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Determination of rigid body registration marker error from edge error

Clark R. Andersen

Division of Research, Department of Orthopaedic Surgery and Rehabilitation, The University of Texas Medical Branch, 301 University Blvd., Galveston, TX 77555-0174, Phone: (409) 747-3221, Fax: (409) 747-3240, clanders@utmb.edu

Abstract

Registration markers affixed to rigid bodies (fixed to bone as opposed to skin) are commonly used when tracking 3D rigid body motion. The measured positions of registration markers are subject to unavoidable errors, both systematic and non-systematic. Prior studies have investigated the error propagated to such derived properties as rigid body positions and helical axes, while others have focused on the error associated with a specific position tracking system under restricted conditions. Theoretical and simulation-based error propagation requires knowledge of the variation due to individual registration markers; however, the variation in registration marker position measurement has previously been either assumed or determined from static cases. The objective of this paper is the introduction of a method for determining individual marker variation irrespective of change in rigid body position or motion by utilizing the distances between the markers (edge lengths), which are invariant under rotation and translation. Simulations were used to validate and characterize the introduced technique, demonstrating that the predictions improve with greater edge length and additional markers, converge on reference values where the edge length is at least 4 times the magnitude of the maximum vertex variation, and that under ideal conditions the confidence interval about the predicted variation is within 7% of the maximum variation associated with that marker set. The introduced technique was tested on the results of a motion tracking experiment to demonstrate the wide disparity in vertex variation between static and non-static measurements of the same registration markers, where the non-static variation exceeded the static variation by an average factor of 12.7.

Keywords

Registration Marker; Vertex Variation; Motion Analysis; Kinematics; Error Propagation

1. Introduction

Arrays of registration marker vertices are routinely utilized in the context of tracking rigid bodies in such fields as motion analysis and kinematics. In kinematics, this would refer to studies in which markers are fixed directly to rigid anatomic structures (bones) (El-Shennawy et al., 2001; Nester et al., 2007), as opposed to those studies in which surface (skin) markers are used (Schmidt et al., 1999; Ren et al., 2008). There is inherent error involved with measured positions of registration markers. This variation has previously been determinable only in static

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positions. However, this fails to account for the additional systematic and non-systematic errors to which a marker in multiple positions, whether static increments or active motion, is susceptible; these errors may include such factors as marker occlusion, capture volume, distance between markers, number of cameras that record the marker, motion blur, and variable marker geometry and reflectivity.

Prior work on marker variation has focused on error classification (Ball and Pierrynowski, 1996) or error propagation from the markers to the associated rigid body positions or helical axes (Woltring et al., 1985; Spoor, 1984); other research has focused on evaluating the accuracy and precision of a specific tracking system (Vander Lindent et al., 1992; Windolf et al., 2008). Establishing the variation of the markers in a rigid body experiment will facilitate a directly relevant simulation of the variation in those bodies and associated parameters such as position and helical axis. Further, determination of individual marker variation provides a means to identify troublesome markers for repair or replacement so as to increase the accuracy of associated measurements. The objective of this paper is the determination of registration marker variation irrespective of the change in position of the associated rigid bodies to which the markers are fixed. While the techniques introduced here may not be directly applicable to the broader error context of skin markers and associated skin movement, they may be applied to pre-calibration of those markers if they are temporarily fixed to a rigid structure. When skin markers are used in the context of tracking associated rigid body segments this technique may be applicable if the skin marker error is normally distributed. Further, the techniques are relevant to any system for measuring the 3D positions of rigidly fixed registration vertices wherein the measurements are normally distributed; these may include optoelectronic, magnetic, and manual digitizer systems.

2. Determining Marker Error from Edge Error

2.1. Propagating marker variation to edge variation

The measured position of a marker vertex is subject to random noise, which is assumed to follow an approximately normal distribution. While it is a simple matter to determine the variation of individual registration markers in a static position through repeated measures of their positions, this is not an option when the markers are in motion. Measurements of static variation are unable to characterize the variation throughout the capture volume and perspective space when the tracked rigid body is recorded in multiple positions or in motion. The solution is to work from the one quantity which should remain constant even in motion, the edge lengths. Edge lengths refer to the distances between linked pairs of markers, which should be invariant following rotational or translational transformations.

Error propagation provides a means to relate the error (as standard deviation) in the edge lengths to the error of individual vertices (as mean component standard deviation), so long as the vertex error is normally distributed and vertices are sufficiently far apart that overlap of their error distributions is negligible. Considering that vertex error is assumed to be normally distributed, and sums or differences of normal distributions are normally distributed, the error in the distances between vertices will be normally distributed. Conversely, when the edge length variation follows a normal distribution, the vertex errors should also be normally distributed.

