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Identification of Material Properties of Orthotropic Elastic Cylinders Immersed in Fluid Using Vibroacoustic Techniques

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Abstract

A numerical study is presented to show the potential for using vibroacoustic-based experiments to identify elastic material properties of orthotropic cylindrical vessels immersed in fluids. Sensitivity analyses and a simulated inverse problem are shown to quantify the potential for material characterization through the use of acoustic emissions. For comparison purposes, the analyses are also shown with the normal component of the velocity at the surface of the cylinder as the measured response in place of the acoustic pressure. The simulated experiment consisted of an orthotropic cylinder immersed in water with an impact force applied to the surface of the cylinder. The material parameters of the cylinder considered in the analyses were the circumferential and longitudinal elastic moduli, and the in-plane shear modulus. The velocity response is shown to provide sufficient information for characterizing all three moduli from a single experiment. Alternatively, the acoustic pressure response is shown to provide sufficient information for characterizing only the two elastic moduli from a single experiment. The analyses show that the acoustic pressure response does not have sufficient sensitivity to the in-plane shear modulus for characterization purposes.

Keywords

Vessels; finite elements; inverse problems; vibroacoustography; vibroacoustics

1. Introduction

Noninvasive methods to characterize material properties are of considerable interest in many fields of science and engineering [1–5]. These methods are of particular interest for non-destructive evaluation of *in-vivo* material behavior of biological structures. For instance, the estimation of local changes in the properties of arterial vessels can assist in the early detection and treatment of diseases such as atherosclerosis [6].

Characterization of biological structures has been shown to pose particular difficulty due to the complexity of the material behavior [7]. In order to characterize these properties, it is necessary to perform tests which contain sufficient information relating to the material without disturbing the current state. Fortunately, several methods which were initially developed for medical imaging have been shown to provide the information necessary to characterize biological material properties [8–11].

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Vibroacoustography is one such imaging technique developed by Fatemi and Greenleaf [12], which offers the potential for the noninvasive characterization of mechanical properties of tissues [8,11,13,14]. In vibroacoustography, the radiation force of ultrasound is used to excite a fluid-immersed structure and the resulting acoustic response in the surrounding fluid is measured. Since the acoustic pressure in the fluid and the deformation of the structure are coupled, the acoustic response of the fluid is dependent on the material properties of the structure.

Several approaches have been used for nondestructive characterization of anisotropic material properties using dynamic testing procedures. Examples include the works by Liu [15] and Chai [16] in which anisotropic elastic constants were inversely determined using the dynamic response of the structure. In this study, the methodology proposed in [8] is applied to the inverse characterization of elastic properties of orthotropic cylindrical vessels using vibroacoustic techniques. A sensitivity analysis is shown first to estimate the information content about material properties available through a vibroacoustic experiment. Then, a simulated inverse problem is considered to display the potential for material characterization through the inverse methodology. For comparison purposes, the sensitivity analyses and inverse problem were repeated with the surface velocity of the cylinder as the measured response in place of the acoustic pressure.

2. Forward problem

A finite element fluid-structure interaction formulation was used to model the dynamic response of a fluid immersed cylindrical vessel subjected to an impact load for a given set of material parameters (see [17] for details). For this work, the surrounding fluid was modeled as a semi-infinite medium using the Sommerfeld radiation boundary condition, and the only excitation in the fluid was assumed to be due to the motion of the cylinder. The fluid was assumed to be compressible with no net flow, small pressure amplitudes, and negligible viscosity (i.e. acoustic medium). For the cylinder behavior, the body forces were assumed to be negligible and no structural damping was considered. In addition, strains and displacements were assumed to be small due to the vibroacoustic excitation and the cylinder material behavior was taken to be linear orthotropic elastic.

The cylinder was modeled using an explicit time integration scheme with 4-node linear general purpose shell finite elements with reduced integration. The three local planes of material symmetry for the orthotropic material behavior were taken to correspond to the longitudinal, circumferential, and radial directions of the cylinder, as shown in Fig. 1. Furthermore, the stress-strain relationship for the shell elements can be described in terms of the plane-stress components, and are shown for the given planes of material symmetry in Voigt notation as

