Published in final edited form as:

Ann Biomed Eng. 2008 July ; 36(7): 1242–1253. doi:10.1007/s10439-008-9491-2.

An Integer Programming Model for Optimizing Shoulder Rehabilitation

Christopher J. Gatti¹, Jason Scibek², Oleg Svintsitski¹, James E. Carpenter¹, and Richard E. Hughes¹

¹Laboratory for Optimization and Computation in Orthopaedic Surgery, University of Michigan, Ann Arbor, MI 48109-2200, USA

²Department of Athletic Training, Duquesne University, Pittsburgh, PA 15282, USA

Abstract

Strength restoration is one goal of shoulder rehabilitation following rotator cuff repair surgery. However, the time spent in a physical rehabilitation setting is limited. The objective of this study was to develop a novel mathematical formulation for determining the optimal shoulder rehabilitation exercise protocol to restore normal shoulder strength given a time-constrained rehabilitation session. Strength gain was modeled using a linear dose-response function and biomechanical parameters of the shoulder musculature. Two different objective functions were tested: (1) one based on a least squares support vector machine using healthy and pathologic shoulder strengths (normative objective function), and (2) one which seeks to match the strength of the contralateral shoulder (contralateral objective function). The normative objective function was subject-independent and the optimal protocol consisted of four sets each of adduction and external rotation. The contralateral objective function was subject-specific and the typical optimal protocol consisted of various set combinations of abduction and internal and external rotation. These results are only partially consistent with current practice. Improvement of the current model is dependent on a better understanding of strength training adaptation and shoulder rehabilitation.

Keywords

Shoulder; Rehabilitation; Optimization; Integer programming; Computer model

INTRODUCTION

Rotator cuff tears represent a significant source of musculoskeletal morbidity, with incidence rates as high as 1% per year in some occupations. The median number of days lost from work for a worker who sustains a rotator cuff injury is 97 days, based on workers' compensation data from Washington state.⁴⁰ One contributing factor to the time spent recovering from rotator cuff pathology is the time necessary for completing a rehabilitation program, which is believed to help restore patients' normal shoulder function. Many health insurance systems are experimenting with ways to limit growth in rehabilitation expenditures, primarily by constraining the amount of time a patient spends in a supervised rehabilitation setting. However, limiting the time spent in treatment may reduce the effectiveness of this important phase of the recovery process and may have a negative impact on an individual's return to

Address correspondence to Richard E. Hughes, Laboratory for Optimization and Computation in Orthopaedic Surgery, University of Michigan, 2019 Biomedical Science Research Building, 109 Zina Pitcher Place, Ann Arbor, MI 48109-2200, USA. Electronic mail: E-mail: rehughes@umich.edu.

normal function. Thus, there is a need to maximize the effectiveness and efficiency of shoulder rehabilitation.

Shoulder strength is an important characteristic of normal shoulder function,^{27,46} and has been shown to be dramatically affected by full-thickness rotator cuff tears.¹⁸ Following surgical repair of the rotator cuff, one component of shoulder rehabilitation is the restoration of strength by performing strengthening exercises. Currently, shoulder rehabilitation is designed to meet a variety of areas depending on the pathology and extent of the injury. Early stages of intervention will tend to focus on pain management and range of motion (ROM) exercises with special emphasis being placed on glenohumeral and scapulothoracic joint mechanics.²⁹ Additionally, clinicians will also look to address deficits in body and postural control to minimize added strain on the shoulder.^{28,47} Patients will also be instructed on strengthening exercises for the peri-scapular and rotator cuff muscles,^{28,35,47} along with general shoulder exercises as progress is made.^{24,28,35,47} Additional emphasis will also be placed on the incorporation of activities designed to enhance neuromuscular control of the shoulder.^{28,47,48} In an effort to restore normal activity, clinicians will also prescribe functional exercises aimed at re-establishing normal movement patterns, skills and functional strength.^{28,35,48}

There is currently little quantitative support for rehabilitation protocols, thus the prescription of such protocols is based on the experience and judgment of the rehabilitation specialist and tradition-based protocols. Some outcomes research involving shoulder rehabilitation does exist;^{8,15,19,29,45} a number of literature reviews have concluded that there is insufficient evidence regarding the efficacy of shoulder rehabilitation.^{2,32,34,41} There continues to be an added emphasis, throughout the medical community, to practice evidence-based medicine and conduct research to substantiate the efficacy of medical practices based on anecdotal evidence.^{30,42} In addition, educational programs in the medical community are being encouraged to emphasize the use of evidence-based medicine in their curricula, thus there is currently a lack of and a need for well-designed experiments and clinical trials to determine the efficacy of various treatment techniques and interventions.^{2,30,34}

Optimization models have been developed for a wide range of applications in musculoskeletal medicine. For example, linear and nonlinear programming models have been developed for predicting muscle and joint contact forces.^{1,7,10,11,38} However, little work has been done using formal optimization methods to improve musculoskeletal rehabilitation techniques.

The objective of this project was to formulate a mathematical model for regaining maximal shoulder strength in patients having undergone surgical repair of full-thickness rotator cuff tears. Specifically, an integer programming model was developed to relate shoulder rehabilitation strengthening exercises to strength gain, and was used to determine the optimal rehabilitation strengthening protocol (consisting of a specific number of sets of different strengthening exercises) to most effectively and efficiently restore normal strength in the involved shoulder subject to a limit on rehabilitation time. The novelty of this model lies in explicitly incorporating a time budget constraint. This article is formatted as follows: second section describes the model and includes the model formulation, objective functions, data, and results procedures; third section presents results; fourth section discusses the results and the model; and fifth section summarizes this study.

