Fourier rebinning algorithm for inverse geometry CT

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Inverse geometry computed tomography (IGCT) is a new type of volumetric CT geometry that employs a large array of x-ray sources opposite a smaller detector array. Volumetric coverage and high isotropic resolution produce very large data sets and therefore require a computationally efficient three-dimensional reconstruction algorithm. The purpose of this work was to adapt and evaluate a fast algorithm based on Defrise's Fourier rebinning (FORE), originally developed for positron emission tomography. The results were compared with the average of FDK reconstructions from each source row. The FORE algorithm is an order of magnitude faster than the FDK-type method for the case of 11 source rows. In the center of the field-of-view both algorithms exhibited the same resolution and noise performance. FORE exhibited some resolution loss (and less noise) in the periphery of the field-of-view. FORE appears to be a fast and reasonably accurate reconstruction method for IGCT. © 2008 American Association of Physicists in Medicine.

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I. INTRODUCTION

Inverse geometry computed tomography (IGCT) is a new volumetric CT geometry that is currently under investigation.¹⁻⁵ Instead of using a single x-ray focal spot to acquire a projection, IGCT employs a large array of focal spots opposite a detector array that has the same axial extent as the source array but is much smaller in the transverse direction (Fig. 1).

It is important to find a reconstruction algorithm for this geometry that is accurate, fast, and signal-to-noise ratio (SNR) efficient. Computational efficiency is important due to the need to process a very large data set that is inherent to IGCT due to the presence of rays at multiple angles with respect to the axis-of-rotation.

Previous work focused on a three-dimensional filtered-backprojection (3DFBP) approach.² Although 3DFBP is accurate and SNR efficient, it is computationally expensive.

Positron emission tomography (PET) has a similar sampling distribution to IGCT (after a rotation). The 3D re-projection algorithm⁶ (3DRP) is a PET algorithm that performs a two-pass reconstruction. The 3DRP algorithm is accurate, however, it is also slow due to a costly reprojection step.

The purpose of this work is to evaluate a much faster algorithm based on a Fourier rebinning (FORE) algorithm developed by Defrise *et al.* for PET,⁷ and also applied to multiple orbit SPECT.⁸

II. METHODS

II.A. FORE algorithm description

FORE was designed for a PET geometry and we will attempt to use the same notation as described in Ref. 7. Each

line-of-response (LOR) can be characterized by four parameters: two in-plane parameters (s, ϕ) and two longitudinal parameters (z, δ) (Fig. 2). s and ϕ are the radial distance and angle, respectively, of the LOR projected onto a transverse plane. z is the midpoint of the LOR and δ is the tangent of the angle the LOR makes with a transverse plane.

Any LOR can then be described as the projection

$$p(s, \phi, z, \delta) = \int_{-\infty}^{\infty} f(s \cos \phi - l \sin \phi, s \sin \phi + l \cos \phi, z + l \delta) dl. \quad (1)$$

For each z and $\delta \neq 0$, we call the two-dimensional (2D) data as a function of s and ϕ an oblique sinogram. The $\delta=0$ subset data are the in-plane sinograms. One can take a 2D Fourier transform of p with respect to s and ϕ to get $P(\omega, k, z, \delta)$, where ω and k are the frequency variables associated with s and ϕ , respectively. One of the main results of Ref. 7 is the approximation

$$P(\omega, k, z, 0) \approx P[\omega, k, z + (k/\omega)\delta, \delta], \quad (2)$$

which suggests that one can rebin all of the oblique sinograms into in-plane sinograms. After the rebinning process, the data is in the form of a stack of in-plane 2D sinograms that can then be reconstructed using any 2D algorithm. The data set is reduced in size, resulting in a very computationally efficient algorithm. The basic steps of the FORE algorithm are as described in Ref. 7:

- (1) Initialize a stack of 2D Fourier-transformed in-plane sinograms $P(\omega, k, z, 0)$ to zero.
- (2) For each (z, δ) sinogram:

FIG. 1. Inverse geometry CT.

- (a) Take the 2D Fourier transform in s and ϕ to form $P(\omega, k, z, \delta)$.
- (b) For each (ω, k) , rebin $P(\omega, k, z, \delta)$ to $P[\omega, k, z, -(k/\omega)\delta, 0]$ using interpolation onto the discrete samples in z .
- (3) Normalize the rebinned stack to account for variable density of contributions by applying the same procedure to unit data ($P=1$).
- (4) Take the inverse 2D Fourier transform to get the stack of in-plane sinograms $p(s, \phi, z, 0)$.
- (5) Perform a series of 2D reconstructions, slice by slice, using any method.