$$\begin{Bmatrix} \sigma_C(\vec{x},t) \\ \sigma_L(\vec{x},t) \\ \sigma_S(\vec{x},t) \end{Bmatrix} = \begin{bmatrix} \frac{E_C^2}{E_C - E_L v_{CL}^2} & \frac{E_C E_L v_{CL}}{E_C - E_L v_{CL}^2} & 0 \\ \frac{E_C E_L v_{CL}}{E_C - E_L v_{CL}^2} & \frac{E_C E_L}{E_C - E_L v_{CL}^2} & 0 \\ 0 & 0 & 2G \end{bmatrix} \begin{Bmatrix} \varepsilon_C(\vec{x},t) \\ \varepsilon_L(\vec{x},t) \\ \varepsilon_S(\vec{x},t) \end{Bmatrix}, \quad (1)$$

where σ_C , σ_L , and σ_S are the circumferential, longitudinal, and in-plane (longitudinal-circumferential) shear stress components; ε_C , ε_L , and ε_S are the circumferential, longitudinal, and in-plane shear strain components; E_L is the longitudinal elastic modulus; E_C is the circumferential elastic modulus; G is the in-plane shear modulus and; v_{CL} is the orthotropic Poisson's ratio, which characterizes the longitudinal strain due to a circumferential strain.

After finite element discretization of the cylinder and the fluid domain, a coupled system of equations is obtained by combining the cylinder equation of motion and the acoustic wave equation through their respective external force vectors. The fluid pressure at the interface between the cylinder and fluid translates to an external traction on the cylinder in the direction of the fluid normal. In addition, the normal components of the structural accelerations at the interface translate into an applied normal derivative of pressure on the fluid. The coupled system of equations is represented as

$$\begin{bmatrix} [M] & [0] \\ \rho_f [S]^T & [M_f] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}(t)\} \\ \{\ddot{p}(t)\} \end{Bmatrix} + \begin{bmatrix} [0] & [0] \\ [0] & [C_f] \end{bmatrix} \begin{Bmatrix} \{\dot{u}(t)\} \\ \{\dot{p}(t)\} \end{Bmatrix} + \begin{bmatrix} [K] & -[S] \\ [0] & [K_f] \end{bmatrix} \begin{Bmatrix} \{u(t)\} \\ \{p(t)\} \end{Bmatrix} = \begin{Bmatrix} \{R(t)\} \\ -\{R_f(t)\} \end{Bmatrix}, \quad (2)$$

where t is time, ρ_f is the fluid density, $[M_f]$ is the fluid mass matrix, $[C_f]$ is the fluid damping matrix, $[K_f]$ is the fluid stiffness matrix, $\{P(t)\}$ is the nodal pressure vector, $\{R_f(t)\}$ is the external fluid force vector, $[M]$ is the solid mass matrix, $[K]$ is the solid stiffness matrix, $\{u(t)\}$ is the nodal displacement vector, $\{R(t)\}$ is the solid external force vector, and $[S]$ represents the interaction matrix.

3. Inverse problem

The inverse problem considered was to determine the elastic material parameters of a cylindrical vessel using the measured response from a vibroacoustic-based testing procedure. To obtain a solution, this inverse problem was cast as a constrained optimization problem described as

$$\begin{aligned} & \underset{\vec{\alpha} \in R^n}{\text{Minimize}} \left(J(\vec{\alpha}) \right) \\ & \text{s.t.} \\ & \vec{a} \leq \vec{\alpha} \leq \vec{b} \end{aligned}, \quad (3)$$

where $\vec{\alpha}$ is the vector of unknown parameters (i.e. E_L , E_C , etc.), $\vec{a}$ and $\vec{b}$ are vectors containing bounds for the unknown parameters, and $J(\vec{\alpha})$ is an error functional which quantifies the distance between the experimental response and the response computed using the Finite Element method for a given set of parameter estimates. For this work, the error functional was defined as

$$J(\vec{\alpha}) = \sum_{i=1}^N \frac{\| r_i^{\text{exp}}(t) - r_i^{\text{sim}}(\vec{\alpha}, t) \|_{L_\infty}}{\| r_i^{\text{exp}}(t) \|_{L_\infty}}, \quad (4)$$

where $r_i^{\text{exp}}(t)$ and $r_i^{\text{sim}}(\vec{\alpha}, t)$ are the experimental and computed responses, respectively, at the i^{th} measurement point, N is the total number of measurement points, and $\| \cdot \|_{L_\infty}$ is the L_∞ -norm which is defined as

$$\| r(t) \|_{L_\infty} = \sup_t | r(t) |. \quad (5)$$

As is typical for inverse problems related to non-destructive evaluation, the above optimization problem has several inherent difficulties. In general, optimization problems of this nature have

non-convex error surfaces with multiple local solutions, requiring the use of non-gradient based solution methods. In addition, properties of biological materials can vary drastically depending on location, age, and presence of disease. Furthermore, the numerical modeling of these complex structures often requires significant computational time. Therefore, to address these potential difficulties, the optimization problem was solved using the Surrogate-Model Accelerated Random Search (SMARS) algorithm introduced by Brigham and Aquino in [18].