MODEL

The goal of shoulder rehabilitation in the current model is to increase subject strength measurements consisting of maximal isometric voluntary contractions in 14 different posture-action combinations (Table 1). The shoulder can produce force in many directions due to its unconstrained geometry, and variations in both arm posture and action create an unlimited number of possible strengthening exercises. Four basic isometric exercises are considered, each

consisting of a different action, as candidates for strengthening: (1) abduction and (2) adduction of the arm in the frontal plane, and (3) internal and (4) external rotation of the humerus. The postures of these isometric exercises are presented in Table 2 and were chosen such that they fall in the midrange of the associated nonisometric exercise. It should be noted that none of these four exercises are identical to any of the 14 strength measurements in terms of the posture-action combinations, thus there is no direct strength transfer from an exercise to a strength measurement, but rather an indirect relationship. In this model, exercises are prescribed in terms of sets, and the number of sets prescribed for each exercise increases the possible combinations of rehabilitation protocols and the ways in which rehabilitation time can be spent.

Each of these four movements are typically incorporated into shoulder rehabilitation programs in an effort to restore overall glenohumeral joint and shoulder strength,^{16,19,24} however, shoulder adduction movements may not be specifically enlisted to enhance the strength of the rotator cuff muscles. In addition to the movements included in the current model, it is not uncommon for shoulder flexion and extension exercises to also be incorporated into shoulder rehabilitation programs. All of these movements are used in conjunction with shoulder girdle and whole-body activities to facilitate strength development within the shoulder.^{16,19,24} However, shoulder flexion and extension were excluded from this initial implementation in order to evaluate the model formulation, and future models would likely include these and other movements to increase the clinical applicability.

Each exercise recruits only a subset of the shoulder musculature due to the positions and orientations of the muscles crossing the shoulder. Performing exercises can elicit strength gains in muscles that contribute to those associated actions. Only a few large muscles primarily contribute to the majority of an action produced at the shoulder, and smaller muscles are primarily responsible for creating stability at the glenohumeral joint. Each muscle crossing the glenohumeral joint has secondary functions in addition to performing a primary mechanical function. For example, muscles that are primarily responsible for arm abduction can also contribute to external rotation, albeit to a lesser degree. We assumed that, by performing a strengthening exercise, the strength gain response of each muscle was dependent on the muscle's contribution to a specific exercise. A gain in muscle strength increases a muscle's force-producing capacity and will subsequently increase the total shoulder strength for any action to which it contributes.

Model Formulation

The current model selects the optimal rehabilitation protocol (the number of sets of each exercise) to most effectively improve shoulder strength subject to a time constraint. This model is based on two assumptions. The first assumption is that the shoulder strength (Y_n , N m) for strength measurements $n = 1 \dots 14$ (Table 1), is proportional to the physiological cross-sectional area (PCSA) of muscle m (A_m , m²):

$$Y_n = \gamma \sum_{m \in \Omega_n} (A_m r_{m,n} \phi_{m,n}) \quad (1)$$

where Ω_n includes the set of indices for muscles which contribute to strength measurement n (based on the muscle moment arms in the specific posture-action combination), $r_{m,n}$ and $\phi_{m,n}$ are the muscle moment arms and length-tension relationships, respectively, for muscle m and strength measurement n , and γ is the specific tension of muscle (4.2×10^5 N m⁻²)¹² which is constant for all muscles and postures. The muscle moment arms and length-tension relationships are muscle- and posture-specific biomechanical parameters, which relate A_m to the isometric moment produced by muscle m . It is important to note that this assumption disregards the contribution of neuromuscular proficiency to strength.

The second assumption is that muscle hypertrophy ($\Delta A_{m,l}$, i.e., increase in PCSA) for muscle m , due to isometric exercise $l = 1 \dots 4$ (Table 2), is proportional to the initial muscle PCSA (A_m^0):

$$\Delta A_{m,l} = k_l A_m^0 \quad \text{for } m \in \Omega_l \quad (2)$$

where k_l is an exercise-specific hypertrophy factor determined by methods presented below and Ω_l includes the set of indices for muscles which contribute to strength exercise l (similarly based on the muscle moment arms). A change in shoulder strength due to muscle hypertrophy from exercise l (ΔY_l), was determined by using the assumption of Eq. (1) for strength exercise l and substituting $\Delta A_{m,l}$ for A_m :

$$\Delta Y_l = \gamma k_l \sum_{m \in \Omega_l} (A_m^0 r_{m,l} \phi_{m,l}) \quad (3)$$

Strength measurement index n and strength exercise index l denote specific posture-action combinations of isometric actions, none of which have the same posture-action combination, as seen in Tables 1 and 2, respectively. Arm posture was defined by 4 degrees of freedom including arm elevation with respect to the torso, humeral rotation, arm plane of elevation, and elbow flexion, which is consistent with the recommendations by the International Society of Biomechanics.⁴⁹ Both strength measurements and exercises are isometric (arm posture does not change), yet muscles are contracting and attempting to produce a specified motion.

The exercise-strength gain dose-response was modeled as a linear function based on the results of prior work by Rhea and colleagues,³⁷ who performed a meta-analysis of strength training literature. This meta-analysis presented pooled estimates of standardized strength gains (strength gains normalized by standard deviations) for one through six sets of exercise for untrained subjects; the analysis grouped all types resistance training (including a variety of exercise protocols), thus the results represent “mean training levels”. It was found that performing more than four sets of an exercise yielded diminishing return on strength gain; thus the current model only incorporated the dose-response up to and including four sets. Data for one to four sets (treatment effect, standard deviation, and number of effect sizes; Table 3 in Rhea *et al.*³⁷) were extracted and the treatment effects were fit to a linear function. The current model assumed that for zero sets of exercise, there was zero strength gain, and that due to the debilitating nature of musculoskeletal pathologies, the subjects were assumed to be untrained. The dose-response was represented as:

$$\psi(x_l) = \begin{cases} 0 & \text{for } x_l = 0 \\ \sigma(a + bx_l) & \text{for } x_l = 1, 2, 3, 4 \end{cases} \quad (4)$$

where $\psi(x_l)$ is the strength change (N m) due to x sets of exercise l , and σ is the standard deviation of strength measurements (N m). Estimated parameters a and b were 0.895 and 0.355 for 1-4 sets, respectively; the fit had a coefficient of determination of 0.954, and the standard deviation was 2.469. An exercise protocol was represented by x_l , or the number of sets for each exercise l .

