omega)|) - 20 \log(|\tilde{U}_s(\omega)|) + 10 \log(T_{ws}^I T_{sw}^I)}{d}, \quad (3)$$

where $|\tilde{U}_w(\omega)|$ and $|\tilde{U}_s(\omega)|$ are the magnitudes of the Fourier transforms of the water and sample path signals, and T_{ws}^I and T_{sw}^I are the intensity transmission coefficients of the water-sample and sample-water interfaces, respectively.

D. Narrowband analysis

The received sample path and reference path narrowband signals were gated with a 10 μ s rectangular window. The narrowband phase velocity measurements of the stepped plastic plate were calculated with a time-domain rf correlation technique using a positive-peak criterion. The narrowband attenuation coefficient calculations were performed using Eq. (3) applied to each individual frequency component transmitted in the total bandwidth.

III. RESULTS

The first column of Fig. 1 illustrates the experimental procedure outlined in Sec. II A. As shown, the six horizontal panels of Fig. 1 encompass the measured apparent ultrasonic properties of the stepped plastic plate as the ultrasonic field produced by the transmitting and receiving transducers moves from the thin side of the plate, across the step, to the thick side of the plate. The thin and thick percentages indicated at the bottom of each panel are intended to give the reader a qualitative measure of how much of the thick and thin portions of the plate are interrogated by the ultrasonic field, and are not meant as a quantitative description of the behavior of the ultrasonic field at this location. The second and third columns of Fig. 1 show the measured attenuation coefficient and measured phase velocity as a function of location along the stepped plastic plate.

Figure 1(a) shows the measured attenuation and phase velocity for investigation of the thin, flat, and parallel portion of the plate. Figures 1(b)–1(e) show the behavior of the measured attenuation coefficient and phase velocity as the stepped region of the plate is moved through the ultrasonic field from the thin side to the thick side. Figure 1(b) shows the measured properties for the ultrasonic field that traverses approximately 70% of the thin part of the plate and approximately 30% of the thick part. Figures 1(c) and 1(d) show the measured attenuation and phase velocity as the incident ultrasonic field is approximately “cut in half” by the step, though more of the thin side of the plate is present in Fig. 1(c) (51% thin 49% thick), whereas more of the thick side is present in the detected ultrasonic field in Fig. 1(d) (49% thin 51% thick). Pictured in Fig. 1(e) are the measured ultrasonic properties of the phase sensitively detected field where ap-

FIG. 2. Broadband and narrowband phase velocity and attenuation coefficient measurements of the thin and thick sides of the stepped plastic plate.

proximately 30% of the field passes through the thin portion of the plate and 70% of the field passes through the thick portion of the plate. In Fig. 1(f), the measured attenuation coefficient and phase velocity for the investigation of the thick, flat, and parallel side of the plate are presented.

Figure 2 shows the comparison of broadband and narrowband measurements made on the 100% thin and 100% thick sides of the stepped plastic plate. As shown, the attenuation and phase velocity properties of both regions agree within the experimental uncertainty for both broadband and narrowband measurements. Error bars for the attenuation

coefficient data are too small to be seen on this plot, and are not shown. The errors bars shown in the phase velocity plot are two standard deviations from the mean of the broadband measurements made by averaging the results of 25 sites. The phase velocity data of the 100% thin and 100% thick regions do not statistically differ. The broadband and narrowband measurements within the stepped region of the plate are compared in Fig. 3. The broadband and narrowband data were collected from the same spatial locations along the stepped plastic plate, and represent the same locations illustrated in Fig. 1.

FIG. 3. Broadband and narrowband phase velocity and attenuation measurements of the stepped region of the plate as a function of insonification location. The rows correspond to panels (b)–(e) of Fig. 1.

IV. DISCUSSION

For the flat and parallel regions of the plate, an attenuation coefficient that rises approximately linearly with frequency, as well as an approximately logarithmically increasing phase velocity was observed, and is shown in Figs. 1(a), 1(e), and 2. The behavior of the ultrasonic field as it passes through the flat and parallel (thick and thin) regions of the plate remains relatively undisturbed when compared with that of the field emerging from the stepped region of the plate. After compensation for the sample thickness, the attenuation coefficient and phase velocity of the thin and thick regions of the plate are nearly identical as shown in Fig. 2. However, a comparison of the ultrasonic properties in Figs. 1(b) and 1(e) with those of Figs. 1(a) and 1(f), indicate that even a modest amount of the step discontinuity introduced to the system results in dramatic changes in measurements of ultrasonic properties obtained with a phase sensitive piezoelectric receiver. The data presented in Figs. 1(c) and 1(d) serve as extreme examples of how a phase aberrating medium affects the apparent properties of a sample when they are detected by a phase sensitive aperture.