The SMARS algorithm is an iterative application of a random search algorithm [19] and a surrogate-model optimization approach [20]. The random search portion of the SMARS algorithm is applied to efficiently search large, non-convex parameter spaces, but typically requires a large number of numerical simulations to converge to a global solution. Therefore, the surrogate-model approach is applied to locally search optimal regions of the parameter space throughout the random search process. The surrogate-model provides computationally inexpensive estimates to the solution, thereby accelerating the search and reducing the number of numerical simulations required. References [8] and [18] provide a complete discussion of the Surrogate-Model Accelerated Random Search algorithm.

4. Examples and discussion

An experiment was simulated using the Finite Element method described above for a cylinder filled and surrounded with water, with an impact force applied to the surface of the cylinder as shown in Fig. 1. The properties of the water were assumed to be known for the inverse problem, with a density of 1000 kg/m³ and a bulk modulus of 2.15 GPa. The geometry and material parameters for the cylinder were based on the experiment shown in [21] for a silicone rubber tube. The cylinder had an outside diameter of 5 mm, a thickness of 1 mm, and a length of 10 cm. The density and orthotropic Poisson's ratio of the cylinder were also assumed to be known for the inverse problem and given as 1180 kg/m³ and 0.44, respectively. Typically biological soft tissues will have approximately the density of water and can be considered nearly incompressible; therefore, for many applications the density and Poisson's ratio can be assumed to be known for the solid materials. Thus, the parameters to be identified for the inverse problem were the longitudinal elastic modulus, the circumferential elastic modulus, and the in-plane shear modulus. Table 1 shows the moduli used for the simulated experiment (Target). The ends of the cylinder were fixed; however, the duration of the analysis was restricted so that the response waves did not reach the ends of the cylinder, therefore, avoiding reflection effects.

The impact force was applied as a circular surface pressure with a 0.7 mm diameter normal to the surface of the cylinder, and the amplitude was modeled as a half-sine wave given as

$$F(t) = \begin{cases} \sin[(2\pi f)t], & 0 \leq t \leq \frac{1}{2f} \\ 0, & \frac{1}{2f} < t \leq t_a \end{cases}, \quad (6)$$

where f is the impact frequency, and t_a is the duration of the computer simulation. Due to the linearity in the representative numerical models, the responses could be normalized to remove the effect of any uncertainty in the load magnitude, and as such, the amplitude of the impact force is irrelevant. The impact frequency for the experiment was 5 kHz and the total analysis duration was 2.1 milliseconds.

Two response quantities were measured during the simulated experiment and applied separately as the experimental data to solve the inverse problem. To simulate the vibroacoustic method, the acoustic pressure response was measured at a point in the surrounding water 10 mm directly above the point of impact. Alternatively, for comparison purposes and to better

understand the information contained in the acoustic signal regarding the mechanical behavior of the cylinder, the normal surface velocity response of the cylinder was measured at a point 10 mm away from the point of impact, in the same direction as the impact.

Random Gaussian noise was added to both responses in order to add realism to the experiment. The noise was introduced discretely into the simulated data as

$$r^n(t_i) = r^o(t_i) \times \left(1 + 0.1\eta \parallel r^o(t) \parallel_{L_\infty}\right), \quad (7)$$

where $r^n(t_i)$ and $r^o(t_i)$ are the simulated response with noise and without noise, respectively, at the i^{th} sampled time, and η is a normally distributed random variable with unit variance and zero mean.

Lastly, the responses for the experiment and the optimization simulations were filtered through a low-pass filter with a cutoff frequency of 2 kHz. Filtering was applied since most laboratory test data is filtered in some way to remove noise from the measured data. In addition, filtering improved the numerical convergence of the finite element models, and the computational expense of the finite element analyses was reduced. Notice that high frequency modes contain small wavelengths, which require fine meshes to produce accurate results.

Several forward simulations were performed before solving the inverse problem to obtain coarse sensitivity estimates of the responses to changes in the moduli. If a response measurement were found to have weak sensitivity to a particular modulus, then the solution of the inverse problem would likely be non-unique. In other words, a modulus with weak sensitivity will not be identifiable in the inverse problem. This analysis was carried out by performing forward simulations (i.e. finite element analyses) of the experiment with each modulus taken, in turn, as the original value (given in Table 1) then one-half, twice, and five-times the original value, while the other moduli were held constant at the original values. The filtered velocity and pressure responses for each forward simulation were then compared to assess the response sensitivities to the variations of each modulus.