A hypertrophy factor (k_l) was then determined by equating the strength change in Eq. (3), ΔY_l , with the strength gain from the dose-response function of Eq. (4), $\psi(x_l)$. Combining these equations and solving for k_l yields:

$$k_l = \frac{\psi(x_l)}{\gamma \sum_{m \in \Omega_l} (A_m^0 r_{m,l} \phi_{m,l})} \quad (5)$$

The hypertrophy of muscle m due to x_l ($\Delta A_{m,l}$) was determined by substituting Eq. (5) into Eq. (2):

$$\Delta A_{m,l} = \left(\frac{\psi(x_l)}{\gamma \sum_{m \in \Omega_l} (A_m^0 r_{m,l} \phi_{m,l})} \right) A_m^0 \quad \text{for } m \in \Omega_l \quad (6)$$

For each exercise protocol x_l , Eq. (6) yields multiple and different values of $\Delta A_{m,l}$, each of which corresponds to exercise l . It was assumed that a muscle would adapt (i.e., hypertrophy) to the maximum exercise stimulus to which it is exposed. A single muscle hypertrophy (ΔA_m) for each exercise protocol was obtained by taking the maximum muscle hypertrophy over all exercises:

$$\Delta A_m = \max_l \{ \Delta A_{m,l} | l = 1, \dots, 4 \} \quad (7)$$

which represents the predicted change in the PCSA of muscle m due to an exercise protocol.

Strength exercises (l) directly affect strength measurements (n) such that changes in PCSAs increase shoulder strength as per our first assumption. The change in strength measurement n (ΔY_n , N m) was determined using Eq. (1) and the muscle hypertrophy ΔA_m :

$$\Delta Y_n = \gamma \sum_{m \in \Omega_n} (\Delta A_m r_{m,n} \phi_{m,n}) \quad (8)$$

The postrehabilitation strength (Y_n^* , N m) was determined by adding the prerehabilitation strength measurements (Y_n^0) and the strength change due to rehabilitation (ΔY_n):

$$Y_n^* = Y_n^0 + \Delta Y_n \quad (9)$$

Combining Eqs. (6-9) yields the postrehabilitation strength measurements:

$$Y_n^* = Y_n^0 + \gamma \sum_{m \in \Omega_n} \left\{ \max_l \left[\left(\frac{\psi(x_l)}{\gamma \sum_{m \in \Omega_l} (A_m^0 r_{m,l} \phi_{m,l})} \right) A_m^0 \right] \times r_{m,n} \phi_{m,n} \right\} \quad (10)$$

Objective Functions

The optimum rehabilitation exercise protocol was determined for two realistic rehabilitation outcome objective functions. The first objective function (normative objective function) used a vector normal to the optimal separating hyperplane between healthy and pathologic shoulder strengths, determined using a least squares support vector machine (LS-SVM); the normal

vector points in the direction of maximum improvement from pathologic to healthy. The second objective function (contralateral objective function) increased shoulder strength to that of the contralateral shoulder.

Normative Objective Function—The normative objective function is based on previous work by Silver and co-workers,³⁹ and a brief description of this work is presented here. An LS-SVM⁴³ was used to classify shoulder strength data from 37 patients (27 males, 10 females) undergoing surgical repair of at least one full-thickness rotator cuff tear. Two classes of data points were constructed: (1) 14 preoperative strength measurements made on each patient's involved shoulder and (2) 14 predicted normal strength measurements based on patient age, gender, and body mass. The 14 strength measurements constitute a 14-dimensional vector, or a strength profile, and consist of the same isometric posture-action combinations as in Table 1. The predicted normal strength measurements for each patient were determined using previously derived regression equations from strength testing 120 asymptomatic subjects ranging in age from 20 to 87.²³ The strength profiles for the involved and for the predicted normal shoulders were included data classes C_1 and C_2 , respectively. The LS-SVM was trained on C_1 and C_2 to determine an optimal separating hyperplane (OSH) that best separated the two classes of strength measurements. The vector normal to the OSH, c , represents the maximum improvement from a pathologic population to a predicted normal population; the coefficients of c corresponding to strength measurements n , c_n , are provided in Table 1 (w_{ROC} from Silver *et al.*³⁹). The LS-SVM was trained with data from patients with rotator cuff tears; thus is specific to this pathology and would have to be retrained for other pathologies.

The optimal rehabilitation protocol can be formulated mathematically as a knapsack integer programming problem with bounded decision variables¹⁴ using the objective function coefficients from c and the postrehabilitation strength measurements Y_n^* .

$$\text{Max } z = \sum_{n=1}^{14} (c_n Y_n^*) \quad (11)$$

$$\text{such that } \sum_{l=1}^4 (dx_l) \leq T \quad (12)$$

$$x_l = 0, 1, 2, 3, 4 \quad (13)$$

where c_n is the objective function coefficient of the strength measurement n , d (min) is the time required to perform one set of exercise and is constant for all exercises, and T (min) is the allotted time budget of the rehabilitation session. The time required for one exercise set, including rest, was 2 min and was based on the clinical experience of one of the co-authors who is a certified athletic trainer (JS), and a single rehabilitation session was chosen to last 30 min. The objective function of Eq. (11) can be restated by substituting Eq. (9):

$$\text{Max } z = \sum_{n=1}^{14} (c_n Y_n^0) + \sum_{n=1}^{14} (c_n \Delta Y_n) \quad (14)$$

Note the first term of Eq. (14) is constant, thus this objective function is independent of the patient's initial strength profile. The optimization problem for the normative objective function consists of maximizing Eq. (14) subject to the constraint Eqs. (12) and (13) and can be simplified:

$$\text{Max } z = \sum_{n=1}^{14} (c_n \Delta Y_n) \quad (15)$$

Contralateral Objective Function—Another possible goal of shoulder rehabilitation could be to increase the involved shoulder strength profile to best match the contralateral (uninvolved) shoulder strength profile. The strength profile of the contralateral shoulder was used as a goal (G_n , N m) for postrehabilitation strength measurements Y_n^* , and the objective function was defined as:

$$\text{Min } z = \sqrt{\sum_{n=1}^{14} (G_n - Y_n^*)^2} \quad (16)$$

subject to the constraint Eqs. (12) and (13). Note that this objective function is dependent on the patient's initial strength profile. To compare the optimal rehabilitation protocols between the two objective functions, the contralateral objective function also used a rehabilitation session lasting 30 min. In addition, the optimal rehabilitation protocols for the contralateral objective function were determined for rehabilitation sessions lasting 10 and 20 min to determine the effects of a reduced time budget.

