Published in final edited form as:

J Opt Soc Am A Opt Image Sci Vis. 2007 December ; 24(12): B136–B150.

Channelized-ideal observer using Laguerre–Gauss channels in detection tasks involving non-Gaussian distributed lumpy backgrounds and a Gaussian signal

Subok Park^{1,*}, Harrison H. Barrett², Eric Clarkson², Matthew A. Kupinski², and Kyle J. Myers¹

¹National Institute of Biomedical Imaging and Bioengineering/Center for Devices and Radiological Health (CDRH), Laboratory for the Assessment of Medical Imaging Systems, Division of Imaging and Applied Mathematics, CDRH, U.S. Food and Drug Administration, Silver Spring, Maryland 20993, USA

²Optical Sciences Center and Department of Radiology, University of Arizona, Tucson, Arizona 85724, USA

Abstract

We investigate a channelized-ideal observer (CIO) with Laguerre–Gauss (LG) channels to approximate ideal-observer performance in detection tasks involving non-Gaussian distributed lumpy backgrounds and a Gaussian signal. A Markov-chain Monte Carlo approach is employed to determine the performance of both the ideal observer and the CIO using a large number of LG channels. Our results indicate that the CIO with LG channels can approximate ideal-observer performance within error bars, depending on the imaging system, object, and channel parameters. The CIO also outperforms a channelized-Hotelling observer using the same channels. In addition, an alternative approach for estimating the CIO is investigated. This approach makes use of the characteristic functions of channelized data and employs an approximation method to the area under the receiver operating characteristic curve. The alternative approach provides good estimates of the performance of the CIO with five LG channels. However, for large channel cases, more efficient computational methods need to be developed for the CIO to become useful in practice.

1. INTRODUCTION

We take a task-based approach to the assessment of image quality [1]. That is, image quality should be assessed based on the average performance of an observer on a task of interest. In this paper, our task of interest is a binary signal-detection task.

Psychophysical studies provide measurements of human performance for a given task, but having humans assess and optimize medical-imaging systems is expensive and time-consuming. In particular, system optimization needs to be performed over a large set of parameters of interest. Therefore, using humans in system optimization is simply infeasible. As an alternative to humans, model observers, such as the Bayesian ideal observer and the Hotelling observer, can be used for the task-based assessment of image quality.

The Hotelling observer uses the mean and covariance of the data for calculating the decision variable; this observer is equivalent to the ideal observer in detection tasks involving Gaussian backgrounds and noise. The Hotelling observer is a useful tool for the assessment of image quality when it is impossible to use the true ideal observer.

The Bayesian ideal observer is a mathematical observer that uses all available statistical information in the raw data to optimally perform a specified task [2]. The test statistic of the ideal observer is defined as the ratio of the conditional probability density functions (PDFs) of the data under the signal-present and signal-absent hypotheses, and it is called the likelihood ratio.

The ideal observer is optimal in the sense that it minimizes the Bayes risk, or the average cost, of making signal-detection decisions using the data [2]. Equivalently, the ideal observer maximizes the diagnostic accuracy as summarized by the area under the receiver operating characteristic curve (AUC). This observer sets an upper bound for any observer, human or model, and it gives a meaningful measure of image quality in terms of diagnostic accuracy. Therefore, the ideal observer should be used for assessing and optimizing medical-imaging hardware whenever possible.

When a task of interest involves realistic and complex backgrounds, it is difficult to estimate the performance of the ideal observer because the PDFs of the data that this observer requires are often unknown. This has been the barrier to the estimation of the likelihood ratio required for the ideal observer for realistic tasks. Barrett *et al.* [3] and Clarkson and Barrett [4] formulated approximations to the performance of the ideal observer in various detection tasks. However, their approximations still require the knowledge of the PDFs of the image data conditional on the signal-present and signal-absent hypotheses.

Recently, Kupinski *et al.* [5] developed a Markov-chain Monte Carlo (MCMC) method for estimating the likelihood ratio in cases where randomly varying lumpy back-grounds and Gaussian signals are used in signal-known-exactly (SKE) tasks. Subsequently, Park *et al.* [6] extended this method for signal-known-statistically (SKS) tasks where Gaussian signals have either location or shape uncertainty. These MCMC methods have made the task of calculating the likelihood ratio in detection tasks using non-Gaussian distributed lumpy backgrounds possible. However, these methods can still take many days to compute the performance of the ideal observer. Further-more, if there are any changes in the configuration of the experiment such as signal magnitude or the mean number of lumps in the background, the entire estimation needs to be repeated.

The addition of a channel mechanism to a model observer was first introduced by Myers and Barrett [7] to reduce the dimensionality of an observer as well as model human performance in detection tasks. Channels are called *efficient* for an observer if a channelized observer reduces the dimensionality of a given observer and approximates the performance of that observer. The choice of efficient channels for an observer is subject to the physics of the imaging system and object statistics used in the task. Gallas and Barrett [8] found that a channelized-Hotelling observer (CHO) with Laguerre–Gauss (LG) channels [9] approximated the performance of the Hotelling observer in detection tasks using various lumpy backgrounds and rotationally symmetric signals.

Recently, Park *et al.* [10–12] introduced a dimensionality-reduction scheme to the full ideal observer in detection tasks using non-Gaussian distributed lumpy backgrounds and a signal of Gaussian profile in order to find efficient channels for the true ideal observer. This approach utilized the characteristic functions (CFs) of channelized data under the signal-present and signal-absent hypotheses instead of the PDFs of the channelized data. The CFs contain as much

statistical information for the channelized-ideal observer (CIO) to perform the detection task as the PDFs.

In this paper, we first discuss an MCMC algorithm that operates on channelized data. This algorithm was previously employed by Park *et al.* [12] to investigate how well the performance of the CIO using large numbers of LG channels approximates that of the ideal observer in the aforementioned detection tasks. A complete description of this algorithm will be presented along with a statistical analysis of channelized data of non-Gaussian distributed lumpy-background images. This MCMC algorithm is an extension of the Kupinski algorithm [5] that operates on image data. The extended algorithm provides good estimates of the performance of the CIO, but the approach does not reduce the computation time of the ideal observer.

Then, we will discuss in detail the analytical formulation of the alternative approach [10] using the CFs of channelized data to the estimation of the CIO. The analytical expressions for the CFs of channelized data under both hypotheses are derived by making use of models of the object, imaging system, and measurement noise. Transforming the characteristic functionals of the object model through the imaging system and channels, with use of the system operator and a channel matrix that consists of rows of channels, yields the CFs of channelized data. The resulting analytical expression of the CIO using this approach was previously presented in [10].

To show the potential of this approach involving the CFs of channelized data, an algorithm combining the Monte Carlo method and the inverse fast Fourier transform (IFFT) algorithm is employed for estimating the CIO using five LG channels in the detection tasks. This algorithm makes use of an approximation method by Barrett *et al.* [3], which we call *the method of $G(0)$ approximation to the AUC*, for estimating the AUC of the CIO. For comparison, the MCMC algorithm is also employed to provide estimates of the performance of the CIO.

To see the effect of image statistics on observer performance, the number and the width parameter of the LG channels and the mean number of lumps in the lumpy object are varied in the simulation. Simulation results from the MCMC algorithm will reveal that the CIO with LG channels approaches the performance of the ideal observer in the detection tasks using non-Gaussian distributed lumpy backgrounds within the error bars, depending on the channel, imaging system, and lumpy-background parameters considered here. They will also show that the CIO with LG channels is a new nonlinear mathematical observer that outperforms the CHO using the same LG channels for the tasks. The results will demonstrate that the Hotelling observer, approximated by the CHO with LG channels, is truly suboptimal with respect to the ideal observer in the tasks involving non-Gaussian distributed images. Comparison of simulation results from the alternative approach and the MCMC algorithm will show that the alternative approach provides good estimates of the performance of the CIO.

