A Numerical Study of the Hydrodynamic Stable Concentration Boundary Layers in a Membrane System Under Microgravitational Conditions

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Abstract On the basis of the classic formula of the concentration Rayleigh number and the Kedem–Katchalsky equation for diffusive membrane transport, we derived the equations of sixteenth order which show the dependence of the thicknesses of the concentration boundary layers on the difference of the solution concentrations, the concentration Rayleigh number, the solute permeability coefficient of the membrane and the diffusion coefficients in the solution, the kinematic viscosity of the solution, the density of solutions, the temperature and gravitational acceleration. The obtained equation has numerical solutions in the first, third and fourth quadrant of a co-ordinate system. However, only two solutions from the first quadrant of the co-ordinate system have physical meaning. Confining ourselves to the set of solutions with physical meaning only, the thicknesses of concentration boundary layers for different parameters occurring in the obtained equation were calculated numerically.

Keywords Membrane transport · Concentration boundary layers · Concentration Rayleigh number

1 Introduction

In most cases, spontaneously occurring transport processes in the external field such as the electric or gravitational field, especially in electro-chemical systems, lead to creation of local and/or global non-homogeneities of various type that modify the membrane transport [1–3]. One of the reasons for forming this non-homogeneity is the concentration

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polarization leading to the temporal–spatial evolution of stimuli and thermodynamic flows [4–6]. Concentration polarization is a well-known phenomenon arising at the interface between an ion-exchange membrane and electrolyte solution when an electric current passes through the system [4]. In its prominent form, concentration polarization is observed by the fact that the transmembrane flux does not increase with increasing driving force and reaches a limiting flux. Chronopotentiometry is an electrochemical characterization method that measures the electric potential response of the system to an imposed current. This method has been used to study diffusion transport in diffusion-controlled boundary layers [7], or to explain overlimiting current behaviour [8, 9]. The theoretical modelling of the concentration polarization phenomenon is based on the Nernst–Planck [4, 10], Poisson [4, 10] and Kedem–Katchalsky [1, 11, 12] equations.

In the case of transport through a neutral membrane, the concentration polarization consists in formation of concentration boundary layers (l_h and l_l) with thicknesses, δ_h and δ_l , respectively, at both sides of the membrane (M) [1, 2, 12, 13]. The layers l_h and l_l can be treated as a liquid membrane with suitable transport properties which can be described by permeability coefficients [14, 15]. One of the effects of the temporal–spatial evolution of the concentration boundary layers is the evolution of the concentration field, which causes solutions concentrations at membrane–solution interfaces in the stationary state (C_i , C_e), having been crucially different from initial concentrations (C_h , C_l), fulfil conditions $C_i < C_h$ and $C_e > C_l$ [3, 15, 16]. Usually the width of the concentration boundary layers (CBLs) was estimated by flux measurements [17, 18]. Alternatively, microelectrodes placed near the membrane surface were used to determine thickness from the time-course of near membrane concentration changes [19, 20]. Concentration profiles were registered with optical methods as well [13, 21, 22]. Thus, these layers, by reducing the concentration gradient across the membrane, limit the osmotic as well as diffusive flows. The kinetics of formation and evolution of these flows, on the phenomenological level, is controlled by the concentration Rayleigh number [6, 23]:

$$R_c = g\alpha_c\beta_c d^4 (D\nu)^{-1} \quad (1)$$

where g is the gravitational acceleration, d the fluid depth along the gravitational (z) direction, $\alpha_c = (\partial\rho/\partial C)/\rho$ the variation of density with concentration, $\beta_c = \partial C/\partial z$ the concentration gradient, D the solute diffusion coefficient and ν kinematic viscosity.