Data

Fourteen isometric strength measurements for each of 12 patients at 6 months following rotator cuff repair surgery were previously collected at the Mayo Clinic.³⁹ Strength measurements were obtained for both the involved and uninvolved (contralateral) shoulders and correspond to Y_n^0 and G_n , respectively. Figure 1 shows the average, over the 12 patients included in the current model, preoperative involved and predicted normal shoulder strengths for each strength measurement. Data collected at 6 months following surgery were used in the current model in order to minimize the possibility that strength was inhibited by pain. The experimental protocol used to collect isometric shoulder strength data was the same as the procedure by Hughes and colleagues²² and was approved by the institution's Institutional Review Board.

Muscle parameters including moment arms (r_m , m) and length-tension relationships (ϕ_m , dimensionless) for the strength measurements (n) and exercises (l), were obtained using a computational model of shoulder muscle, tendon, and bone geometry developed by Holzbaur and colleagues²⁰ and implemented in Software for Interactive Musculoskeletal Modeling (MusculoGraphics Inc., USA).⁹ The initial muscle PCSAs (A_m^0) used in the current model were those used in the Holzbaur shoulder model, which were obtained from various sources in the literature and represent average population values. The Holzbaur shoulder model uses 18 muscle elements to represent the musculature crossing the shoulder and the current optimization models include these 18 muscle elements in the formulations.

Solution

The optimization problems were implemented in MATLAB (The MathWorks, Inc., Natick, MA, USA) and run on a 2.4 GHz Pentium 4 personal computer. For each objective function,

all combinations of 0-4 sets for each of the four exercises were iterated through for all subjects. The primary results of interest consisted of the optimal rehabilitation exercise protocols for each subject with respect to the associated objective function and time constraint. In addition, the effects of the optimum rehabilitation protocols on strength measurements and muscle PCSAs were also analyzed.

RESULTS

Normative Objective Function

The solutions to the normative objective function were found to be four sets of adduction and four sets of external rotation, with 0 sets for both abduction and internal rotation for all subjects. In other words, a rehabilitation sessions consisting of four sets each of adduction and external rotation is predicted to most directly increase subject strength to that of a healthy population. This objective function depends only on the model formulation and not the initial subject strength, thus the solution is subject-independent. This solution yields a total of eight sets lasting 16 min, which is well under the 30 min time constraint. Though more exercises could be performed during the rehabilitation session, the model predicts that strength would increase less effectively to that of the normal population strength data used in the current model.

Contralateral Objective Function

The solutions to the contralateral objective function were found to vary among subjects, and are presented in Table 3 for the 10, 20, and 30 min rehabilitation session time constraints. For some subjects (7, 8, 10, 11, and 12), the optimal rehabilitation protocol varied as the time constraint was changed. The remaining subjects had the same solution for all three time constraints. Thus for these subjects, a 20 or 30 min rehabilitation session is predicted to be no more beneficial than a 10 min rehabilitation session for attempting to reach the contralateral shoulder strength. With the addition of more exercises just to use up the remaining rehabilitation session, these subjects' strength profile would increase less directly toward the goal strength profile. A subject's rehabilitation protocol changed for the different time constraints depending on the differences between the initial strength and the goal strength, such that greater strength deficits require additional sets of exercise to best match the contralateral shoulder strength.

The rehabilitation exercise protocols increase strength measurements due to muscle hypertrophy, and the postrehabilitation strength measurements and muscle PCSAs are of interest. To show the relationships between the rehabilitation protocols, strength measurements, and muscle hypertrophies, a single case was analyzed, specifically subject 10 with the contralateral objective function for a 30 min rehabilitation session. The results of the strength measurement improvements and muscle hypertrophies are presented in Figs. 2 and 3, respectively. The optimal rehabilitation protocol for this case was four sets each of abduction, internal rotation, and external rotation, and 0 sets of adduction. The model predicted that, for some strength measurements, the involved shoulder strength is predicted to increase to, or beyond, that of the contralateral shoulder, while other strength measurement goals will not be attained. It would be expected that abduction, internal rotation, and external rotation strength measurements would increase to some degree due to the inclusion of these actions in the rehabilitation protocol despite the differences between the postures of the strength measurements and the exercises. It is also seen that adduction strength was increased (beyond that of the goal in one strength measurement) without adduction exercises included in the rehabilitation protocol. This is because muscles that are active in one action (for example, internal rotation), can also contribute to orthogonal actions (adduction), and therefore have the potential to hypertrophy due to their secondary and tertiary moment-producing capabilities. Muscle hypertrophies further illustrate this effect and are shown in Fig. 3a, which shows

nominal muscle hypertrophies, and Fig. 3b, which shows muscle hypertrophies as percentages of the initial PCSAs. The model predicted that muscles which are primarily responsible for certain actions hypertrophied greater than muscles which are not primarily responsible for any actions. This effect can be seen by the hypertrophy of the middle deltoid, the subscapularis, and the infraspinatus, which are primarily responsible for abduction, internal rotation, and external rotation, respectively, and is due to their moment-producing capabilities with respect to the exercise postures and because of their large initial PCSAs. The model did not prescribe any sets of adduction, yet muscles that are primarily adductors were still predicted to hypertrophy; some of these muscles include the three portions of the latissimus dorsi, the three portions of the pectoralis major, and the coracobrachialis. These muscles demonstrate the fact that muscles can indirectly hypertrophy from actions for which they are not primarily responsible. In addition, Fig. 3b shows that the model predicts three levels of hypertrophy, which are attributed to the three exercises subject 10 was prescribed.