For a given linked pair of vertices U and V , the error of the individual vertices (ξ_U and ξ_V) may be propagated through the distance between U and V ($\|U - V\|$) using partial derivatives. This simplifies to yield the error of the edge length τ_{UV} in terms of the Euclidean norm of ξ_U and ξ_V :

$$\tau_{uv}^2 = \left(\left(\frac{\partial \|U-V\|}{\partial U_x} \right)^2 + \left(\frac{\partial \|U-V\|}{\partial U_y} \right)^2 + \left(\frac{\partial \|U-V\|}{\partial U_z} \right)^2 \right) \xi_u^2 + \left(\left(\frac{\partial \|U-V\|}{\partial V_x} \right)^2 + \left(\frac{\partial \|U-V\|}{\partial V_y} \right)^2 + \left(\frac{\partial \|U-V\|}{\partial V_z} \right)^2 \right) \xi_v^2 = \xi_u^2 + \xi_v^2 \quad (1.1)$$

2.2. Solving for marker variation in the 3-marker case

In the minimal case of a set of 3 registration marker vertices defining a triad ABC, the errors of the 3 edge lengths permit an ideal solution. Building upon (1.1), a system of 3 equations can be constructed and solved for the 3 errors of each vertex of the triad as:

$$\begin{aligned} \tau_{AB}^2 &= \xi_A^2 + \xi_B^2 \\ \tau_{BC}^2 &= \xi_B^2 + \xi_C^2 \\ \tau_{CA}^2 &= \xi_C^2 + \xi_A^2 \end{aligned} \quad (1.2)$$

The solution of this set of equations yields the individual vertex errors as:

$$\begin{aligned} \xi_A &= \sqrt{\frac{\tau_{AB}^2 - \tau_{BC}^2 + \tau_{CA}^2}{2}} \\ \xi_B &= \sqrt{\frac{\tau_{AB}^2 + \tau_{BC}^2 - \tau_{CA}^2}{2}} \\ \xi_C &= \sqrt{\frac{-\tau_{AB}^2 + \tau_{BC}^2 + \tau_{CA}^2}{2}} \end{aligned} \quad (1.3)$$

2.3. Solving for marker variation in all cases

Solving for vertex errors in the general case of N vertices presents a more complex problem.

The set of edges between each vertex and every other vertex yields $E = \frac{N!}{2(N-2)!}$ edges, as combinations of size 2 taken from the set N. When N exceeds 3, this results in more equations than vertices, requiring a least squares approach.

The least squares solution to a set of linear equations is determinable using the Moore-Penrose pseudoinverse (Penrose, 1956). Rewriting (1.1) as,

$$\tau_{jk}^2 = \sum_{i=1}^N C_i \xi_i^2, \quad \begin{cases} C_i = 1, & (i=j)|(i=k) \\ C_i = 0, & (k \neq j) \& (i \neq k) \end{cases} \quad (1.4)$$

and defining an E-row column-vector **b** as the set of all squared edge errors τ_{jk}^2 , an E-row by N-column matrix **A** filled with the corresponding coefficients C_i , and an N-row column-vector **x** as the set of all corresponding squared vertex errors ξ_i^2 , the system of equations in matrix form becomes **b** = **Ax**. The general least squares solution for the vertex errors of a set N vertices is then,

$$\mathbf{x} = \mathbf{A}^+ \mathbf{b}, \quad (1.5)$$

where $\mathbf{A}^+$ is the Moore-Penrose pseudoinverse of $\mathbf{A}$. When $N=3$, (1.5) yields (1.3).

For the commonly used case of tetrad (4-marker) registration markers, the matrix form appears as follows:

$$\begin{bmatrix} \xi_1^2 \\ \xi_2^2 \\ \xi_3^2 \\ \xi_4^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}^+ \begin{bmatrix} \tau_{12}^2 \\ \tau_{13}^2 \\ \tau_{14}^2 \\ \tau_{23}^2 \\ \tau_{24}^2 \\ \tau_{34}^2 \end{bmatrix}. \quad (1.6)$$

Once the pseudoinverse has been found, this provides the solutions for the 4 vertex errors as:

$$\begin{aligned} \xi_1 &= \sqrt{\frac{2\tau_{12}^2 + 2\tau_{13}^2 + 2\tau_{14}^2 - \tau_{23}^2 - \tau_{24}^2 - \tau_{34}^2}{6}} \\ \xi_2 &= \sqrt{\frac{2\tau_{12}^2 - \tau_{13}^2 - \tau_{14}^2 + 2\tau_{23}^2 + 2\tau_{24}^2 - \tau_{34}^2}{6}} \\ \xi_3 &= \sqrt{\frac{-\tau_{12}^2 + 2\tau_{13}^2 - \tau_{14}^2 + 2\tau_{23}^2 - \tau_{24}^2 + 2\tau_{34}^2}{6}} \\ \xi_4 &= \sqrt{\frac{-\tau_{12}^2 - \tau_{13}^2 + 2\tau_{14}^2 - \tau_{23}^2 + 2\tau_{24}^2 + 2\tau_{34}^2}{6}} \end{aligned} \quad (1.7)$$