2. MATHEMATICAL BACKGROUND

A linear imaging system can be mathematically represented by

$$\mathbf{g} = \mathcal{H}\mathbf{f} + \mathbf{n}, \quad (1)$$

where $\mathcal{H}$ is a continuous-to-discrete imaging operator that maps $\mathbf{f}(\mathbf{r})$, a function of continuous variables in a Hilbert space of square-integrable functions, to an $M \times 1$ data vector $\mathbf{g}$; $\mathbf{r}$ is a two-dimensional (2D) spatial coordinate; and $\mathbf{n}$ is associated with measurement noise. In this work, an image is 64×64 and hence $M = 64^2$.

The operator $\mathcal{H}$ consists of a set of sensitivity functions $\{h_m(\mathbf{r})\}$. In our simulation, each sensitivity function is modeled as a Gaussian function, i.e.,

$$h_m(\mathbf{r}) = \frac{h}{2\pi w^2} \exp \left[-\frac{(\mathbf{r} - \mathbf{p}_m)'(\mathbf{r} - \mathbf{p}_m)}{2w^2} \right], \quad (2)$$

where $\{\mathbf{p}_m\}_{m=1}^M$ are pixel locations in the field of view (FOV), width $w = 0.5$, and height $h = 40$.

In this work, the task of interest is binary signal detection, which requires the following two hypotheses, signal-absent and signal-present:

$$H_0 : \mathbf{g} = \mathcal{H} \mathbf{f}_b + \mathbf{n}, \quad (3)$$

$$H_1 : \mathbf{g} = \mathcal{H} (\mathbf{f}_b + \mathbf{f}_s) + \mathbf{n}, \quad (4)$$

where $\mathbf{f}_b$ and $\mathbf{f}_s$ represent a random-background object and a signal object, respectively. The background and the signal are assumed to be additive and statistically independent of each other. For notational convenience, $\mathbf{b}$ and $\mathbf{s}$ denote the noise-free background image $\mathbf{f}_b$ and the noise-free signal image $\mathcal{H} \mathbf{f}_s$, respectively.

We model the measurement noise $\mathbf{n}$ as an i.i.d. Gaussian with zero mean and covariance $\mathbf{K}_n = \sigma_n^2 \mathbf{I}$. Then the PDF of $\mathbf{n}$ under each hypothesis is given by

$$\begin{aligned} pr(\mathbf{n}|H_j) &= [(2\pi)^M \det(\mathbf{K}_n)]^{-1/2} \exp \left[-\frac{1}{2} \mathbf{n}' \mathbf{K}_n^{-1} \mathbf{n} \right] \\ &= [2\pi\sigma_n^2]^{-M/2} \exp \left[-\frac{\mathbf{n}' \mathbf{n}}{2\sigma_n^2} \right], \end{aligned} \quad (5)$$

where $\mathbf{n} = \mathbf{g} - \mathbf{b} - \mathbf{j}\mathbf{s}$ and j indicates which hypothesis $\mathbf{g}$ comes from. Then, the m th component of the data vector $\mathbf{g}$ is generated by

$$g_m = \int_s d\mathbf{r} h_m(\mathbf{r}) f(\mathbf{r}) + n_m, \quad (6)$$

where n_m is the m th component of $\mathbf{n}$.

For the background object $\mathbf{f}_b$, the lumpy background model [13] is used, given by

$$\mathbf{f}_b \equiv f_b(\mathbf{r}) = \sum_{n=1}^N l_b(\mathbf{r} - \mathbf{c}_n | a_b, \sigma_b), \quad (7)$$

where N is a random number of lumps in the object, N follows a Poisson distribution with mean $\bar{N}$, $l_b(\cdot)$ is the lump function, $\mathbf{c}_n$ is the center of the n th lump and is uniformly distributed in the FOV, and a_b and σ_b are the magnitude and width of the lump function, respectively. A circularly symmetric Gaussian profile is used for the lump function $l_b(\cdot)$, given by

$$l_b(\mathbf{r}-\mathbf{c}_n|a_b, \sigma_b) = a_b \exp \left(-\frac{(\mathbf{r}-\mathbf{c}_n)^T(\mathbf{r}-\mathbf{c}_n)}{2\sigma_b^2} \right). \quad (8)$$

The signal object $\mathbf{f}_s$ is also Gaussian, given by

$$\mathbf{f}_s = l_s(\mathbf{r}-\mathbf{c}_s|a_s, \sigma_s) = a_s \exp \left(-\frac{(\mathbf{r}-\mathbf{c}_s)^T(\mathbf{r}-\mathbf{c}_s)}{2\sigma_s^2} \right), \quad (9)$$

where $\mathbf{c}_s$ is the center, and a_s and σ_s are the magnitude and width of the signal object, respectively.

In Table 1, the values for the object and system parameters used in our simulation are summarized. The image data is generated using Eq. (6), the imaging operator $\mathcal{H}$ with a Gaussian blur function $h_m(\mathbf{r})$ given in Eq. (2), and the background and signal objects given in Eq. (8) and Eq. (9). The width and the magnitude of this signal image are about 3.04 and 7.78, respectively. The location $\mathbf{c}_s$ of the signal is the center of the FOV of each signal-present LB image. See Fig. 1 below for a Gaussian-signal image and lumpy-background images with $N=5$ or 12.

3. MODEL OBSERVERS

In this section, we first discuss how to compute the performance of the ideal observer. Then, channelization of the image data using LG channels is discussed. Finally, we discuss how to compute channelized observers such as the CIO and the CHO.

A. Ideal Observer

For a binary detection task, the test statistic of the ideal observer is the likelihood ratio, given by [2]

$$\Lambda(\mathbf{g}) = \frac{\text{pr}(\mathbf{g}|H_1)}{\text{pr}(\mathbf{g}|H_0)}, \quad (10)$$

where $\text{pr}(\cdot|H_j)$ is the PDF of the image data under the hypothesis H_j .

To incorporate the randomness in the background into the ideal-observer test statistic, we define $\boldsymbol{\theta} = \{N, c_1, \dots, c_N\}$ to represent the random background parameters. Then, the likelihood ratio in Eq. (10) can be rewritten as [5]

$$\Lambda(\mathbf{g}) = \int d\boldsymbol{\theta} \Lambda_{\text{BKE}}(\mathbf{g}|\mathbf{b}(\boldsymbol{\theta})) \text{pr}(\boldsymbol{\theta}|\mathbf{g}, H_0), \quad (11)$$

where Λ_{BKE} represents the likelihood ratio for background-known-exactly (BKE) cases and $\text{pr}(\boldsymbol{\theta}|\mathbf{g}, H_0)$ represents the posterior density, given by

$$\text{pr}(\boldsymbol{\theta}|\mathbf{g}, H_0) = \frac{\text{pr}(\mathbf{g}|\mathbf{b}(\boldsymbol{\theta}), H_0) \text{pr}(\boldsymbol{\theta})}{\int d\boldsymbol{\theta}' \text{pr}(\mathbf{g}|\mathbf{b}(\boldsymbol{\theta}'), H_0) \text{pr}(\boldsymbol{\theta}')} \quad (12)$$