DISCUSSION

The purpose of this study was to develop an initial formulation of a mathematical model that determines the optimal shoulder rehabilitation exercise protocol to restore normal shoulder strength in patients recovering from rotator cuff repair surgery. Two different objective functions were tested, and it was found that one solution selected a patient-independent and nontraditional exercise protocol (normative objective function), while the other selected patient-specific and more traditional exercise protocols (contralateral objective function). The model formulation is based on a number of assumptions which are important to consider when evaluating the results of the model. Some assumptions are directly relevant to the model formulation (previously stated in “Methods” section) while others are intrinsic to the practice of physical rehabilitation and human physiology (expanded upon below).

The innovation of this model lies in understanding the clinical reality there is limited time available for rehabilitation and explicitly incorporating that fact into our modeling approach. Previous shoulder models have solely focused on estimating internal tissue loads^{21,25,44} rather than optimally utilizing time. The two formulations presented here build on previously published investigations,^{23,39} but they significantly extend our shoulder rehabilitation research by focusing on the efficient utilization of time available for rehabilitation.

Model Evaluation

The normative objective function, which increased strength to that of a healthy normal population, is patient-independent and the solution of this model depends only on the model formulation; thus, all subjects have the same optimal rehabilitation protocol. The contralateral objective function is dependent on initial strength measurements of the involved and uninvolved shoulders, and this objective function yielded patient-specific rehabilitation protocols. These two objective functions also differ in that the normative objective function is linear whereas the contralateral objective function is nonlinear.

The solutions of the normative objective function prescribed external rotation and adduction. The reason for this is evident from Fig. 1, where the external rotation (#11-14) and adduction (#4-6) strength measures have the largest average strength deficits between the involved and predicted normal values. While few would argue against external rotation,^{16,19,24} the inclusion of adduction is of notable interest due to the number of muscles around the shoulder which contribute to adduction. Failure to address shoulder adduction strength deficits would result in failure to restore normal shoulder function. There is relatively little published literature concerning shoulder adduction strength, and even less with regards to its role in shoulder rehabilitation, thus a comparison of our results is not possible. There is some evidence though, with clinical implications, to suggest that performing shoulder adduction against resistance, as

compared to abduction, results in minimizing superior migration of the humeral head on the glenoid, resulting in widening of the subacromial space.¹⁷ It should be noted that the normative objective function is based on a LS-SVM with a small dataset without cross-validation; therefore the predictions from the normative objective function should be viewed with some caution.

The solutions of the contralateral objective function were similar to the current practice of shoulder rehabilitation, which typically includes abduction and internal and external rotation. With regards to the optimal rehabilitation protocols as determined by the model, it could be said that current practice is optimal if the goal of shoulder rehabilitation is to attempt to restore shoulder strength to that of the contralateral shoulder.

The strength measurement data used as inputs to each model has an effect on the solution of each objective function. By grouping the subjects and averaging their strength measurements, as with the normative objective function, critical individual information is lost, and this can be seen by the large standard deviations in Fig. 1. Thus the prescribed rehabilitation protocol (as suggested by the normative objective function) may not address important individual deficits. By individualizing the model and using the contralateral objective function, the grouping effects are no longer present, and individual protocols are likely to address the individual deficits. Considering these two points, the initial strength data and the prescribed protocols support the practice of individualized protocols.

Limitations

There is no clearly accepted outcome objective for strength restoration during shoulder rehabilitation. Additionally, what constitutes a 'healthy' or 'normal' shoulder is open to discussion. The two objective functions implemented in the current model are both realistic clinical goals. However, other possible objective functions could include increasing strength to obtain specific agonist/antagonist strength ratios or increasing strength specific to a patient's activities. The objective function selected in practice is dependent on the experience and views of the clinician and the needs of the patient.⁴⁵ To increase the clinical applicability of the current model, clearer definitions of shoulder rehabilitation outcome measures are necessary.

The current model is limited by the incomplete knowledge of adaptation to strength training. We represented strength gains as an increase muscle PCSA. Neural adaptations have been found to play a role in strength gain,³¹ but their precise relationship to strength training protocols remains unknown,¹³ and have thus been excluded from the current model. Additionally, strength gains, due to either muscle hypertrophy or neural adaptation or both, from similar training protocols, are another topic of discussion. One such example is the single-vs. multi-set debate which concerns the dose-response relationship between the number of sets of an exercises and strength gain.^{4-6,33,36}

Another topic of uncertainty concerns the amount of strength gain over time; our model is time-independent. The exercise-strength gain does-response used in the current model was from a meta-analysis, and the time frame for the strength gains of the optimal rehabilitation protocols could be considered the average duration of the protocols used in the studies included in the meta-analysis. In addition, the meta-analysis included all types of studies in the strength training literature and did not determine a specific dose-response for different exercise prescription variables (e.g., type of contraction, contraction duration, contraction intensity, repetitions per set). Further specificity of other exercise parameters is not included in the current model because there is currently insufficient data regarding the effects of these variables with respect to strength gain. Moreover, while there are more variables in exercise prescription than sets alone, the current model is based on sets because this is a primary variable affecting the duration (the constraint of the current model) of a rehabilitation session.

The biomechanical parameters used in the current model are not patient-specific and were calculated using the Holzbaur shoulder model and are based on average population data. These parameters, including muscle PCSAs, moment arms, and length-tension relationships, play a role in muscle activation and the production of muscle forces and joint torques and vary among individuals. Such variations are likely to have an effect on the inter-patient variability of muscle hypertrophy to the same strengthening exercises. Absolute muscle size, in particular, vary widely among individuals,²⁶ and are likely even more variable in patients with shoulder pathologies due to functional limitations leading to disuse and subsequent muscle atrophy.³ Muscle moment arms and length-tension relationships are dependent on the Holzbaur shoulder model to accurately represent the musculoskeletal shoulder anatomy in various postures. Variations in any of these muscle parameters, due to individual variations or the effects of rehabilitation, could yield different muscle hypertrophies and different optimal rehabilitation protocols than that of the current solutions.