A. Attenuation coefficient

The trend in the frequency dependence of the attenuation coefficient in Figs. 1(a)–1(f) can be explained by considering the phase fronts of the incident ultrasonic field as they pass through the step. When the two transducers are transmitting and receiving a broadband pulse through the flat and parallel sides of the plate, the frequencies contained in the pulse (with a –6 dB bandwidth of 3–7 MHz) are attenuated in a uniform way, commensurate with the native attenuation properties of the sample. Because these surfaces are nominally flat and parallel, the planes of constant phase entering the sample remain constant as they emerge from the plastic plate. If, instead, the ultrasonic field begins to traverse the stepped region of the plate, the phase fronts of the field passing through the thin portion of the plate will be out of phase with those passing through the thick portion of the plate by an amount, $\Delta\varphi = (k_w - k_s)\Delta x$, where k_w and k_s are the wave numbers of the water and sample, respectively, and Δx is the step thickness. This step thickness was chosen so that (approximately) the 5 MHz component of the incident ultrasonic field, when emerging from the stepped region of the plate, would be nominally cancelled upon detection with a phase sensitive aperture.²¹ Because the phase of the wave fronts passing through the thicker part of the plate are advanced by an odd multiple of π radians relative to the phase fronts emerging from the thinner region, the charge distribu-

tion induced on the receiving piezoelectric transducer, when summed across the receiver, nominally cancels the 5 MHz component of the incident field. The frequency components above and below 5 MHz are progressively less and less out of phase with each other. The apparent attenuation coefficient therefore reaches a maximum near 5 MHz and decreases slowly on either side of this frequency to that of the native attenuation coefficient of the sample.

B. Phase velocity

As the transmitted field progresses from the thin region of the plate to the thick region of the plate, the behavior of the apparent phase velocity appears to be more complicated than that of the apparent attenuation coefficient. The apparent phase velocity starts as an approximately logarithmically increasing function of frequency [Fig. 1(a)], but begins to deform as a result of the step's influence on the transmitted ultrasonic field. As the insonifying field further progresses towards the step [Fig. 1(b)], this deformation increases and the apparent phase velocity goes through its maximum in dispersion with a specific curvature [Fig. 1(c)]. As the field continues towards the thick side of the plate, the sign of the curvature of the apparent phase velocity changes [Fig. 1(d)]—the data are reflected about a horizontal axis—and then the apparent phase velocity gradually returns to its logarithmic dependence on frequency, the native dispersion of the sample [Fig. 1(f)].

This trend can be accounted for by writing the ultrasonic field incident on the stepped plastic plate as two signals: one signal, Ψ_{Thin} , describing the portion of the field traversing the thin region of the plate, and another signal, Ψ_{Thick} , that describes the portion of the field traveling through the thick region of the plate. The ultrasonic field, Ψ , is then a superposition of these two signals:

$$\Psi = \Psi_{\text{Thin}} + \Psi_{\text{Thick}}. \quad (4)$$

This superimposed signal will be referred to as a mixed signal. The role each signal plays in the detected ultrasonic field, and ultimately the phase velocity, becomes apparent by writing Ψ_{Thin} and Ψ_{Thick} as a complex phasor in polar (magnitude and phase) form,

$$\tilde{\Psi}(x=L, \omega) = |\tilde{\Psi}_{\text{Thin}}(\omega)|e^{i\varphi_{\text{Thin}}} + |\tilde{\Psi}_{\text{Thick}}(\omega)|e^{i\varphi_{\text{Thick}}}, \quad (5)$$

where $|\tilde{\Psi}_{\text{Thin}}(\omega)|$ and $|\tilde{\Psi}_{\text{Thick}}(\omega)|$ are the complex magnitudes of the thin and thick signals, and φ_{Thin} and φ_{Thick} are the phases of the thin and thick signals, respectively.

After some algebraic manipulation, the overall phase of the mixed signal can be written in terms of its real and imaginary parts as

$$\varphi_{\text{Mixed}}(\omega) = a \tan \left[\frac{|\tilde{\Psi}_{\text{Thin}}(\omega)|\sin(\varphi_{\text{Thin}}(\omega)) + |\tilde{\Psi}_{\text{Thick}}(\omega)|\sin(\varphi_{\text{Thick}}(\omega))}{|\tilde{\Psi}_{\text{Thin}}(\omega)|\cos(\varphi_{\text{Thin}}(\omega)) + |\tilde{\Psi}_{\text{Thick}}(\omega)|\cos(\varphi_{\text{Thick}}(\omega))} \right] + 2\pi m, \quad (6)$$

where the factor of $2\pi m$ comes from the ambiguity resulting from a four quadrant inverse tangent operation.²⁹ The phases of the thin and thick signals are themselves also ambiguous to an integer multiple of 2π . As noted in Trousil, *et al.*,²⁹ an improper choice of m results in dramatic changes in the apparent dispersion, so it is often possible to infer the proper choice of m based on some a priori knowledge about the system under investigation.