Since Gaussian noise with covariance $\mathbf{K}_n$ is used in this work, the likelihood ratio for BKE cases is

$$\Lambda_{\text{BKE}}(\mathbf{g}|\mathbf{b}(\theta)) = C^{-1} \exp(\{\mathbf{g} - \mathbf{b}(\theta)\}^t \mathbf{K}_n^{-1} \mathbf{s}), \quad (13)$$

where $C = \exp((\mathbf{s}/2)^t \mathbf{K}_n^{-1} \mathbf{s})$. We employ the MCMC algorithm presented by Kupinski *et al.* [5] to estimate this likelihood ratio. Then, the integral in Eq. (11) is approximated via

$$\widehat{\Lambda}(\mathbf{g}) = \frac{1}{J - J_0} \sum_{i=J_0+1}^J \Lambda_{\text{BKE}}[\mathbf{g}|\mathbf{b}(\theta^{(i)})], \quad (14)$$

where $\{\theta^{(i)}\}_{i=1}^J$ are sampled from a symmetric proposal density and acceptance probability combination. The quantity J is the number of sample backgrounds $\{\mathbf{b}(\theta^{(i)})\}$ that form the Markov chain. The quantity J_0 , which is often called a burn-in, is the number of sample backgrounds that would not be used for estimating the likelihood ratio. This ensures that initial, unstable samples will be discarded from the estimation of the likelihood ratio [5]. In our MCMC simulation, $J = 200,000$ and $J_0 = 1000$ are used. These numbers are chosen based on the work by Kupinski *et al.* [5,14]. In Eq. (13), the quantity C is a positive-valued constant for an SKE signal, and hence $\Lambda_{\text{BKE}}[\mathbf{g}|\mathbf{b}(\theta^{(i)})] = \exp(\{\mathbf{g} - \mathbf{b}(\theta^{(i)})\}^t \mathbf{K}_n^{-1} \mathbf{s})$ can be used in the estimation of the likelihood ratio in Eq. (14).

B. Channelized Observers

We define $\mathbf{T}$ as an $N_c \times M$ channel matrix that consists of N_c rows of channel vectors. Each channel vector consists of sampled values of a continuous channel function at the pixel locations in the FOV. This channel matrix $\mathbf{T}$ maps the image vector $\mathbf{g}$ to an N_c -dimensional vector $\mathbf{v}$ via

$$\mathbf{v} = \mathbf{T}\mathbf{g} = \mathbf{T}(\mathcal{H}\mathbf{f} + \mathbf{n}). \quad (15)$$

In our simulation, LG functions [9] are employed as the channel functions. LG functions are the product of La-guerre polynomials and Gaussian functions, i.e.,

$$u_p(r|a_u) = \frac{\sqrt{2}}{a_u} \exp\left(\frac{-\pi r^2}{a_u^2}\right) L_p\left(\frac{2\pi r^2}{a_u^2}\right), \quad (16)$$

where a_u is the width of the Gaussian function and

$$L_p(x) = \sum_{k=0}^p (-1)^k \binom{p}{k} \frac{x^k}{k!}. \quad (17)$$

These continuous functions form an orthogonal set. Each LG function $u_p(r|a_u)$ is sampled to yield a row vector in an $N_c \times M$ channel matrix $\mathbf{T}$. As the dimension N_c of the channel matrix becomes large, the rows of this matrix lose orthogonality. In our simulation, LG channels are used, and they are orthogonal only if N_c is smaller than 10. The Gaussian functions in the LG channels are centered at the signal location, and the values for the width parameter are chosen

relative to the width of the signal. Figure 1 shows how big the signal is in the task, and Fig. 2 presents intensity plots of LG channels with $a_u = 5, 10, 16$, and 23 , and $p = 0, 4$, and 9 .

The CIO likelihood ratio is defined as the ideal observer that optimally detects the signal seen in the channelized data, given by

$$\Lambda(\mathbf{v}) = \frac{\text{pr}(\mathbf{v}|H_1)}{\text{pr}(\mathbf{v}|H_0)}, \quad (18)$$

where the vector $\mathbf{v}$ represents channelized data. In a fashion similar to the ideal-observer likelihood ratio, the likelihood ratio of the CIO can be rewritten as an integral, using the parameter vector $\boldsymbol{\theta}$ for the lumpy object:

$$\Lambda(\mathbf{v}) = \int d\theta \Lambda_{\text{BKE}}(\mathbf{v}|\mathbf{b}(\theta)) \text{pr}(\theta|\mathbf{v}, H_0), \quad (19)$$

where $\Lambda_{\text{BKE}}(\mathbf{v}|\mathbf{b}(\theta))$ is

$$\Lambda_{\text{BKE}}(\mathbf{v}|\mathbf{b}(\theta)) = \frac{\text{pr}(\mathbf{v}|\mathbf{b}(\theta), H_1)}{\text{pr}(\mathbf{v}|\mathbf{b}(\theta), H_0)} \quad (20)$$

and the posterior density is

$$\text{pr}(\theta|\mathbf{v}, H_0) = \frac{\text{pr}(\mathbf{v}|\mathbf{b}(\theta), H_0) \text{pr}(\theta)}{\int d\theta' \text{pr}(\mathbf{v}|\mathbf{b}(\theta'), H_0) \text{pr}(\theta')} \quad (21)$$

The likelihood ratio $\Lambda(\mathbf{v})$ in Eq. (19) can be approximated by an MCMC approach:

$$\widehat{\Lambda}(\mathbf{v}) = \frac{1}{J - J_0} \sum_{i=J_0+1}^J \Lambda_{\text{BKE}}(\mathbf{v}|\mathbf{b}(\theta^{(i)})), \quad (22)$$

where $\{\mathbf{b}(\theta^{(i)})\}$ constitute a Markov chain.

Since the measurement noise $\mathbf{n}$ is an i.i.d. Gaussian with zero mean and covariance $\mathbf{K}_n$, applying a channel matrix $\mathbf{T}$ to the vector $\mathbf{n}$ yields an i.i.d. Gaussian distribution with zero mean and covariance $\mathbf{TK}_n\mathbf{T}^t$. Hence, the PDFs of channelized data under the hypothesis H_j conditional upon the background $\mathbf{b}(\boldsymbol{\theta})$

$$\text{pr}(\mathbf{v}|\mathbf{b}(\theta), H_j) = \frac{\exp\left(-(\mathbf{2}\sigma_n^2)^{-1}[\mathbf{T}(\mathbf{g} - \mathbf{b}(\theta) - j\mathbf{s})]^t [\mathbf{TT}^t]^{-1} [\mathbf{T}(\mathbf{g} - \mathbf{b}(\theta) - j\mathbf{s})]\right)}{\sqrt{(2\pi)^{N_c} \det(\mathbf{TK}_n\mathbf{T}^t)}}. \quad (23)$$

Then, the likelihood ratio of the CIO for BKE cases is

$$\Lambda_{\text{BKE}}(\mathbf{v}|\mathbf{b}(\theta)) = \exp\left(\frac{1}{\sigma_n^2} \left[\mathbf{T} \left(\mathbf{g} - \mathbf{b}(\theta) - \frac{\mathbf{s}}{2}\right)\right]^t [\mathbf{TT}^t]^{-1} [\mathbf{Ts}]\right). \quad (24)$$

In the extended MCMC algorithm, the proposal functions employed by Kupinski *et al.* [5] are used to sample background images. Then, the channel matrix is applied to the sampled-background images to generate samples of channelized data. Kupinski *et al.* [5] used the acceptance ratio $\alpha(\tilde{\theta}, \theta^{(i)})$ to determine whether or not a sampled-parameter vector $\tilde{\theta}$ of the background is acceptable to add to a Markov chain of background images

$$\alpha(\tilde{\theta}, \theta^{(i)}) = \min \left[1, \frac{\text{pr}(\tilde{\theta}|\mathbf{g}, H_0)q(\tilde{\theta}|\theta^{(i)})}{\text{pr}(\theta^{(i)}|\mathbf{g}, H_0)q(\theta^{(i)}|\tilde{\theta})} \right], \quad (25)$$

where $q(\theta|\theta^{(i)})$ is a symmetric proposal function. In this work, $\text{pr}(\theta|\mathbf{g}, H_0)$ is replaced with $\text{pr}(\theta|\mathbf{v}, H_0)$, given in Eq. (21), to create the acceptance ratio α_v for generating a Markov chain of channelized data. Then, the acceptance ratio becomes a ratio of $\text{pr}(\mathbf{v}|\mathbf{b}(\theta), H_0)\text{pr}(\theta)$ evaluated at $\tilde{\theta}$ and $\theta^{(i)}$.

