Published in final edited form as:

Atherosclerosis. 2009 March ; 203(1): 47–48. doi:10.1016/j.atherosclerosis.2008.07.003.

Variability of arterial wall shear stress, its dependence on vessel diameter and implications for Murray's Law

Morton H. Friedman

Department of Biomedical Engineering, Duke University, Durham NC 27708 USA, E-mail address: mort.friedman@duke.edu

Keywords

Wall shear stress; Arteries; Variability; Species Effects; Murray's Law

Dear Editor

Cheng et al [1] carried out a thorough literature review to generate a valuable compilation of experimental wall shear stress (WSS) data demonstrating the variability of this important hemodynamic variable in man and among species. We agree with the authors that, given this apparent variability, care should be used in generalizing results from a vessel in one anatomical location to vessels elsewhere. They furthermore conclude from their compilation that (1) shear stress in man varies inversely with vessel diameter and (2) the variation among species of carotid artery WSS violates Murray's Law relating vessel diameter and flow. However, using their data, we have been unable to come to the same conclusions.

The human WSS data compiled by Cheng et al are summarized in their Table 1. Twenty-seven data from 15 papers quantifying the WSS in seven arteries or arterial segments are presented (18 references are cited in the text; three of these relate to the coronary arteries, do not include any shear measurements, and are absent from the table). Corresponding lumen diameter data were provided for 16 of the 27 shear data; no diameter data were available for two of the seven segments. Thus the data available to test the null hypothesis that shear is independent of vessel diameter consists of 16 paired values of shear and diameter in five segments. The p-value of a linear regression of WSS vs. diameter ($N=16$) is 0.23; $r^2 = 0.10$.

Cheng et al plot the data differently in their Fig. 3A. They show seven paired values of shear and diameter, fewer than the 16 individual data and more than the five segments for which paired data are provided in their Table 1. Five of the points do seem to match the averages for the latter segments; the remaining two correspond to diameters in the 2–3 mm range, presumably coronary arteries. For the five segments for which data are provided, the p-value of a regression of the average shear stress for each artery versus the average diameter ($N=5$), is 0.26; $r^2 = 0.39$.

The correlation in Fig. 3A in Cheng et al is driven by a single data point in the infrarenal aorta (IRA). A weighted least squares regression, taking into account the larger number of data for

several of the other arteries, was not attempted because of the known sensitivity of such analysis to outliers, which the IRA datum might be.

If there is a relationship between wall shear and wall diameter, it might be masked by the variability in measurement technique and in the methods used by each group to estimate wall shear from near-wall velocity data, as well as individual variability. The effects of technique differences can be eliminated by examining the results from investigators who made measurements of shear and diameter in more than one artery. Viewed this way, the data do not generally support an inverse relation between WSS and diameter. C.P. Cheng et al [2], the source of the single IRA datum, also made measurements in the supraceliac aorta (abbreviated SCRA in Cheng et al). Processing C.P. Cheng's SCRA data in the same way that Cheng et al processed their IRA data, we find a value for the diameter of the SCRA that is more than 3 mm greater than that of the IRA, while the WSS in the SCRA obtained by C.P. Cheng et al was more than twice that in the IRA. Similarly, measurements by Dammers et al [3] show that the shear in the 6.7-mm common carotid artery is 2.4 times that in the 3.7-mm brachial artery. Wu et al [4] measured the shear stress in the superficial femoral, brachial and common carotid arteries, and found that the shear in the last of these was more than $2\frac{1}{2}$ times the shear in the first, although the average diameters of the two vessels were virtually identical. The brachial artery experienced an intermediate shear stress although its diameter was only slightly more than half of that of the other two vessels. The only pairwise comparison that demonstrates an inverse relationship between shear and diameter is that of Gaenger et al [5], who used high-resolution echo Doppler to compare the brachial (4.2 mm, 7.8 dynes/cm²) and common femoral (7.8, 2.9) arteries. However, Silber et al [6] compared the same two arteries using MRI phase contrast, and found no differences of consequence in either diameter or shear. The murine experiments reported by Cheng et al also demonstrate large shear variations unaccompanied by significant differences in diameter: the wall shear stress in the murine femoral artery was four times that in the common carotid artery, but the vessels are nearly identical in size.

In the absence of near-wall velocity data, arterial WSS is often estimated from mean velocity

or flow and vessel diameter: $\langle \text{WSS} \rangle = \frac{32\mu \langle q \rangle}{\pi d^3}$ where μ is fluid viscosity, q is flow rate, d is vessel diameter, and the brackets denote averages over a cardiac cycle.. This estimate is based on several idealizations of the vascular flow field, and Cheng et al accordingly included in their Table 1 only shear data derived from near-wall velocity measurements. The major idealizations are that the artery is straight and uniform and that the flow is fully developed. It has been known for many years that this is an approximation, owing to the irregular shape and frequent branching of the arterial tree. However, steady flow is not assumed: for fully developed and parallel pulsatile flow, the relation between time-average shear and time-average flow is the same as that between their steady flow counterparts. Of course, the most interesting parts of the vasculature for fluid mechanics researchers in atherosclerosis are branches and bends, where the approximation is certain to fail.

Cheng et al also examined the variation among species of WSS and diameter for the common carotid artery. Here, shear stress appears to have been estimated from mean flow and diameter, presumably using the approximation discussed above. The inverse dependence of shear stress on vessel diameter is clear, in spite of considerable variability among the estimates for individual species. Data from multiple sources for each species appear to have been averaged – in their Fig. 3B, each species is represented by a single data point – and their regression analysis yields $\text{WSS} \sim d^{-0.5}$ across species. This result is claimed to contradict Murray's law [7].