Although maximal isometric strength was the goal in the current model, shoulder rehabilitation is a multi-faceted problem which focuses on many aspects of shoulder function. Other goals of shoulder rehabilitation include the reestablishment of active and passive ROM, proper musculoskeletal kinematics, scapular stabilization, muscular endurance, and functionality specific to an individual's activities of daily living among others. Depending on the status of the patient, the rehabilitation specialist must make time management decisions based on their experience and the best interest of the patient. The current model could be adapted to optimize rehabilitation sessions with respect to multiple functional aspects by formulating the problem as a multiobjective optimization problem.

CONCLUSIONS

We presented a novel mathematical formulation of an optimization model for maximizing the effectiveness of a time-constrained rehabilitation session for increasing shoulder strength. Optimal rehabilitation protocols for two different, and both clinically realistic, shoulder strength rehabilitation outcome objective functions were determined. One objective function yielded atypical results while the other yielded results that were similar to those of current shoulder rehabilitation practices. As our understandings regarding strength training and shoulder rehabilitation expand, subsequent model formulations would likely be improved to accurately predict adaptations to strength training for use in a clinical setting. In addition, future work could include a validation study to evaluate the performance of the current model.

ACKNOWLEDGMENTS

The authors would like to thank Marjorie Johnson, Kai-Nan An, and Shawn O'Driscoll of the Mayo Clinic for sharing shoulder strength data with us, and David Gabriel for identifying the necessary meta-analysis paper. We also thank the National Institutes of Health for financial support via grant AR048540.

GLOSSARY OF TERMS

- A_m , physiological cross-sectional area of muscle m , m^2
- A_m^0 , initial (prerehabilitation) physiological cross-sectional area of muscle m , m^2
- ΔA_m , muscle hypertrophy for muscle m due to an entire exercise protocol, m^2
- $\Delta A_{m,l}$, muscle hypertrophy for muscle m due to an individual exercise l , m^2
- a , intercept of strength dose-response, 0.895 dimensionless
- b , slope of strength dose-response, 0.355 sets^{-1}
- c_n , objective function coefficient for strength measurement n , dimensionless
- d , time required to perform one strength exercise set including rest, 2 min
- G_n , contralateral shoulder strength (goal) for strength measurement n , N m

k_l , muscle hypertrophy factor due to strength exercise l , dimensionless
 l , strength exercise index
 m , muscle index
 n , strength measurement index
 $r_{m,l}$, muscle moment arm for muscle m for strength exercise l , m
 $r_{m,n}$, muscle moment arm for muscle m for strength measurement n , m
 T , time budget allotted for the rehabilitation session, min
 x_l , number of sets of strength exercise l , sets
 Y_n , strength for strength measurement n , N m
 Y_n^0 , prerenhabilitation strength for strength measurement n , N m
 Y_n^* , postrenhabilitation strength for strength measurement n , N m
 ΔY_l , increase in strength for strength exercise l , N m
 ΔY_n , increase in strength for strength measurement n , N m
 γ , specific tension of muscle, constant for all muscles and postures, N m⁻²
 σ , standard deviation of strength measurements from dose-response, 2.895 N m
 $\phi_{m,l}$, length-tension relationship for muscle m for strength exercise l , dimensionless
 $\phi_{m,n}$, length-tension relationship for muscle m for strength measurement n , dimensionless
 $\psi(x_l)$, increase in strength for x sets of strength exercise l from dose-response, N m
 Ω_l , set of indices for muscles which contribute to strength exercise l based on muscle moment arms
 Ω_n , set of indices for muscles which contribute to strength measurement n based on muscle moment arms

REFERENCES

1. Bean JC, Chaffin DB, Schultz AB. Biomechanical model calculation of muscle contraction forces: a double linear programming method. *J. Biomech* 1988;21:59–66. [PubMed: 3339029]
2. Beckerman H, Bouter LM, van der Heijden GJMG, de Bie RA, Koes BW. Efficacy of physiotherapy for musculoskeletal disorders: what can we learn from research? *Br. J. Gen. Prac* 1993;43:73–77.
3. Booth FW. Physiologic and biochemical effects of immobilization on muscle. *Clin. Orthop. Rel. Res* 1987;219:15–20.
4. Byrd R, Chandler TJ, Conley MS, Fry AC, Haff GG, Koch A, Hatfield F, Kirksey KB, McBride J, McBride T, Newton H, O'Bryant HS, Stone MH, Pierce KC, Plisk S, Ritchie-Stone M, Wathen D. Strength training: single versus multiple sets. *Sports Med* 1999;27:409–416. [PubMed: 10418075]
5. Carpinelli RN, Otto RM. Strength training. Single versus multiple sets. *Sports Med* 1998;26(2):73–84. [PubMed: 9777681]
6. Carpinelli RN, Otto RM, Winett RA. A critical analysis of the ACSM position stand on resistance training: insufficient evidence to support recommended training protocols. *JEP online* 2004;7(3):1–60.
7. Crowninshield RD. Use of optimization techniques to predict muscle forces. *J. Biomech* 1978;100:88–92.
8. Curtis KA, Tyner TM, Zachary L, Lentell G, Brink D, Didyk T, Gean K, Hall J, Hooper M, Klos J, Lesina S, Pacillas B. Effect of a standard exercise protocol on shoulder pain in long-term wheelchair users. *Spinal Cord* 1999;37(6):421–429. [PubMed: 10432262]
9. Delp SL, Loan JP. A graphics-based software system to develop and analyze models of musculoskeletal systems. *Comput. Biol. Med* 1995;25(1):21–34. [PubMed: 7600758]
10. Dul J, Johnson GE, Shiavi R, Townsend MA. Muscular synergism—II. A minimum-fatigue criterion for load sharing between synergistic muscles. *J. Biomech* 1984;17:675–684. [PubMed: 6501327]
11. Dul J, Townsend MA, Shiavi R, Johnson GE. Muscular synergism—I. On criteria for load sharing between synergistic muscles. *J. Biomech* 1984;17:663–673. [PubMed: 6501326]
12. Edgerton VR, Apor P, Roy R. Specific tension of human elbow flexor muscles. *Acta. Physiol. Hung* 1990;75(3):205–216. [PubMed: 2392943]