II.B. Adaptation to IGCT

The IGCT sampling geometry is similar to that of PET. After a 2π rotation, the IGCT projection data can be represented as a set of in-plane and oblique sinograms. We call a line of focal spots at the same axial coordinate a source row and similarly we call a line of detector elements with the same axial coordinate a detector row. Using the same notation as in Sec. II A, we can treat each ray measurement from source to detector like a LOR and thus represent the data set in the same form $p(s, \phi, z, \delta)$. There are five deviations from the PET case:

FIG. 3. Reconstructions of the slice at $z=-3$ mm windowed to 400 HU around 0. (a) FDK and (b) FORE reconstructions of the torso phantom. (c) FDK and (d) FORE reconstructions of the truncated torso phantom. The boxes in (a) depict the regions where noise was estimated.

- (1) In IGCT, if the source and detector arrays are both flat and parallel to each other, all ray measurements from a fixed source row to a fixed detector row have the same δ . In PET, the δ values of LORs between two detector rings depend on s . In Ref. 7, it is assumed that s is always much smaller than the detector ring radius so that δ is approximately uniform. This approximation does not need to be made in IGCT.
- (2) PET data are sampled uniformly in z . IGCT data sampling in z is nonuniform due to the different sample spacings within the source and detector arrays in z . Because Ref. 7 used a gridding scheme for axial interpolation of the rebinned data, no changes for the IGCT sampling scheme were necessary.
- (3) If the magnification factor for IGCT is not 2, we need to redefine z from its original definition as the midpoint of the LOR. Let z_A and z_B denote the endpoints of a ray measurement from source spot A to detector element B in IGCT. Let SDD and SID be the IGCT source-to-detector and source-to-isocenter distances. We redefine $z = [(SDD-SID)/SDD]z_A + (SID/SDD)z_B$. This is consistent with the definition for the PET geometry where the effective magnification is 2 (i.e., $SDD=2SID$).
- (4) In Ref. 7, it was noted that PET data are sampled over a range $\phi \in [0, \pi)$. Therefore, to properly form the Fourier

FIG. 2. (a) Transverse view and (b) longitudinal view of a PET system. A LOR is characterized by four parameters. s and ϕ are in-plane parameters, z and δ are longitudinal parameters.

FIG. 4. (a) FDK and (b) FORE reconstructions of the Shepp-Logan phantom.

FIG. 5. Profile plots along the central vertical axis of the Shepp-Logan reconstructions.

transformed oblique sinogram, one has to combine the data from the two half-sinograms (z, δ) and $(z, -\delta)$ using the symmetry relation $p(s, \phi + \pi, z, \delta) = p(-s, \phi, z, -\delta)$. This step is not necessary if one views the data as being sampled over the full range $\phi \in [0, 2\pi)$.

- (5) For low frequencies ($|\omega| < \omega_{\text{lim}}$ and $|k| < k_{\text{lim}}$), Ref. 7 used the approximation $P(\omega, k, z, 0) \approx P(\omega, k, z, \delta)$, with ω_{lim} and k_{lim} set to small nonzero values. We did not apply this approximation because we did not observe any benefit from its use (i.e., we used $\omega_{\text{lim}} = 0$ and $k_{\text{lim}} = 0$).

The FORE algorithm requires that each projection in a sinogram be composed of parallel rays that are equally spaced in s . As in PET, rebinning is required for IGCT data. A 2D gridding algorithm⁹ was employed to resample the rays from each source/detector row pair into parallel ray oblique sinograms with uniform sample spacing in s and ϕ using a 2D Kaiser-Bessel kernel with parameter $\beta = 9.1375$,¹⁰ followed by postdensity compensation using unit data as the input.

II.C. FDK-like algorithm for comparison

To compare FORE to another reconstruction algorithm that similarly does not require a reprojection step, we developed an algorithm based on FDK.¹¹

In IGCT, the data from each source row is very analogous to the data from a circular source trajectory. We can therefore treat the entire data set as multiple circular source trajectories, one for each source row. As in FORE, we first grid the data from each source row and detector row pair into parallel rays forming a 2D oblique sinogram. All sinograms corresponding to a single source row are then one-dimensional (1D) filtered and backprojected into the 3D volume. To account for a variable number of contributions to each voxel, unit data were backprojected using the same process and the result is used to normalize the final reconstruction.