The test statistic of the CHO is

$$t_v = (\mathbf{K}_v^{-1} \mathbf{v}_s)^T \mathbf{v}, \quad (26)$$

where $\mathbf{v}_s = \mathbf{T}\mathbf{s}$ is the channelized signal. The matrix $\mathbf{K}_v$ is the covariance of the channelized data, given by

$$\mathbf{K}_v = \mathbf{T}\mathbf{K}_b\mathbf{T}^T + \sigma_n^2 \mathbf{T}\mathbf{T}^T, \quad (27)$$

where $\mathbf{K}_b$ is the covariance of the noiseless background. In our simulation, $\mathbf{K}_v$ is directly estimated using 5000 channelized signal-absent noisy images.

4. CIO USING CFs

In Subsections 3.A and 3.B, we discussed the MCMC methods for estimating the test statistics of the ideal observer and the CIO. In this section, we investigate an alternative approach involving the CFs of channelized data and the method of $G(0)$ approximation to the AUC for estimating the performance of the CIO. We discuss first the derivation of the CFs of channelized data and then the estimation of the performance of the CIO.

A CF of a vector $\mathbf{v}$, $\psi_v(\omega)$, is the Fourier transform of $\mathbf{v}$, given by

$$\psi_v(\omega) = \left\langle \exp(-2\pi i \omega^\dagger \mathbf{v}) \right\rangle_v, \quad (28)$$

where $\omega^\dagger \mathbf{v} = \sum_{k=1} \omega_k^* v_k$ and ω_k and v_k are the k th elements of the vectors ω and $\mathbf{v}$, respectively. A CF of a function is called a characteristic *functional* (CFL). In this work, the object $\mathbf{f}$ is a random function, and hence its Fourier transform is the CFL of the object $\mathbf{f}$ [15], given by

$$\Psi_f(\xi) = \left\langle \exp(-2\pi i \xi^\dagger \mathbf{f}) \right\rangle_f, \quad (29)$$

where $\xi^\dagger \mathbf{f}$ is the L_2 inner product, given by $\xi^\dagger \mathbf{f} = \int d\mathbf{r} \xi(\mathbf{r})^* f(\mathbf{r})$. Since the object in this work is real-valued, $\xi^\dagger \mathbf{f} = \int d\mathbf{r} \xi(\mathbf{r}) f(\mathbf{r})$.

Using the Poisson statistics of the random variable N and the Taylor expansions [16] of an exponential function, the CFL of the LB can be written as

$$\Psi_{f_b}(\xi) = \exp(-\bar{N}[1 - \Psi_{l_b}(\xi)]), \quad (30)$$

where $\Psi_{l_b}(\xi)$ is the CFL of one randomly located lump at center $\mathbf{c}$, given by

$$\Psi_{l_b}(\xi) = \left\langle \exp \left(-2\pi i \int d\mathbf{r} \xi(\mathbf{r}) l_b(\mathbf{r} - \mathbf{c} | a_b, \sigma_b) \right) \right\rangle_{\mathbf{c}}. \quad (31)$$

Since the Gaussian signal can be considered as an LB with one lump, the CFL of the Gaussian signal is similar to $\Psi_{l_b}(\xi)$. However, for an SKE task, a circularly symmetric Gaussian signal $\mathbf{f}_s$ is fixed at the center of each signal-present image, so averaging over the center is unnecessary. Thus, the CFL of $\mathbf{f}_s$ for an SKE task is given by

$$\Psi_{f_s}(\xi) = \exp \left(-2\pi i \int d\mathbf{r} \xi(\mathbf{r}) l_s(\mathbf{r} - \mathbf{c}_s | a_s, \sigma_s) \right). \quad (32)$$

The simulations in this work do not include SKS tasks, but signal uncertainty can also be considered in the approach involving the CFs of channelized data for estimating the CIO. For the CFL of an SKS task, see Appendix A.

A complete description of how to transfer the CFL of the object through a noisy, continuous-to-discrete imaging system to arrive at the CF of image data is given by Barrett and Myers [15] and Clarkson *et al.* [17]. The necessary relations for this work are summarized in Appendix B.

The CFs of channelized data, $\psi_{\mathbf{v}}(\cdot)$, can be derived by transforming the CFs of image data, $\psi_{\mathbf{g}}(\cdot)$, through a set of channels using the transpose of $\mathbf{T}$ [17]:

$$\psi_{\mathbf{v}}(\omega) = \psi_{\mathbf{g}}(\mathbf{T}^t \omega). \quad (33)$$

Note that $\psi(\cdot)$ is used for the CF, and $\Psi(\cdot)$ indicates the CFL. Using Eq. (33) and Eq. (B2), Eq. (B5), and Eq. (B7) from Appendix B yields the CFs of channelized data under both hypotheses, $\Psi_{\mathbf{v}|H_j}(\omega)$:

$$\psi_{\mathbf{v}|H_j}(\omega) = \Psi_{\mathbf{v}|H_j}[\mathcal{H}^\dagger \mathbf{T}^t \omega] \psi_{\mathbf{n}}(\mathbf{T}^t \omega), \quad j=0, 1, \quad (34)$$

where

$$\Psi_{\mathbf{v}|H_0}[\mathcal{H}^\dagger \mathbf{T}^t \omega] = \Psi_{f_b}[\mathcal{H}^\dagger \mathbf{T}^t \omega] \quad (35)$$

and

$$\Psi_{\mathbf{v}|H_1}[\mathcal{H}^\dagger \mathbf{T}^t \omega] = \Psi_{f_b}[\mathcal{H}^\dagger \mathbf{T}^t \omega] \Psi_{f_s}[\mathcal{H}^\dagger \mathbf{T}^t \omega]. \quad (36)$$

The analytical expressions for the CFs of channelized data with the LB model and the Gaussian signal are obtained by replacing ξ in Eq. (30)–Eq. (32) with $\mathcal{H}^\dagger \mathbf{T}' \omega$. Then, the CFL of the LB object through the system and channels is

$$\Psi_{f_b}[\mathcal{H}^\dagger \mathbf{T}' \omega] = \exp[-\bar{N} \{1 - \Psi_{l_b}[\mathcal{H}^\dagger \mathbf{T}' \omega]\}], \quad (37)$$

where $\Psi_{l_b}[\mathcal{H}^\dagger \mathbf{T}' \omega]$ is the CFL of one random lump through the system and channels, given by

$$\Psi_{l_b}[\mathcal{H}^\dagger \mathbf{T}' \omega] = \left\langle \exp \left(-2\pi i \int d\mathbf{r} \mathcal{H}^\dagger \mathbf{T}' \omega(\mathbf{r}) l_b(\mathbf{r} - \mathbf{c} | a_b, \sigma_b) \right) \right\rangle_c \quad (38)$$

and the CFL of the signal through the system and channels is

$$\Psi_{f_s}[\mathcal{H}^\dagger \mathbf{T}' \omega] = \exp \left(-2\pi i \int d\mathbf{r} \mathcal{H}^\dagger \mathbf{T}' \omega(\mathbf{r}) l_s(\mathbf{r} - \mathbf{c}_s | a_s, \sigma_s) \right). \quad (39)$$

With knowledge of the CFs of channelized data, $\Psi_{\mathbf{v}} | H_j(\omega)$, the PDFs of channelized data can be recovered via the inverse Fourier transform:

$$\text{pr}(\mathbf{v} | H_j) = \int d^{N_c} \omega \psi_{\mathbf{v} | H_j}(\omega) \exp(2\pi i \omega^\dagger \mathbf{v}), \quad j=0, 1, \quad (40)$$

where ω is assumed to be real, i.e., $\omega^\dagger \mathbf{v} = \sum_{k=1} \omega_k v_k$.