13. Enoka RM, Gandevia SC. Neural changes associated with training. *J. Appl. Physiol* 2006;101(4): 1009–1010. [PubMed: 16973817]
14. Garfinkel, RS.; Nemhauser, GL. *Integer Programming*. Wiley; New York: 1972. p. 214-216.
15. Ginn K, Herbert R, Khouw W, Lee R. A randomized, controlled trial of treatment of shoulder pain. *Phys. Ther* 1997;77:802–811. [PubMed: 9256868]
16. Goldberg BA, Nowinski RJ, Matsen FA. Outcome of nonoperative management of full-thickness rotator cuff tears. *Clin. Orthop. Relat. Res* 2001;382:99–107. [PubMed: 11154011]
17. Grachien H, Hinterwimmer S, von Eisenhart-Rothe R, Vogl T, Englmeier KH, Eckstein F. Effect of abducting and adducting muscle activity on glenohumeral translation, scapular kinematics and subacromial space width in vivo. *J. Biomech* 2005;38:755–760. [PubMed: 15713296]
18. Harryman I, Douglas T, Mack LA, Wang KY, Jackins SE, Richardson ML, Matsen I, Frederick A. Repairs of the rotator cuff: correlation of functional results with integrity of the cuff. *J. Bone Joint Surg. Am* 1991;73-A:982–989. [PubMed: 1874784]
19. Hayes K, Ginn KA, Walton JR, Szomor ZL, Murrell GA. A randomized clinical trial evaluating the efficacy of physiotherapy after rotator cuff repair. *Aust. J. Physiother* 2004;50:77–83. [PubMed: 15151491]
20. Holzbaur KRS, Murray WM, Delp SL. A model of the upper extremity for simulating musculoskeletal surgery and analyzing neuromuscular control. *Ann. Biomed. Eng* 2005;33(6):829–840. [PubMed: 16078622]
21. Hughes RE, An KN. Force analysis of rotator cuff muscles. *Clin. Orthop. Rel. Res* 1996;330:75–83.
22. Hughes RE, Johnson ME, O'Driscoll SW, An KN. Age-related changes in normal isometric shoulder strength. *Am. J. Sports Med* 1999;27:651–657. [PubMed: 10496585]
23. Hughes RE, Johnson ME, O'Driscoll SW, An KN. Normative values of agonist-antagonist shoulder strength ratios of adults aged 20-78 years. *Arch. Phys. Med. Rehabil* 1999;80:1324–1326. [PubMed: 10527095]
24. Jackins S. Postoperative shoulder rehabilitation. *Phys. Med. Rehabil. Clin. N. Am* 2004;15:643–683.
25. Karlsson D, Peterson B. Towards a model for force predictions in the human shoulder. *J. Biomech* 1992;25:189–199. [PubMed: 1733994]
26. Kawakami Y, Abe T, Kanehisa H, Fukunaga T. Human skeletal muscle size and architecture: variability and interdependence. *Am. J. Hum. Biol* 2006;18(6):845–848. [PubMed: 17039483]
27. Kell RT, Bell G, Quinney A. Musculoskeletal fitness, health outcomes and quality of life. *Sports Med* 2001;31(12):863–873. [PubMed: 11665913]
28. Kibler WB, McMullen J, Uhl T. Shoulder rehabilitation strategies, guidelines, and practice. *Orthop. Clin. North Am* 2001;32(3):527–538. [PubMed: 11888148]
29. McClure PW, Bialker J, Neff N, Williams G, Karduna A. Shoulder function and 3-dimensional kinematics in people with shoulder impingement syndrome before and after a 6-week exercise program. *Phys. Ther* 2004;84(9):832–848. [PubMed: 15330696]
30. Merrick MA. I can't believe we don't know that! *J. Athl. Train* 2006;41(3):231–232. [PubMed: 17043688]
31. Narici MV, Roi GS, Landoni L, Minetti AE, Cerretelli P. Changes in force, cross-sectional area and neural activation during strength training and detraining of the human quadriceps. *Eur. J. Appl. Physiol* 1989;59:310–319.
32. Oh LS, Wolf BR, Hall MP, Levy BA, Marx RG. Indications for rotator cuff repair. *Clin. Orthop. Relat. Res* 2006;455:52–63. [PubMed: 17179786]
33. Otto RM, Carpinelli RN. A critical analysis of the single versus multiple set debate. *JEP online* 2006;9(1):32–57.
34. Philadelphia Panel. Philadelphia panel evidence-based clinical practice guidelines on selected rehabilitation interventions for shoulder pain. *Phys. Ther* 2001;81:1719–1730. [PubMed: 11589645]
35. Prentice, WE. *Rehabilitation Techniques for Sports Medicine and Athletic Training*. McGraw-Hill; New York: 2004. p. 406-408.
36. Rhea MR, Alvar BA, Burkett LN. Single versus multiple sets for strength: a meta-analysis to address the controversy. *Res. Q. Exerc. Sport* 2002;73(4):485–488. [PubMed: 12495252]