TABLE I. Numerical torso phantom simulation parameters.

SDD (Source-to-detector distance)	950 mm
SID (Source-to-isocenter distance)	550 mm
Detector axial coverage	1.0 mm $\times$ 50 rows
Source axial coverage	5.0 mm $\times$ 11 rows
Radial coverage (s)	1.0 mm $\times$ 512 samples
View coverage (ϕ)	2π (1024 samples)
Number of rebinned slices	50

II.D. Computational analysis

Let N and M denote the number of detector rows and source rows, respectively. Let P denote the number of radial samples per oblique sinogram, and assume that the number of views is also P . There are therefore NM oblique sinograms, each with P^2 samples. Further, we assume that the reconstructed volume will have on the order of N slices and P^2 pixels per slice.

The FDK-like algorithm consists of $2NMP$ 1D Fourier transforms of length $2P$ to filter the sinogram data. The factor of two in $2NMP$ comes from the need to perform both forward and inverse transforms. The factor of 2 in $2P$ comes from the need to zero pad the data in s prior to filtering. This is followed by 3D backprojection for all source rows requiring MP operations for each of the NP^2 pixels. Therefore, the total computational effort of the FDK-like algorithm is on the order of $4NMP^2 \log(2P) + NMP^3$, where we have assumed that a $2P$ -length FFT requires $2P \log(2P)$ operations. If we divide by the number of reconstructed pixels, NP^2 , then the per-pixel effort is $4M \log(2P) + MP$. In the backprojection-limited case where $P \gg M$, the FDK-like algorithm requires on the order of MP operations per reconstructed pixel.

The FORE algorithm requires NM forward and N inverse 2D transforms. After zero padding in s , each sinogram has $2P \times P$ samples. Each $2P \times P$ 2D Fourier transform can be decomposed into P 1D transforms of length $2P$ (s direction) and $2P$ 1D transforms of length P (ϕ direction). As in the FDK analysis, the s direction transforms require a total of $2NMP^2 \log(2P)$ operations for the forward direction and $2NP^2 \log(2P)$ for the inverse direction. Similarly, the ϕ direction transforms require an additional $2NMP^2 \log P + 2NP^2 \log P$ operations. The Fourier rebinning step requires $2NMP^2$ operations (one for each data point). Finally, N 2D reconstructions require NP^3 backprojection operations. Thus, FORE requires $2N(M+1)P^2 \log(2P) + 2N(M+1)P^2 \log P + 2NMP^2 + NP^3$ operations. If we again divide by the number of reconstructed pixels, NP^2 , FORE requires $2(M+1)\log(2P) + 2(M+1)\log P + 2M + P$ operations per pixel. In the backprojection-limited case where $P \gg M$, FORE requires on the order of P operations per pixel.

The FDK-like algorithm therefore requires roughly M times as many operations per reconstructed pixel as the FORE algorithm.

FIG. 6. Theoretical and simulated resolution from the FDK and FORE algorithms. (a) In-plane MTF at 10 mm and (b) 100 mm away from the z axis. (c) Fourier transform along the z axis of the point spread function at 10 and (d) 100 mm away from the axis.

II.E. Simulations

Projection data of a numerical torso phantom (uniformly sampled in s and ϕ , but sampled under the IGCT geometry in z and δ) were simulated, modeling finite source and detector blurring, under the geometry specified in Table I.

Noisy data were generated using a Poisson model. The entrance intensity, I_0 was set to 20 000 photons per ray.

A second simulation was performed with the torso phantom truncated in z at the central slice for a more stringent test of the 3D performance of the algorithms. We call this phantom the truncated torso phantom.

To test low contrast performance, a 3D Shepp–Logan phantom was simulated.

The noise in the reconstructed volumes was evaluated by calculating standard deviations within 32×32 pixel regions both near and far from the axis-of-rotation across all slices. Additionally, standard deviations were calculated from difference images between the FORE and FDK reconstructions.

To compare the resolution performance of the algorithms, oversampled images of two $200 \mu\text{m}$ spheres at 10 and 100 mm away from the axis-of-rotation were simulated.

To evaluate cone-beam effects from high spatial frequencies in z , a numerical “Defrise” phantom was simulated which consisted of 13 ellipsoids stacked along the z axis. All ellipsoids had x , y , and z radii of 50, 50, and 1 mm, respectively. Their centers were separated by 4 mm along z .