Then, the performance of the CIO can be estimated by the method of $G(0)$ approximation to the AUC [3,4], i.e.,

$$\text{AUC}_\Lambda \approx \frac{1}{2} + \frac{1}{2} \text{erf} \left(\frac{1}{2} \text{SNR}_{G(0)} \right), \quad (41)$$

where the AUC_Λ indicates the AUC of an observer using the likelihood ratio, $\text{erf}(\cdot)$ represents the error function, and $\text{SNR}_{G(0)} = \sqrt{2G(0)}$. The function $G(\cdot)$ is called the likelihood-generating function, which contains all statistical properties of the likelihood ratio [3]. The quantity $G(0)$ is defined by

$$G(0) = -4 \log \left(\left\langle \sqrt{\Lambda(\mathbf{v})} \right\rangle_{H_0} \right). \quad (42)$$

Equation (42) can be rewritten as

$$\begin{aligned} G(0) &= -4 \log \left(\int d^{N_c} \mathbf{v} \sqrt{\frac{\text{pr}(\mathbf{v} | H_1)}{\text{pr}(\mathbf{v} | H_0)}} \text{pr}(\mathbf{v} | H_0) \right) \\ &= -4 \log \left(\int d^{N_c} \mathbf{v} \sqrt{\text{pr}(\mathbf{v} | H_1) \text{pr}(\mathbf{v} | H_0)} \right), \end{aligned} \quad (43)$$

where the first equality holds by the definition of the like-lihood ratio, $\Lambda(\mathbf{v})$.

5. NUMERICAL COMPUTATION

In this section, first we discuss how we computed the CFs of channelized data, of which the forms are derived in Section 4. Then, we discuss how to compute observer performance including the FFT algorithm to estimate the quantity $G(0)$. Finally, the variance analysis for quantifying the precision of our observer performance estimates is discussed.

A. Computing the CFs of Channelized Data

In this simulation, the method of Monte Carlo integration was used to estimate $\Psi_{l_b}[\mathcal{H}^\dagger \mathbf{T}^t \omega]$ and hence $\Psi_{f_b}[\mathcal{H}^\dagger \mathbf{T}^t \omega]$. Lump centers $\{\mathbf{c}_k\}_{k=1}^{N_{mc}}$ were chosen from a uniform distribution in the FOV of the LB image and were used to approximate Eq. (38), as follows:

$$\Psi_{l_b}[\mathcal{H}^\dagger \mathbf{T}^t \omega] \approx \frac{1}{N_{mc}} \sum_{k=1}^{N_{mc}} \exp\left(-2\pi i \int d\mathbf{r} \mathcal{H}^\dagger \mathbf{T}^t \omega(\mathbf{r}) \times l_b(\mathbf{r} - \mathbf{c}_k | a_b, \sigma_b)\right) \quad (44)$$

$$= \frac{1}{N_{mc}} \sum_{k=1}^{N_{mc}} \exp\left(-2\pi i \sum_{m=1}^M \{\mathbf{T}^t \omega\}_m \times \int d\mathbf{r} h_m(\mathbf{r}) l_b(\mathbf{r} - \mathbf{c}_k | a_b, \sigma_b)\right) \quad (45)$$

$$= \frac{1}{N_{mc}} \sum_{k=1}^{N_{mc}} \exp(-2\pi i \omega^t \mathbf{T} \mathbf{y}_k), \quad (46)$$

where $\mathcal{H}^\dagger$ is the adjoint of $\mathcal{H}$ with $\mathcal{H}^\dagger \mathbf{T}^t \omega(\mathbf{r}) = \sum_{m=1}^M h_m(\mathbf{r}) \times \{\mathbf{T}^t \omega\}_m$, ω is an N_c -dimensional vector, and N_{mc} is the number of sampled centers $\mathbf{c}_k$. Equation (44) results in Eq. (45) with use of $\mathcal{H}^\dagger$. In Eq. (46), $\mathbf{y}_k$ is an M -dimensional vector, given by

$$\mathbf{y}_k = \left\{ \int d\mathbf{r} h_m(\mathbf{r}) l_b(\mathbf{r} - \mathbf{c}_k | a_b, \sigma_b) \right\}_m, \quad m=1, \dots, M. \quad (47)$$

The CFL of the SKE Gaussian signal through the system and channels given in Eq. (32) can be calculated similarly:

$$\Psi_{f_s}[\mathcal{H}^\dagger \mathbf{T}^t \omega] = \exp(-2\pi i \omega^t \mathbf{T} \mathbf{x}), \quad (48)$$

where $\mathbf{x}$ is an M -dimensional vector:

$$\mathbf{x} = \left\{ \int d\mathbf{r} h_m(\mathbf{r}) l_s(\mathbf{r} - \mathbf{c}_s | a_s, \sigma_s) \right\}_m, \quad m=1, \dots, M. \quad (49)$$

Then, $\{\mathbf{T} \mathbf{y}_k\}_{k=1}^{N_{mc}}$ and $\mathbf{T} \mathbf{x}$ are $N_c \times N_{mc}$ and $N_c \times 1$ matrices, respectively. These matrices may be precalculated to reduce memory requirements and hence speed up computation of the CIO. In our simulation, $N_{mc}=5000$.

B. Computing Observer Performance

For estimating the performance of the CIO using the method of $G(0)$ approximation to the AUC, Eq. (43) was employed with evenly sampled grids in every dimension of $\mathbf{v}$:

$$G(0) \approx -4 \log \left(\sum_p^{(N_v)^{N_d}} \sqrt{\text{pr}(\mathbf{v}_p | H_1) \text{pr}(\mathbf{v}_p | H_0)} \right). \quad (50)$$

The number of dimensions of $\mathbf{v}_p$, N_d , is the same as the number of channels, N_c , used in the CIO, and the quantity N_v is the number of grid points in each direction of $\mathbf{v}$.

In order to provide estimates of $\text{pr}(\mathbf{v}_p | H_j)$ in Eq. (50), the multidimensional IFFT algorithm was employed to calculate the integrals in Eq. (40), using estimates of the CFs of channelized data, $\psi_{\mathbf{v}_p | H_j}(\boldsymbol{\omega})$, as discussed in Subsection 5.A. The number of operations for the IFFT algorithm is on the order of $N_d N_\omega^{N_d} \log_2(N_\omega)$, where N_ω is the number of grid points in each dimension of $\boldsymbol{\omega}$. In our simulation, $N_c = N_d = 5$ and $N_\omega = N_v = 32$.

In this simulation, for each case of $\{\bar{N}, a_u\}$, the domain $[-L_\omega, L_\omega]$ of $\psi_{\mathbf{v}_p | H_j}(\boldsymbol{\omega})$, $j=0,1$, was predetermined manually by visual inspection, and the IFFT algorithm was applied to the estimated CFs with the predetermined domain. For example, see Fig. 3–Fig. 6 for the cases of $\bar{N} = 12$ and $a_u = 5$.

Figure 3 and Figure 4 show plots of profiles of the real and imaginary parts of $\psi_{\mathbf{v} | H_0}(\boldsymbol{\omega})$ calculated at surfaces where only $\omega_k \neq 0, k=1, \dots, 5$. Similarly, plots given in Fig. 5 and Fig. 6 show profiles of the real and imaginary parts of $\psi_{\mathbf{v} | H_1}(\boldsymbol{\omega})$ calculated at surfaces where only $\omega_k \neq 0, k=1, \dots, 5$. In this case, $L_\omega = 0.025$ was used for the domain of the CFs.