It is clear from Eq. (6) that the phase of the overall mixed signal is a function of the magnitudes and phases of the individual thin and thick signals. In the limit that the transmitting and receiving transducers are only interrogating the thin region, the detected ultrasonic field is only a function of the magnitude and phase of the thin signal, and similarly for the thick signal. The interesting behavior arises as a result of the superposition, or interference, of these two signals as the magnitude of one signal dominates that of the other in their relative contributions to the mixed signal. The reflection in the curvature of the measured apparent phase velocity occurs because the phasor of the mixed signal at 5 MHz undergoes an abrupt 180° rotation in the complex phase plane as the ultrasonic field traverses from the thin side of the plate to the thick side [Figs. 1(c) and 1(d)]. As the field approaches the step discontinuity, the mixed signal becomes composed approximately of equal parts thin and thick signal, with a slightly greater contribution from the thin signal [Fig. 1(c)]. As the beam crosses the step, the mixed signal suddenly receives a greater contribution from the thick signal [Fig. 1(d)]. The 5 MHz phasor of the mixed signal therefore immediately rotates 180° in the complex phase plane as the second signal becomes more dominant. Because the thin and thick signals are nominally π radians out of phase at 5 MHz, higher frequencies contained within the experimental bandwidth have rotated almost completely around the complex phase plane, thus almost moving from one phase sheet (e.g., the phase sheet of the lower frequencies) to the next (the phase sheet of the higher frequencies). In the context of the phase sheet offset discussed in Trousil *et al.*,²⁹ the frequencies from ~ 5 to 7 MHz have a phase sheet offset of $+1/2$ in Fig. 1(c) and $-1/2$ in Fig. 1(d) relative to the frequencies from 3 to 5 MHz.

The results of Fig. 1 indicate that given two measurements of the phase velocity in this stepped plastic plate, one could measure two different phase velocity values. For instance, in Figs. 1(c) and 1(d), at 6 MHz, one could measure a phase velocity of 2690 m/s while insonifying one site, whereas another measurement taken 100 μm away would yield a result of 2809 m/s. This apparent inconsistency is due to the standard method used to measure the phase velocity of a sample supporting two temporally overlapping signals with broadband spectroscopic techniques on data collected with phase sensitive devices.

If the two waves were of finite extent, and were able to be temporally gated from one another, then this apparent inconsistency could be resolved. By isolating the two signals from one another (e.g., by using a gate applied in the time domain), the native ultrasonic properties underlying each wave could be determined. However, when a single rf signal is detected, regardless of the constituent waves or modes

superposed to create it, one is left with little choice but to use traditional phase spectroscopy (i.e., treating the received signal as a single signal).

C. Narrowband

Because of the initially puzzling behavior of the broadband phase velocity data, narrowband measurements were carried out over the same spatial locations as the broadband measurements. As a check on the consistency of the broadband measurements, Figs. 2 and 3 demonstrate the marked agreement between the broadband and narrowband data over the flat and parallel sides of the plate, as well as over the stepped portion, respectively. For all narrowband frequency components used to interrogate the sample, a positive peak correlation method was used to determine the appropriate time shift between the sample path and water path rf signals. The 4.5 and 5 MHz components of the 51% thin 49% thick and 49% thin 51% thick narrowband data are not shown in Fig. 3. For these four points the positive peak correlation criterion gave results consistent with a phase sheet offset of 2π .²⁹ That is, if the next nearest peak in the correlation wave was chosen as the appropriate time shift, these four data points would lie directly on top of the broadband data, in line with the other narrowband data points. Even without these four points, the narrowband results agree quite well with the broadband data collected over this phantom.

D. Negative dispersion

The results of measuring selected locations across the phantom used in this study (over a slightly different bandwidth than that of Figs. 1–3) are shown in Fig. 4. The bandwidth of 2.25 to 4.5 MHz was chosen to illustrate a specific behavior that is characteristic of many measurements on bone. Over this bandwidth, spanning a factor of 2 in frequency, the data exhibit many apparent attenuation coefficients, all of which increase approximately linearly with frequency. Yet over this same bandwidth, at exactly the same sites, an apparent phase velocity is measured that appears to decrease with increasing frequency (i.e., to exhibit negative dispersion.)