II.F. Physical experiments

The experimental setup for the table-top experiments is the same as in Ref. 4. We used a C-arm angiography system (NovaRay, Palo Alto, CA) that employs a large-area scanned anode x-ray source opposite a small-area fast photon-counting detector array.¹² Data from 100×100 focal spots and 48×48 CdZnTe detectors were used in the experiment. An anthropomorphic torso phantom was scanned in a step-and-shoot mode.

The data for each source row and detector row pair were regridded in s and ϕ into parallel data corresponding to sinograms as explained in Sec. II B.

Dose measurements were not conducted. However, one way to gauge the dose level for the experiment is to calculate the mAs product of a conventional CT scanner that would result in an equivalent total photon flux through isocenter. This analysis resulted in an effective mAs of 14.

III. RESULTS

III.A. Simulations

Fifty slices (each 512×512 pixels) were reconstructed with the FORE algorithm at a computation time of 13.6 s/slice while FDK ran at 194 s/slice. Figure 3 shows the reconstructed images of both the noisy torso phantom and truncated torso phantom at slice location $z = -3$ mm

FIG. 7. Standard deviation from two 32×32 pixel regions across the volume. (a) Near the axis-of-rotation. (b) 100 mm away from the axis.

(3 mm from the truncation point), all displayed with a window width of 400 HU. The truncated torso phantom exhibited cupping artifacts in both the FORE and FDK reconstructions. The discrete nature of the cupping in the FDK reconstruction is due to the transition when a source row suddenly “sees” a pixel. The pixels closer to the axis-of-rotation are seen by sources that have larger cone angles and therefore produce more cupping error. Compared to FDK, the FORE cupping artifact is more gradual, but more severe at the edges. Figures 4 and 5 depict the Shepp–Logan phantom reconstructions and profiles along the central vertical axis, respectively, demonstrating similar low contrast performance. The axial resolution degradation in FORE (see below) is evident in the periphery.

Figure 6 shows the spatial resolution at 10 and 100 mm away from the axis-of-rotation. At locations away from the axis-of-rotation, FORE exhibits some loss of resolution in the z direction.

Figure 7 shows the standard deviation as a function of z in the regions depicted in Fig. 3(a). Near the axis-of-rotation, FDK and FORE reconstructions yield similar noise as can be seen from their standard deviation and also from the low standard deviation of the difference between the FDK and FORE images. At 100 mm from the axis-of-rotation FORE exhibited approximately 10% less noise across all slices. This may be related to the loss of axial spatial resolution in this region.

FIG. 8. FDK (top) and FORE (bottom) reconstructions of the numerical Defrise phantom. In this case the vertical direction is the longitudinal (z) direction.

Figure 8 shows FDK and FORE reconstructions of the numerical Defrise phantom. Although both suffer from cone-beam effects in the center of the field-of-view, FORE does not have the long range streaking artifacts that are prominent in the FDK reconstruction.

FIG. 9. (a) A 2D FBP reconstruction of the central slice of the anthropomorphic torso phantom using rays from the central source row and detector row only. (b) FDK and (c) FORE reconstructions.

III.B. Physical experiments

Figure 9(a) shows a 2D FBP reconstruction of the central slice of the anthropomorphic torso phantom using only in-plane rays. Figures 9(b) and 9(c) are FDK and FORE reconstructions of the same slice which show the advantage of utilizing all ray measurements in reducing noise.

IV. DISCUSSION AND CONCLUSIONS

FORE's superior speed is primarily due to the rebinning step. It reduces the 3D reconstruction problem into a series of 2D reconstruction problems. Also, no interpolation is necessary with respect to the in-plane parameters s , ϕ , w , or k (assuming the data are parallel and uniform in s and ϕ). However, FORE loses some axial resolution and has an asymmetric point spread function in the periphery. This is compensated by lower noise in these regions.

Like FDK, FORE suffers from cone-beam effects. Unlike FDK, FORE did not suffer from streak artifacts in the De-frise phantom reconstruction. FORE is an approximate algorithm, and we believe that the residual cone-beam effects are due to the approximate nature of FORE. An exact version of the FORE algorithm for PET, FORE-J,¹³ is currently being investigated for application to IGCT.

In conclusion, FORE is a reasonably accurate algorithm for IGCT data with significantly lower reconstruction times compared to a FDK-like algorithm based on 3D backprojection.

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