Then, $\text{pr}(\mathbf{v} | H_j)$, $j=1,0$, were estimated by the IFFT algorithm using the estimates of $\psi_{\mathbf{v} | H_j}(\boldsymbol{\omega})$ including the estimates shown in Fig. 3–Fig. 6. Figure 7 shows estimates of the marginal PDFs of v_k , $k=1, \dots, 5$, which were obtained respectively by integrating the resulting estimates of $\text{pr}(\mathbf{v} | H_j)$, $j=0,1$, over $\{v_n | v_n \neq v_k, n=1, \dots, 5\}$. Figure 8 presents plots of empirical histograms of 5000 channelized, signal-absent lumpy-background images. Note that the empirical histograms of v_k and the marginal PDFs of v_k , $k=1, \dots, 5$, match each other closely. Also note that the IFFT delivers the full multivariate PDF estimates, whereas the empirical histograms provide only the univariate marginal PDF estimates.

The MCMC algorithms were also employed to estimate the likelihood ratios for the CIO and the ideal observer. For the performance of these observers and the CHO, the Wilcoxon–Mann–Whitney statistic was employed to calculate the empirical estimate of the AUC.

For a comparison between the MCMC algorithm and the numerical approach involving the method of $G(0)$ approximation to the AUC and the IFFT algorithm, five LG channels were used in the CIO.

C. Variance Analysis

To quantify the precision of observer performance, the method of multiple-reader multiple-case variance analysis [18], in particular the one-shot method by Gallas [19], was employed. The multiple-reader multiple-case variance estimates generalize to a similar experiment in which a new set of readers and a new set of cases are drawn.

The test statistics of the ideal observer and CIO are random variables based on the Markov-chain sequence using a random-number seed. In our simulation, five different random-number

seeds were used to start the Markov-chain sequence. This way, five different readers were generated for each ideal observer and CIO experiment, and the average of the five AUC estimates was taken to be our AUC estimate for each observer. Each Markov chain was used to calculate the likelihood ratio for 200 pairs of signal-absent and signal-present images. For the CHO, two different covariance matrices using 5000 images were calculated to serve as two different readers. Each covariance matrix was used to calculate the decision variable for 1000 image pairs.

The approach using the method of $G(0)$ approximation to the AUC and the IFFT algorithm does not yield individual decision variables that are necessary in our variance analysis. The randomness in this approach arises from the fact that we use randomly sampled centers to estimate the CFL of the lumpy object given in Eq. (38), as discussed in Subsection 5.A. In our simulation, five different sets of 5000 randomly chosen lump centers were used to repeat the computation of the CFs of channelized data. In addition, the predetermined domains for the CFs, as discussed in Subsection 5.B, were fixed for the repetition of the computation using different sets of lump centers. Then, the average and standard deviations of the five AUC estimates were used to indicate the precision of the performance of the CIO, estimated by this approach.

6. RESULTS

A. Performance of the Ideal Observer, CIO, and CHO Using LG Channels

Figure 9 and Figure 10 show the performance of the CIO using from 5 to 40 LG channels in comparison with that of the ideal observer, while Fig. 11 and Fig. 12 do that for the CHO. Error bars are ± 2 standard deviations, estimated by the one-shot method. These figures show how the number, N_c , and width, a_u , of LG channels affect observer performance. Figure 9 and Figure 10 indicate that the performance of the CIO can approach that of the ideal observer in detection tasks using non-Gaussian distributed lumpy backgrounds, dependent on the lumpy object, imaging system, and channel parameters used in the simulation. In particular, for the case of $N = 5$, the performance of the CIO approaches that of the ideal observer within the error bars as the number of LG channels with $a_u = 16$ increases. On the other hand, Fig. 11 and Fig. 12 indicate that the CHO with LG channels, which approximates the performance of the Hotelling observer in these tasks, does not approach the performance of the ideal observer. The results provide evidence that the Hotelling observer, approximated by the CHO in this work, is suboptimal with respect to the ideal observer in the tasks involving non-Gaussian distributed images.

Table 2 presents a comparison of the ideal observer, CIO, and CHO. Different numbers of LG channels were used for the CIO and CHO in each case to see how they affect the ability of channelized observers to approximate the performance of the ideal observer. The results indicate that the CIO with LG channels outperforms the CHO using the same set of LG channels for the tasks considered in this work.

B. Performance of the Method of $G(0)$ Approximation to the AUC

In Fig. 13 and Table 3, we compare the MCMC algorithm and the alternative approach using the method of $G(0)$ approximation to the AUC and the IFFT algorithm for estimating the performance of the CIO. Error bars in Fig. 13 and Table 3 are ± 2 standard errors, and they were calculated from the repeated runs with five different sets of 5000 lump centers as explained in Subsection 5.C. The results show that AUC estimates for the CIO from the alternative approach are consistent with those from the MCMC approach.

7. DISCUSSION

While the CIO with LG channels is more efficient than the CHO for the ideal observer, the efficiency of the CIO relative to the ideal observer is still less than 1 (see Table 4). Factors that may have caused this outcome can include a choice of channels and computational methods for estimating the PDFs of channelized data such as the implementation of the FFT algorithm. In addition, it is not clear whether there is an interaction between the imaging system and the channel model and how important it is to take into account this information in optimizing system parameters with the CIO. If the CIO is to be used for evaluating imaging systems, these factors are worth investigating.

For the CIO with LG channels, at least ten channels are needed for the CIO to approach its asymptote, as indicated in Fig. 9 for the cases of $a_u = 16$ and $N = 5$. However, different types of channels may not require as many channels as LG channels for the CIO to approach ideal-observer performance. To improve the efficiency of the CIO, we are currently investigating different types of channels as candidate efficient channels for the CIO. In particular, we are looking into singular vectors of the imaging system to extract salient information of the signal and the background.

For the CIO using the method of $G(0)$ approximation, as the number of channels increases, the size of the input and output files for the usual IFFT algorithm becomes huge. For instance, in our simulation using five channels and 32 grid points in each dimension of the Fourier conjugate ω , the size of the input file was 32^5 bytes, which is 32 megabytes. However, if ten channels and sixteen grid points in each dimension were used, the size of the input file would be 16^{10} bytes, equivalently, 2^{10} gigabytes, which is too large for the clusters used in the simulation. The high-dimensional IFFTs and large input and output files necessitate vast computation resources. Therefore, if the CIO is to become useful in practice, more efficient computational methods for calculating the performance of the CIO need to be developed.

The simulation using the method of $G(0)$ approximation to the AUC and the IFFT algorithm for estimating the CIO with five channels was performed on clusters of 64 and 32 single-processor nodes, respectively, for calculating the CFs and PDFs of channelized data. These processors were 32-bit Intel Pentium 4 running at either 2.66 GHz with 1 GB of RAM or 3.0 GHz with 2 GB of RAM. In each set of the object and channel parameters, $\{N, a_u, N_c\}$, the

simulation took about $\frac{1}{2}$ hour to calculate one AUC estimate. About a third of the computation time was spent on managing input and output files; thus, the 20 minutes spent in running the IFFT algorithm was not impractical in the case of five channels. We repeated this simulation for the five different sets of lump centers to calculate the error bars of the AUC estimates. On the other hand, for each set of the object and channel parameters, the MCMC simulation took about six hours on a cluster of 32-single processors for estimating one AUC estimate using 400 likelihood ratios of 200 pairs of signal-present and signal-absent lumpy images. We repeated the MCMC simulation using the five different random-number seeds to calculate the multiple-reader multiplecase variance of the AUC estimates.