Finally, the methodology described in Section 3 was applied with the simulated experiments to inversely identify the moduli for which the responses were sensitive. Table 1 shows the minimum and maximum parameter values considered for the optimization problems. Due to the stochastic nature of the SMARS algorithm, five optimization runs were performed for each measured response, and the mean and standard deviation of the results were calculated.

4.1 Results from surface velocity measurements

Fig. 2, Fig. 3, and Fig. 4 show the sensitivity results for the normal surface velocity response of the cylinder with respect to the circumferential elastic modulus, the longitudinal elastic modulus, and the shear modulus, respectively. The results clearly show that the velocity response is sensitive to changes in all three moduli, with the response being slightly less sensitive to the circumferential elastic modulus than for the other two moduli.

Fig. 5 shows the mean and standard deviation of the surface velocity responses computed from the results of the five optimization trials and compared to the simulated experiment with noise. In all cases, the optimization algorithm found solutions that matched the experimental response accurately.

Table 2 shows the mean and standard deviation of the parameters found in the five optimization trials. The three moduli were all found with high accuracy. However, as was expected from

the sensitivity results, the circumferential elastic modulus deviated from the target solution slightly more than the solutions for the other two moduli.

4.2 Results from acoustic pressure measurements

Fig. 6, Fig. 7, and Fig. 8 show the sensitivity results for the acoustic pressure response in the surrounding water with respect to the circumferential elastic modulus, the longitudinal elastic modulus, and the shear modulus, respectively. The sensitivity results for the acoustic pressure response show a far weaker sensitivity to all three of the elastic moduli than was found for the velocity response. However, this reduction in sensitivity was expected for the acoustic response, as the acoustic pressure field is not directly affected by the entire mechanical response of the cylinder, since it is only coupled to the normal accelerations of the cylinder wall. In particular, the shear modulus had essentially no effect on the acoustic pressure response, which is likely due to the lack of displacements occurring normal to the cylinder wall in modes of vibration affected by in-plane shear. The discrepancy between the sensitivity results for the two elastic moduli is more likely due to the specifics of the given experiment (e.g. response measurement point and/or impact frequency), since both circumferential and longitudinal modes of vibration do contribute to normal accelerations.

Based on the sensitivity results, it is expected that the circumferential elastic modulus can be accurately identified through the inverse problem, whereas, a less accurate approximation would be expected for the longitudinal elastic modulus. Due to the lack of sensitivity, the shear modulus can be considered to be non-observable through the acoustic pressure response. Therefore, only the two elastic moduli were considered for the inverse problem.

Fig. 9 shows the mean and standard deviation of the acoustic pressure responses obtained from the results of the five optimization trials, and compared to the simulated experiment with noise. Again, in all cases the SMARS algorithm was able to locate solutions matching the experimental response with high accuracy and almost zero standard deviation.

Table 3 shows the mean and standard deviation of the parameters found in the five optimization trials. As was expected from the sensitivity results, the circumferential elastic modulus was found with reasonable accuracy, although with lower accuracy than the results for the surface velocity response. The longitudinal elastic modulus was found with a relatively high deviation; however, the accuracy of the solutions was reasonable in consideration of the weak sensitivity found for the acoustic response. In addition, the results for the acoustic pressure response are promising in that, for all trials, the algorithm was able to identify the anisotropy of the material, and in particular identify which directional component was stiffer.

5. Additional comments

Additional forward simulations were conducted to investigate the sensitivity of the acoustic pressure response to changes in the moduli. Two different loading cases were proposed in which the frequency or the location of the load was changed. For the first case considered, a load frequency of 1 kHz was used to impact the cylinder. This was done to create different vibrations modes that could produce an increase in the sensitivity of the longitudinal elastic modulus. In the second case, an impact load was applied at a point above the longitudinal axes of the cylinder and parallel to the original load (shown in Figure 1). This load case was intended to create torsion modes and verify that indeed shear modulus cannot be identified from acoustic pressure. Notice that torsion modes are expected to be sensitive to in-plane shear modulus.