37. Rhea MR, Alvar BA, Burkett LN, Ball SD. A meta-analysis to determine the dose response for strength development. *Med. Sci. Sports Exerc* 2003;35:456–464. [PubMed: 12618576]
38. Schultz A, Haderspeck K, Warwick D, Portillo D. Use of lumbar trunk muscles in isometric performance of mechanically complex standing tasks. *J. Orthop. Res* 1983;1:77–91. [PubMed: 6679579]
39. Silver AE, Lungren MP, Johnson ME, O'Driscoll SW, An KN, Hughes RE. Using support vector machines to optimally classify rotator cuff strength data and quantify post-operative strength in rotator cuff patients. *J. Biomech* 2006;39:973–979. [PubMed: 16488235]
40. Silverstein B, Welp E, Nelson N, Kalat J. Claims incidence of work-related disorders of the upper extremities: Washington state, 1987 through 1995. *Am. J. Public Health* 1998;88:1827–1833. [PubMed: 9842381]
41. Smidt N, de Vet HCW, Bouter LM, Dekker J. Effectiveness of exercise therapy: a best-evidence summary of systematic reviews. *Aust. J. Physiother* 2005;51:71–85. [PubMed: 15924510]
42. Snyder AR, McLeod TCV, Sauers EL. Defining, valuing, and teaching clinical outcomes assessment in professional and post-professional athletic training education programs. *Athletic Train. Ed. J* 2007;2(2):31–41.
43. Suykens JAK, Vandewalle J. Least squares support vector machine classifiers. *Neural Process. Lett* 1999;9:293–300.
44. van der Helm FCT. A finite element musculoskeletal model of the shoulder mechanism. *J. Biomech* 1994;27:551–569. [PubMed: 8027090]
45. Wang SS, Trudelle-Jackson EJ. Comparison of customized versus standard exercises in rehabilitation of shoulder disorders. *Clin. Rehab* 2006;20:675–685.
46. Warburton DER, Gledhill N, Quinney A. The effects of changes in musculoskeletal fitness on health. *Can. J. Appl. Physiol* 2001;26(2):217–237. [PubMed: 11312417]
47. Wilk KE, Andrews JR, Arrigo CA. The physical examination of the glenohumeral joint: emphasis on the stabilizing structures. *J. Orthop. Sports Phys. Ther* 1997;25(6):380–389. [PubMed: 9168345]
48. Wilk KE, Arrigo C. Current concepts in the rehabilitation of the athletic shoulder. *J. Orthop. Sports Phys. Ther* 1993;18(1):365–378. [PubMed: 8348138]
49. Wu G, van der Helm FCT, Veeger HEJ, Makhous M, Van Roy P, Anglin C, Nagels J, Karduna AR, McQuade K, Wang X. ISB recommendation on definitions of joint coordinate systems of various joints for the reporting of human joint motion-Part II: shoulder, elbow, wrist and hand. *J. Biomech* 2005;38:981–992. [PubMed: 15844264]

FIGURE 1.

Preoperative involved (*Involved*) and predicted normal (*Predicted*) strength measurements averaged over all 12 subjects. Strength measurement indices (*n*) correspond to those listed in Table 1; error bars represent one standard deviation.

FIGURE 2.

Effect of the optimal rehabilitation protocol on pre- and postrehabilitation strength measurements for subject 10 for the contralateral objective function (30 min time constraint). Strength measurement indices (n) correspond to those listed in Table 1.

FIGURE 3.

Effect of the optimal rehabilitation protocol on nominal (a) and percent of initial (b) muscle physiological cross-sectional areas for subject 10 for the contralateral objective function (30 min time constraint). *Abbreviations:* BicL = long head of biceps, BicS = short head of biceps, Corb = coracobrachialis, Delt 1/2/3 = anterior/middle/posterior portions of deltoid, Infsp = infraspinatus, Lat1/2/3 = superior/middle/inferior portions of latissimus dorsi, Pec1 = clavicular head of pectoralis major, Pec2/3 = superior/ inferior portions of sternocostal head of pectoralis major, Supra = supraspinatus, Subsc = subscapularis, TMaj = teres major, TMin = teres minor, TriL = long head of triceps.

TABLE 1
Isometric strength measurement indices (*n*) corresponding to arm postures, actions, and the LS-SVM objective coefficients

<i>n</i>	Arm posture				Action ^{<i>b</i>}	Objective coefficient (<i>c</i>)
	Elevation angle (°)	Humeral rotation ^{<i>a</i>} (°)	Elevation plane (°)	Elbow flexion (°)		
1	30	0	0	0	Abduction	0.0130
2	60	0	0	0	Abduction	-0.0372
3	90	0	0	0	Abduction	0.0264
4	30	0	0	0	Adduction	0.0227
5	60	0	0	0	Adduction	-0.0386
6	90	0	0	0	Adduction	0.0114
7	15	0	0	90	Internal rotation	-0.0044
8	15	-30	0	90	Internal rotation	0.0079
9	90	-30	0	90	Internal rotation	-0.0454
10	90	-60	0	90	Internal rotation	0.0142
11	15	0	0	90	External rotation	-0.1236
12	15	30	0	90	External rotation	0.1443
13	90	0	0	90	External rotation	-0.0291
14	90	-30	0	90	External rotation	0.0903

^aPositive humeral rotation angles correspond to internal rotation.
^bRotation strength measurements consist of humeral rotation.

Isometric exercise indices (*I*) corresponding to arm postures and actions

<i>I</i>	Arm posture				Action ^a
	Elevation angle (°)	Humeral rotation (°)	Elevation plane (°)	Elbow flexion (°)	
1	45	0	0	0	Abduction
2	45	0	0	0	Adduction
3	0	0	0	90	Internal rotation
4	0	0	0	90	External rotation

^aRotation exercises consist of humeral rotation.

TABLE 3
Optimal rehabilitation protocols for the contralateral objective function for 10, 20, and 30 min time constraints; exercise indices (*l*) correspond to Table 2

Subject																		
1				2				3				4						
Exercise index (<i>l</i>)	Action	10	20	30	10	20	30	10	20	30	10	20	30	10	20	30		
1	ABD	0	0	0	2	2	2	4	4	4	1	1	1	1	1	1		
2	ADD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
3	IR	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1		
4	ER	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0		
Subject																		
5				6				7				8						
10	20	30		10	20	30		10	20	30		10	20	30		10	20	30
4	4	4		1	1	1		0	2	4		0	2	4		0	2	4
0	0	0		0	0	0		0	0	0		0	0	0		0	0	0
1	1	1		0	0	0		4	4	4		4	4	4		4	4	4
0	0	0		4	4	4		1	4	4		1	4	4		4	4	4
Subject																		
9				10				11				12						
Exercise index (<i>l</i>)	Action	10	20	30	10	20	30	10	20	30	10	20	30	10	20	30		
1	ABD	0	0	0	0	4	4	0	0	4	0	2	4	0	1	1		
2	ADD	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1		
3	IR	0	0	0	4	2	4	4	4	4	4	4	4	1	0	0		
4	ER	2	2	2	1	4	4	1	4	4	1	4	4	4	4	4		