8. CONCLUSIONS

We have shown that the CIO using LG channels is a new mathematical observer that outperforms the CHO in detection tasks using non-Gaussian distributed lumpy backgrounds and a signal of Gaussian profile. We have also shown that the CIO can approach the performance of the ideal observer within the error bars, dependent on the object, imaging system, and channel parameters. Our simulation results indicate that the approach with the method of $G(0)$ approximation to the AUC and the IFFT algorithm provides good estimates

of the performance of the CIO in the five-channel case. In order for the CIO to become useful in practice, the development of efficient computational methods for estimating the performance of the CIO as well as efficient channels for the ideal observer is necessary.

Appendix

APPENDIX A: CFL OF THE SIGNAL WITH UNCERTAINTY

For an SKS task with signal-location uncertainty, the location of the signal is randomly chosen in the FOV, and the signal is marginalized over its random center as required for the CFL of $\mathbf{f}_s$. Thus, the CFL of $\mathbf{f}_s$ is equivalent to the CFL of one lump in Eq. (31), i.e.,

$$\Psi_{\mathbf{f}_s}(\xi) = \left\langle \exp \left(-2\pi i \int d\mathbf{r} \xi(\mathbf{r}) l_s(\mathbf{r} - \mathbf{c}_s | a_s, \sigma_s) \right) \right\rangle_{\mathbf{c}_s}. \quad (\text{A1})$$

For other kinds of signal uncertainty such as shape, the treatment is similar in that marginalizing the signal over the uncertainty is required to obtain the CFL of the signal.

APPENDIX B: TRANSFORMATION OF THE CFL OF THE OBJECT THROUGH A NOISY, DISCRETE IMAGING SYSTEM

We summarize the necessary relations for transferring the CFL of the object through a noisy, continuous-to-discrete imaging system to arrive at the CF of image data [15,17].

First, the CF of the noiseless image is mathematically written using the CFL of the object. Let $\bar{\mathbf{g}}$ denote the noiseless image, i.e., $\bar{\mathbf{g}} = \mathcal{H}\mathbf{f}$. This vector is regarded as the mean image of all measurements of the measurement process of the object $\mathbf{f}$:

$$\bar{\mathbf{g}} = \langle \mathbf{g} \rangle_{\mathbf{n}}, \quad (\text{B1})$$

where $\mathbf{n}$ is an $M \times 1$ vector of measurement noise with zero mean. The CF of the mean image, $\psi_{\bar{\mathbf{g}}}(\cdot)$, is related to the CFL of the object, $\Psi_{\mathbf{f}}(\cdot)$ [15]:

$$\begin{aligned} \psi_{\bar{\mathbf{g}}}(\rho) &= \left\langle \exp(-2\pi i \rho^\dagger \{\mathcal{H}\mathbf{f}\}) \right\rangle_{\bar{\mathbf{g}}} = \left\langle \exp(-2\pi i \{\mathcal{H}^\dagger \rho\}^\dagger \mathbf{f}) \right\rangle_{\mathbf{f}} \\ &= \Psi_{\mathbf{f}}(\mathcal{H}^\dagger \rho), \end{aligned} \quad (\text{B2})$$

where $\mathcal{H}^\dagger$ is the adjoint of $\mathcal{H}$ with $\mathcal{H}^\dagger \rho = \sum_{m=1}^M h_m(\mathbf{r}) \rho_m$.

Then, since $\mathbf{n}$ is considered additive and statistically independent of the object,

$$\psi_{\mathbf{g}}(\rho) = \left\langle \exp(-2\pi i \rho^\dagger \{\mathcal{H}\mathbf{f} + \mathbf{n}\}) \right\rangle_{\mathbf{g}} \quad (\text{B3})$$

$$= \left\langle \exp(-2\pi i \rho^\dagger \bar{\mathbf{g}}) \right\rangle_{\bar{\mathbf{g}}} \left\langle \exp(-2\pi i \rho^\dagger \mathbf{n}) \right\rangle_{\mathbf{n}} \quad (\text{B4})$$

$$= \psi_{\bar{\mathbf{g}}}(\rho) \psi_{\mathbf{n}}(\rho), \quad (\text{B5})$$

where $\psi_n(\rho)$ represents the CF of $\mathbf{n}$.

To obtain the CF of the signal-absent image, Eq. (B2) and Eq. (B5) may be used with $\mathbf{f}_b$ in place of $\mathbf{f}$ in Eq. (B2). The CF of the signal-present image, $\Psi_{\mathbf{f}_b + \mathbf{f}_s}(\mathcal{H}^\dagger \rho)$, is

$$\psi_{\mathbf{g}}(\rho) = \Psi_{\mathbf{f}}(\mathcal{H}^\dagger \rho) = \Psi_{\mathbf{f}_b + \mathbf{f}_s}(\mathcal{H}^\dagger \rho) = \Psi_{\mathbf{f}_b}(\mathcal{H}^\dagger \rho) \Psi_{\mathbf{f}_s}(\mathcal{H}^\dagger \rho), \quad (\text{B6})$$

where the first equality is from Eq. (B2), and the last equality holds owing to the statistical independence of $\mathbf{f}_b$ and $\mathbf{f}_s$. Hence, the CF of signal-present image data, using Eq. (B5) and Eq. (B6), is

$$\psi_{\mathbf{g}}(\rho) = \Psi_{\mathbf{f}_b}(\mathcal{H}^\dagger \rho) \Psi_{\mathbf{f}_s}(\mathcal{H}^\dagger \rho) \psi_n(\rho). \quad (\text{B7})$$

ACKNOWLEDGMENTS

This work was supported in part by the National Institute of Biomedical Imaging and Bioengineering at the National Institutes of Health intramural grant to the Center for Devices and Radiological Health as well as grants to the University of Arizona under grants R37EB000803 and P41EB002035.

REFERENCES

1. Barrett HH. Objective assessment of image quality: Effects of quantum noise and object variability. *J. Opt. Soc. Am. A* 1990;7:1266–1278. [PubMed: 2370589]
2. Van Trees, HL. Detection, Estimation, and Modulation Theory (Part 1). Academic; 1968.
3. Barrett HH, Abbey CK, Clarkson E. Objective assessment of image quality III: ROC metrics, ideal observers, and likelihood-generating functions. *J. Opt. Soc. Am. A* 1998;15:1520–1535.
4. Clarkson E, Barrett HH. Approximation to ideal-observer performance on signal-detection tasks. *Appl. Opt* 2000;39:1783–1794. [PubMed: 18345075]
5. Kupinski MA, Hoppin JW, Clarkson E, Barrett HH. Ideal observer computation using Markov-chain Monte Carlo. *J. Opt. Soc. Am. A* 2003;20:430–438.
6. Park, S.; Kupinski, MA.; Clarkson, E.; Barrett, HH. Ideal-observer performance under signal and background uncertainty. In: Taylor, CJ.; Noble, JA., editors. *5A in Information Processing in Medical Imaging*, Vol. 2732 in Lecture Notes in Computer Science. Springer-Verlag; 2003. p. 342-353.
7. Myers KJ, Barrett HH. Addition of a channel mechanism to the ideal observer model. *J. Opt. Soc. Am. A* 1987;4:2447–2457. [PubMed: 3430229]
8. Gallas BD, Barrett HH. Validating the use of channels to estimate the ideal linear observer. *J. Opt. Soc. Am. A* 2003;20:1725–1738.
9. Barrett HH, Abbey CK, Gallas BD, Eckstein M. Stabilized estimates of Hotelling-observer detection performance in patient-structured noise. *Proc. SPIE* 1998;3340:27–43.
10. Park S, Kupinski MA, Clarkson E, Barrett HH. Efficient channels for the ideal observer. *Proc. SPIE* 2004;5372:12–21.
11. Park, S. Ph.D. dissertation. The University of Arizona; 2004. Signal detection with random backgrounds and random signals.
12. Park S, Clarkson E, Barrett HH, Kupinski MA, Myers KJ. Performance of a channelized-ideal observer using Laguerre-Gauss channels for detecting a Gaussian signal at a known location in different lumpy backgrounds. *Proc. SPIE* 2006;6146:61460P.
13. Rolland JP, Barrett HH. Effect of random background inhomogeneity on observer detection performance. *J. Opt. Soc. Am. A* 1992;9:649–658. [PubMed: 1588452]
14. Clarkson E, Kupinski MA, Hoppin JW. Assessing the accuracy of estimates of the likelihood ratio. *Proc. SPIE* 2003;5034:135–143.
15. Barrett, HH.; Myers, KJ. Foundations of Image Science. Wiley; 2004.