The results for both cases showed no improvement in the sensitivity of the signal to changes in the shear modulus (G). In addition, for the case where the load frequency was decreased, an increase in sensitivity for the longitudinal modulus was detected, while the sensitivity of

the circumferential modulus decreased. This behavior can be explained by noticing that lower frequency loading can excite global bending modes that can be more sensitive to the longitudinal modulus, while the higher frequency loading can excite local bending modes that may be more sensitive to the circumferential modulus. Hence, it is conceivable that multiple experiments may be needed to accurately characterize elastic moduli in anisotropic vessels using the proposed approach.

6. Conclusions

Through a simulated experiment, the capability to characterize orthotropic elastic properties for fluid immersed cylinders using vibroacoustic-based testing procedures was analyzed. Several numerical simulations of the experiment were performed in which the surface velocity response at a point on the cylinder due to an impact excitation was shown to be strongly sensitive to changes in the circumferential elastic modulus, the longitudinal elastic modulus, and the in-plane shear modulus of the cylinder. Alternatively, the acoustic response at a point in the surrounding fluid was shown to have comparatively mild sensitivity to changes in the circumferential and longitudinal elastic moduli, and almost no sensitivity to the in-plane shear modulus.

The response sensitivities were further validated through simulated inverse problems. For a single experiment and measurement point on the cylinder, the velocity response was sufficient for identifying all three moduli of the cylinder with high accuracy. For the same experiment with a single measurement point in the fluid, the acoustic response was sufficient for identifying the two elastic moduli of the cylinder with moderate accuracy, but was insufficient for identifying the in-plane shear modulus.

Vibroacoustic-based testing procedures were shown to have significant promise for non-destructive evaluation of anisotropic mechanical properties of fluid immersed cylinders. However, due to the information loss which occurs through the fluid-structure coupling, multiple experiments and/or multiple measurement locations may be necessary to fully and accurately characterize the mechanical behavior through acoustic emissions.

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Figure 1. Schematic of the simulated experiment (not to scale). The axes C, L, and R represent the circumferential, longitudinal, and radial directions of the cylinder, respectively.

Figure 2. Sensitivity results for the surface velocity response with respect to the circumferential elastic modulus, ($—$ $0.5 E_C$, $-□-$ E_C , $-○-$ $2 E_C$, $-×-$ $5 E_C$).

Figure 3. Sensitivity results for the surface velocity response with respect to the longitudinal elastic modulus, ($—$ $0.5 E_L$, $-□-$ E_L , $-○-$ $2 E_L$, $-×-$ $5 E_L$).

Figure 4. Sensitivity results for the surface velocity response with respect to the shear modulus, (— 0.5 G, -□- 1 G, -○- 2 G, -x- 5 G).

Figure 5. Mean and standard deviation (error bars) of the surface velocity responses for the results of the five optimization trials compared to the simulated experiment, (— Experiment with noise, • Optimization results).

Figure 6.

Sensitivity results for the acoustic pressure response with respect to the circumferential elastic modulus, ($—$ $0.5 E_C$, $-□-$ E_C , $-○-$ $2 E_C$, $-×-$ $5 E_C$).

Figure 7.
Sensitivity results for the acoustic pressure response with respect to the longitudinal elastic modulus, (— $0.5 E_L$, $\text{--}\square\text{--}$ E_L , $\text{--}\circ\text{--}$ $2 E_L$, $\text{--}\times\text{--}$ $5 E_L$).

Figure 8.
Sensitivity results for the acoustic pressure response with respect to the shear modulus, (— 0.5 G, —□— 2 G, —×— 5 G).

Figure 9. Mean and standard deviation (error bars) of the acoustic pressure responses for the results of the five optimization trials compared to the simulated experiment, (— Experiment with noise, • Optimization results).

Table 1

Simulated experimental material properties and optimization search range.

Optimization Parameter	Target Value (MPa)	Optimization Minimum (MPa)	Optimization Maximum (MPa)
E_L	1.09	0.75	7.5
E_C	5.45	0.75	7.5
G	0.378	0.10	1.0

Table 2

Mean and standard deviation of the optimization results for the five trials with the surface velocity response.

	E_C (Pa)	E_L (Pa)	G (Pa)
Target	5.45×10^6	1.09×10^6	3.78×10^5
Mean	4.85×10^6	1.07×10^6	3.94×10^5
Std. Dev.	4.94×10^4	2.86×10^3	1.34×10^3

Table 3

Mean and standard deviation of the optimization results for the five trials with the acoustic pressure response.

	E_C (Pa)	E_L (Pa)
Target	5.45×10^6	1.09×10^6
Mean	5.50×10^6	1.16×10^6
Std. Dev.	4.88×10^5	3.43×10^5