Considering that obtaining the pseudoinverse may be computationally difficult, a general solution for the error of each vertex of a set of N vertices, in terms of the errors of the associated edge lengths, emerges from patterns observed in the symbolic solutions to (1.5) for cases of $N \in \{3, 4, 5, 6, 7\}$ (not shown in the interest of conciseness). The following general solution for the error of each vertex of a set of N vertices is offered as a conjecture, without proof.

$$\xi_i = \sqrt{\frac{\sum_{j=1}^{N-1} \sum_{k=j+1}^N \begin{cases} (N-2)\tau_{jk}^2, & (j=i) \vee (k=i) \\ -\tau_{jk}^2, & (j \neq i) \& (k \neq i) \end{cases}}{(N-1)(N-2)}}, i \in \{1, 2, 3, \dots, N\} \quad (1.8)$$

Here τ_{jk} is the error of the length of the edge spanning vertices j and k , and ξ_i is the error of vertex i . This formula yields results equivalent to the symbolic solutions to (1.5) for cases of $N \in \{3, 4, 5, 6, 7\}$ and has been numerically validated up to $N=20$ (see online supplement, section 3).

3. Discussion

It is essential to determine the inherent error in any experimental procedure. A technique for determining the error associated with individual registration vertices fixed to rigid bodies has been introduced, wherein vertex error may be determined irrespective of the change in position or motion of the associated rigid body, based upon the error of the edge lengths (distances between the vertices) associated with those vertices. This technique is based upon the

assumption that vertex error follows a normal distribution, and that there is sufficient distance between the vertices such that there is negligible overlap in their error distributions. Normality may be indirectly established by validating that the edge length errors are normally distributed; data transformations to improve normality may be appropriate. Departures from normality will degrade the accuracy of the predictions. Knowledge of the error contributed by individual registration vertices provides a means of identifying excessively noisy markers for subsequent adjustment or elimination to facilitate associated measurements of higher accuracy. When applied to the markers used in calibrating a tracking system, such as those attached to calibration cubes or wands, it could facilitate improved calibration of the tracking system. It can also form the basis for error propagation and simulations of the effect of the variation of the registration markers on parameters that are less susceptible to direct error propagation, including variation in the position of a point on the object to which the registration markers are fixed, variation in distances between points on different objects each with its own associated fixed registration markers while those objects are in relative motion, or to explore the variation in rotation axis and angle. This technique provides a means of quantifying and optimizing marker errors for applications in such areas as motion analysis and kinematics, and provides a mechanism for determining the error associated with a specific measurement.

Additionally, simulations (online supplement, sections 1 and 2) demonstrated that the accuracy of predicted registration vertex variation improves both with greater edge length and additional registration vertices, and provided a numerical validation of the introduced techniques. An application of these techniques to experimental data (online supplement, section 3) demonstrated that the variation of registration marker vertices is much larger when the positions of the markers are measured when they are in motion as opposed to when they are measured in a static position and also showed how marker variation can vary with position and orientation within the capture volume.

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Fig. 1.

Effect of edge length on prediction of vertex error. Predictions converge to reference values as the equilateral triad's edge length increases, with convergence effectively complete when the edge length is approximately 4X the largest vertex error. Points represent the predicted vertex error for an equilateral triad for each of 1500 edge lengths ranging from 0.01 to 15 units, based upon 5000 random variates of the vertices of the reference triad at each edge length.

Fig. 2.

Effect of registration vertex count on accuracy of predicted vertex error; the 95% confidence interval is shown. Prediction accuracy improves as the number of vertices increases.

Confidence intervals are averages from 13 simulations of 100 sets of 3 to 20 spherically distributed vertices, wherein the vertices from each simulation were assigned vertex errors from a subinterval between 0.1 and 14 units, and results from each subinterval were normalized to the maximum vertex error of that subinterval.

Fig. 3.

Single-marker variation from experimental data (marker #6 in Table 1). Both static and dynamic data were acquired over 60 seconds at 60 Hz. The marker error was calculated over a running 17-frame window.

- a) Variation of single-marker error in static and dynamic conditions as a function of frame number (time).
- b) Variation of single-marker dynamic error as a function of rotation angle.
- c) Variation of single-marker dynamic error as a function of marker velocity.

Comparison of Static and Dynamic Marker Variations for 9 Markers. The ratio values were calculated prior to rounding the static and dynamic MSDs.

MSD	Vertex								
	1	2	3	4	5	6	7	8	9
Static, mm	0.005	0.005	0.006	0.005	0.007	0.006	0.006	0.009	0.006
Dynamic, mm	0.022	0.023	0.040	0.102	0.119	0.092	0.113	0.081	0.133
Ratio (Dyn/Stat)	4.1	4.7	6.8	19.4	15.9	16.2	17.7	9.1	20.8