16. Kaplan, W. Advanced Calculus. Vol. 4th ed.. Addison-Wesley; 1993.
17. Clarkson E, Kupinski MA, Barrett HH. Transformation of characteristic functionals through imaging systems. *Opt. Express* 2002;10:536–539. [PubMed: 19436394]
18. Clarkson E, Kupinski MA, Barrett HH. A probabilistic development of the MRMC method. *Acad. Radiol* 2006;13:1410–1421. [PubMed: 17070460]
19. Gallas BD. One-shot estimate of MRMC variance: AUC. *Acad. Radiol* 2006;13:353–362. [PubMed: 16488848]

Fig. 1.

The left picture shows a 64×64 noiseless Gaussian-signal image. The middle and right pictures show 64×64 LB images with $\bar{N}=5$ and 12, respectively.

Fig. 2.

The four rows show 64×64 LG channels with $a_u=5, 10, 16$, and 23 from top to bottom. The three columns show 64×64 LG channels with $p=0$ (left), $p=4$ (middle), and $p=9$ (right).

Fig. 3.

Plots (a)–(e) show profiles of the real part of an estimate of the CF under the signal-absent hypothesis, $\psi_{\mathbf{v}|H_0}(\boldsymbol{\omega})$, calculated at surfaces where only $\omega_k \neq 0, k=1, \dots, 5$. For these plots, $N=12$, $a_u=5$, $N_c=5$, and $L_\omega=0.025$.

Fig. 4.
As Fig. 3, for the imaginary part.

Fig. 5.
As Fig. 3, but for $\psi_{v|H_1}(\omega)$.

Fig. 6.
As Fig. 4, but for $\psi_{v|H_1}(\omega)$.

Fig. 7. (Color online) Plots (a)–(e) show estimates of the marginal PDFs of v_k , $k=1, \dots, 5$, obtained by integrating estimates of the PDF functions, $\text{pr}(\mathbf{v}|H_j)$, $j=0,1$, respectively, over $\{v_n|n \neq k, n=1, \dots, 5\}$. For these plots, $N=12$, $a=5$, and $N_c=5$.

Fig. 8.

Plots (a)–(e) show empirical histograms of v_k , $k=1, \dots, 5$, obtained from 5000 channelized signal-absent lumpy-background images. For these plots, $\bar{N} = 12$, $a = 5$, and $N_c = 5$.

Fig. 9.

(Color online) Performance of the ideal observer and the CIO using 5 to 40 LG channels with the MCMC algorithm. The lumpy images with $\bar{N}=5$ were used. The channel width parameters are 5 and 10 for (a) and 16 and 23 for (b). The dotted curve represents ± 2 standard errors for ideal-observer performance. All error bars are ± 2 standard errors by the one-shot method.

Fig. 10.
(Color online) As Fig. 9, but for $\bar{N} = 12$.

Fig. 11.

(Color online) As Fig. 9, but for the ideal observer and the CHO.

Fig. 12.
(Color online) As Fig. 11, but for $\bar{N} = 12$.

Fig. 13.

(Color online) Comparison of the MCMC algorithm and the approach with the method of G (0) approximation to the AUC and the IFFT algorithm for estimating the performance of the CIO using five LG channels. The mean number $\bar{N}$ of lumps is 5 and 12, respectively, for (a) and (b). Values for the width parameter a_u of LG channels are 5, 10, 16, and 23. Error bars for MCMC estimates are ± 2 standard errors by the one-shot method. For the approach with the G (0) approximation, errors are approximation errors.

Table 1

Object and System Parameters

Lumpy object	$a_b=1.0$	$\sigma_b=7$	$N=5,12$
Gaussian signal	$a_s=0.2$	$\sigma_s=3$	—
Gaussian noise	$\mu=0.0$	$\sigma_n=20$	—
Imaging system	$h=40$	$w=0.5$	—

Table 2Performance of the Ideal Observer, the CIO, and the CHO^a

AUC_{ideal}	a_u	$AUC_{CIO:N_c}$	$AUC_{CHO:N_c}$
$N=5$		AUC ± 2 STD	
0.91 $\pm$ 0.02	5	0.87 $\pm$ 0.03:40	0.81 $\pm$ 0.02:40
	10	0.88 $\pm$ 0.03:40	0.81 $\pm$ 0.02:40
	16	0.88 $\pm$ 0.03:40	0.81 $\pm$ 0.02:40
	23	0.88 $\pm$ 0.03:20	0.81 $\pm$ 0.02:40
$N=12$		AUC ± 2 STD	
0.87 $\pm$ 0.02	5	0.82 $\pm$ 0.03:40	0.78 $\pm$ 0.02:40
	10	0.83 $\pm$ 0.03:40	0.78 $\pm$ 0.02:40
	16	0.83 $\pm$ 0.03:20	0.79 $\pm$ 0.02:40
	23	0.83 $\pm$ 0.03:20	0.78 $\pm$ 0.02:40

^aThe MCMC algorithm was used to determine the performance of the ideal observer and the CIO. The number of LG channels is represented by N_c . Errors are ± 2 standard errors by the one-shot method.

Table 3

Comparison of the Markov-Chain Monte Carlo Algorithm and the Approach Using the Method of $G(0)$ Approximation to the AUC and the IFFT Algorithm to Determine the Performance of the CIO Using Five LG Channels^a

a_u	Markov-Chain Monte Carlo	Alternative with $G(0)$
<hr/>		
$N=5$	AUC ± 2 STD	
5	0.80 $\pm$ 0.03	0.82 $\pm$ 0.006
10	0.84 $\pm$ 0.03	0.85 $\pm$ 0.004
16	0.86 $\pm$ 0.03	0.85 $\pm$ 0.003
23	0.85 $\pm$ 0.03	0.83 $\pm$ 0.002
<hr/>		
$N=12$	AUC ± 2 STD	
5	0.75 $\pm$ 0.04	0.74 $\pm$ 0.02
10	0.79 $\pm$ 0.03	0.78 $\pm$ 0.01
16	0.82 $\pm$ 0.04	0.79 $\pm$ 0.01
23	0.75 $\pm$ 0.04	0.74 $\pm$ 0.01

^a Error bars for both methods are ± 2 standard errors, and the one-shot method was used for the Markov-chain Monte Carlo algorithm. For the approach with the $G(0)$ approximation, errors are approximation errors.

Table 4Efficiencies of the CIO and the CHO Relative to the Ideal Observer^a

Efficiency	$N=5$	$N=12$
CIO - LG	77%	72%
CHO - LG	43%	51%

^aEfficiency of a model observer is calculated as $[\text{SNRAUC}(\text{model obs}) / \text{SNRAUC}(\text{ideal obs})]^2$, where $\text{SNRAUC} = 2 \text{erf}^{-1}(2\text{AUC}-1)$. The notations CIO-LG and CHO-LG represent a channelized-ideal observer with LG channels of $a_H=16$ and a channelized-Hotelling observer with LG channels of $a_H=16$, respectively.