Murray derived his eponymous law by constructing an expression for the power required to pass blood through a vessel of specified length at a specified flow rate. Two power requirements

were considered: (1) the cost of overcoming frictional losses in the vessel and (2) the cost of maintaining the blood that occupies the vessel volume. He found that the power requirement was a minimum when $q/r^3 \sim (b/\mu)^{1/2}$, where r is vessel radius and b is the unit cost of blood in ergs/s-cc. Thus, for an individual, in whom b and μ are uniform, q/r^3 is predicted to be the same for all vessels. This might reasonably be extended to apply to all vessels in a *single species*. We saw earlier that, for a given viscosity, and for fully developed parallel flow, this ratio is proportional to WSS; hence the oft-quoted result that Murray's law predicts a uniformity of shear stress in the vasculature.

If we start with the parallel flow solution that $WSS \sim q/r^3$, the authors' finding that, across species, the carotid artery WSS is proportional to $r^{-0.5}$ suggests that $q \sim r^{2.5}$. This caused Cheng et al to question Murray's Law, but the authors' conclusion that their result supports $q \sim r^2$ rather than Murray's $q \sim r^3$ does not seem to follow. The confidence interval for the exponent is unspecified and no error bars are provided in their Fig. 3B.

It would seem that the greater challenge to Murray's law presented in Cheng et al is the apparent variability of WSS in geometrically uncomplicated regions of the human and murine vasculatures, a conclusion also reached in a recent paper by Reneman and Hoeks [8]. Reneman et al [9] pointed out that shear stress in large arteries is influenced by distal features such as peripheral resistance and wave reflections; were Murray's Law determining in the vasculature, one would expect these vessels to remodel to diameters such that, notwithstanding downstream effects, a common shear stress would be achieved. Reneman and Hoeks also reviewed earlier studies of the relationship between flow and vessel size in a variety of vessels ranging from large arteries to arterioles. The exponent n in $q \sim r^n$, which would equal 3 throughout the vasculature according to Murray's Law, varies from 2, in branches off the human aortic arch, to 3, in rat arterioles. These variabilities suggest that other factors besides those proposed by Murray play a role in determining wall shear stress in relatively straight and unbranched arterial segments.

The variation of WSS *among species* presented in Cheng et al and reported earlier by Greve et al [10] may reflect an allometric variation of b (species variations in μ alone are insufficient explanation) and can accordingly be rationalized without abandoning Murray. For instance, the "unit cost" of maintaining blood could be greater for smaller species. The species variation of the infrarenal aortic shear data presented in Greve et al can be explained in terms of Murray's Law if $b \sim M^{-0.76}$, where M is body mass. Of course, this variation can also reflect an entirely distinct species-dependent phenomenon. In any event, Murray's paper explicitly "center(s) around oxygen transport in man" and his Law makes no prediction regarding the variation of shear among species, so such data neither supports nor challenges its validity.

References

1. Cheng C, Helderma F, Tempel D, et al. Large variations in absolute wall shear stress levels within one species and between species. *Atherosclerosis* 2007;195:225–235. [PubMed: 17169362]
2. Cheng CP, Herfkens RJ, Taylor CA. Abdominal aortic hemodynamic conditions in healthy subjects aged 50–70 at rest and during lower limb exercise: in vivo quantification using MRI. *Atherosclerosis* 2003;168:323–331. [PubMed: 12801616]
3. Dammers R, Stiff F, Tordoir JH, et al. Shear stress depends on vascular territory: comparison between common carotid and brachial artery. *J Appl Physiol* 2003;94:485–489. [PubMed: 12391066]
4. Wu SP, Ringgaard S, Oyre S, et al. Wall shear rates differ between the normal carotid, femoral, and brachial arteries: an in vivo MRI study. *J Magn Reson Imaging* 2004;19:188–193. [PubMed: 14745752]
5. Gaenzer W, Neumayr G, Marschang P, et al. Flow-mediated vasodilation of the femoral and brachial artery induced by exercise in healthy nonsmoking and smoking men. *J Am Coll Cardiol* 2001;38:1313–1319. [PubMed: 11691501]

6. Silber HA, Ouyang P, Bluemke DA, et al. Why is flow-mediated dilation dependent on arterial size? Assessment of the shear stimulus using phase-contrast magnetic resonance imaging. *Am J Physiol Heart Circ Physiol* 2005;288:H822–H828. [PubMed: 15345491]
7. Murray CD. The physiological principle of minimum work. I. The vascular system and the cost of blood volume. *Proc Nat Acad Sci US* 1926;12:207–214.
8. Reneman RS, Hoeks APG. Wall shear stress as measured in vivo: consequences for the design of the arterial system. *Med Bio Eng Comput* 2008;46:499–507. [PubMed: 18324431]
9. Reneman RS, Arts T, Hoeks APG. Wall shear stress – an important determinant of endothelial cell function and structure – in the arterial system in vivo. *J Vasc Res* 2006;43:251–269. [PubMed: 16491020]
10. Greve JM, Les AS, Tang BT, et al. Allometric scaling of wall shear stress from mice to humans: quantification using cine phase-contrast MRI and computational fluid dynamics. *Am J Physiol Heart Circ Physiol* 2006;291:H1700–H1708. [PubMed: 16714362]