The slope of the attenuation coefficient plotted as a function of frequency is often used to characterize biological materials. Researchers who investigate bone refer to this parameter as normalized broadband ultrasound attenuation, or nBUA. Fig. 4 clearly shows that one can measure many different apparent slopes of attenuation, or nBUA, for which one finds an apparently negative dispersion even though the material itself exhibits a positive dispersion as shown in Fig. 2. (For clarity, only a few locations from the total observed on the plate displaying negative dispersion are shown in Fig. 4.)

In Waters and Hoffmeister,³ rf data from trabecular bone samples exhibiting three different mass densities are shown (low, medium, and high). It was observed that even for low-density specimens, for which a seemingly undistorted rf is obtained, one could still measure negative dispersion. In Fig. 5 of the present manuscript, three different time-domain ultrasonic signals (typical of data collected for this study) are

FIG. 4. Broadband measurements of regions of the stepped plastic plate exhibiting a linearly increasing attenuation coefficient and negative dispersion for the selected bandwidth.

shown. The left-most panel shows an rf signal acquired from the 100% thin side of the plate. To the right are two different rf signals acquired as one insonifies locations closer and closer to the step, 80% thin 20% thick and 51% thin 49% thick, respectively. Even when a modest amount of the step is interrogated by the ultrasonic field, and then when the observed rf signal appears to be relatively undistorted, one still measures a linearly increasing attenuation coefficient with an apparently negative dispersion. Hence the presence of only a small amplitude of a second signal (in bone, for example, a fast wave mode in a signal dominated by a slow wave mode, or *visa versa*) could account for the presence of negative dispersion observed in many measurements of bone.¹⁻⁶ This hypothesis is the basis for a model introduced by our laboratory that might account for the measurements of (anomalous) negative dispersion observed in studies of cancellous bone.¹¹ More sophisticated models, such as the stratified model³⁰⁻³² or extensions of the porous media model of Biot,³³⁻³⁵ will be necessary to account for the detailed behav-

ior of bone. But such models are likely to include the simultaneous existence of multiple modes of propagation, which is the fundamental feature of the unsophisticated phantom employed in the present experiment to demonstrate the underlying physics resulting in apparent negative dispersion.

E. Limitations of the present study

The ultrasonic frequencies used to investigate the phantom described in this study are substantially higher than those typically used in the quantitative ultrasonic study of bone. However, the range of frequencies (3–7 MHz) represents a fractional bandwidth similar to that employed in many studies of bone (300–700 kHz).

The sample used in the present study was not capable of propagating fast and slow modes, nor did it exhibit the ultrasonic attenuation and speed of sound of cancellous bone. The purpose of this study was to address the frequency dependence of an apparent attenuation coefficient and an apparent phase velocity of a sample that mimicked the anomalous properties of those often reported for cancellous bone: a linearly increasing attenuation coefficient and a negatively decreasing phase velocity. The sample with which we chose to pursue this goal was designed to be the simplest possible type of phantom that could propagate two temporally overlapping signals. The goal was to illustrate the fact that when one detects two overlapping signals, analogous to the superimposed fast and slow modes common to studies of cancellous bone, it is possible for traditional phase spectroscopy to yield negative dispersion.

We suggest that for some signals transmitted through cancellous bone, one often cannot distinguish whether there are, in fact, two superimposed signals. For low and medium density cancellous bone,³ the acquired signal through the bone can be a single rf signal that appears relatively undistorted, yet this signal might be comprised of two modes. The potential misinterpretations of this undistorted signal are the focus of this manuscript.

The mechanism described in this manuscript may not be the sole explanation for the observation of negative dispersion. Scattering and mode conversion within the stratified microarchitecture of the trabeculae presumably result in complex signals that may contribute to other sources of negative dispersion. Although the totality of possible sources of the causal inconsistency between a linearly increasing attenuation coefficient and a simultaneously decreasing with frequency phase velocity remains unclear, evidence in this

FIG. 5. Representative time-domain rf signals for the 100% thin, 80% thin 20% thick, and 51% thin 49% thick locations measured along a stepped plastic plate illustrating progressively more overlap in the received signals.

study suggests that the anomalous dispersion observed in ultrasonic studies of cancellous bone can arise from an interference between two waves, such as the fast and slow waves predicted by Biot theory.

V. SUMMARY

Broadband and narrowband measurements were made on two overlapping signals arising from propagation through a simple phantom. The phase sensitively measured broadband phase velocity and attenuation coefficient were observed to undergo drastic changes in frequency dependence as a function of location on the sample. Despite this behavior, narrowband measurements agreed very well with broadband data taken at the same spatial locations. Further, it was possible to measure negative dispersion at sites exhibiting a linearly increasing attenuation coefficient, in contrast to what might be expected based on the nearly local form of the once-subtracted Kramers–Kronig equations.

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