Concentration boundary layers near membranes play the role of additional kinetic barriers in transport processes of rapidly permeating substances through natural and artificial membranes [2, 24–26]. The rate and effectiveness of chemical transformations within the CBLs are affected by the availability of the reactants. Boundary layers near the membrane are the source of an inaccurate Michaelis constant in membrane transport [27]. The size of CBLs seems to have regulatory functions. Variations in epithelial function or luminal stirring can for example readily influence the absorption of small molecules [28, 29]. In terrestrial conditions, the concentration boundary layers can be destroyed by natural convection and sedimentation [3, 16, 23]. Lack of these phenomena under microgravity conditions has generated an influence on nutrition, oxygen and by-product gradients, because in states of microgravity these elements are produced only by diffusion. In this connection, in conditions of microgravity in the cell–solution border, membrane potential changes could occur, causing alterations of solution concentration, hence leading to formation of stationary boundary layers around the cell [3].

The present paper presents considerations aiming to prove the influence of the concentration gradient and the concentration Rayleigh number on the thickness of concentration boundary

layers. In order to do this, using the classic definition of the concentration Rayleigh number [6, 23] and the second Kedem–Katchalsky equation [1] for diffusive membrane transport, we will present the equations of sixteenth order for the thicknesses of the concentration boundary layers (δ_h or δ_l). On the basis of these equations, δ_h can be calculated for different values of the gravitational acceleration (g), concentration Rayleigh number (R_{Ch}), solute permeability of the membrane (ω_m) and difference of the solution concentration (C_h).

2 Theory

Let us consider the single-membrane system presented in Fig. 1. This membrane system contains non-mechanically stirred solutions whose density decreases with concentration (aqueous ethanol or ammonia solution), separated by an isotropic and symmetric membrane (M). δ_l and δ_h denote the thicknesses of the concentration boundary layers l_l and l_h , which originate in this system. The layers l_l and l_h are characterised by the solute permeability coefficients ω_l and ω_h , respectively. J_l and J_h denote the solute fluxes through the layers, respectively, l_l and l_h . The membrane is characterised by the solute permeability coefficient ω_m , and J_m is the solute flux through the membrane. C_i and C_e ($C_i > C_e$) are the concentrations at interfaces l_h/M and M/l_l , respectively, while C_h and C_l ($C_h > C_l$) denote the concentrations of solutions outside the layers. In this configuration only diffusive transport occurs and this configuration is gravitationally stable [13, 23].

In order to calculate the thickness δ_l and δ_h of concentration boundary layers l_l and l_h for a stationary state, we will make the following considerations on the basis of (1). If we denote the thickness δ_l and δ_h with d , then using the relations $\alpha_{Cl}\beta_{Cl} = g(\rho_e - \rho_l)(\rho_l\delta_l)^{-1}$ and $\alpha_{Ch}\beta_{Ch} = g(\rho_h - \rho_l)(\rho_h\delta_h)^{-1}$, (1) can be written in the following forms [23]:

$$R_{Cl} = g(\rho_e - \rho_l)\delta_l^3(D_l\rho_l\nu_l)^{-1} \quad (2)$$

$$R_{Ch} = g(\rho_h - \rho_l)\delta_h^3(D_h\rho_h\nu_h)^{-1} \quad (3)$$

where D_l and D_h are the solute diffusion coefficients in layers l_l and l_h , ν_l and ν_h are the kinematic viscosity coefficients, ρ_e and ρ_l are the densities of solutions at the membrane/CBL interfaces, ρ_l and ρ_h are the solution densities.

Fig. 1 Sketch of the single-membrane system (see description in the text)

Differences $(\rho_e - \rho_l)$ and $(\rho_h - \rho_i)$ can be calculated by using quite simple analytical methods. Calculation of density differences $(\rho_e - \rho_l)$ and $(\rho_h - \rho_i)$ for a stationary state involves calculating the concentration differences $(C_e - C_l)$ and $(C_h - C_i)$ because

$$\rho_e - \rho_l = \frac{\partial \rho}{\partial C} (C_e - C_l) \quad (4)$$

$$\rho_h - \rho_i = \frac{\partial \rho}{\partial C} (C_h - C_i) \quad (5)$$

where $\partial \rho / \partial C$ is the change of density with the change of the concentration in the solution. In order to calculate these concentration differences, we will carry out the following considerations on the basis of the second Kedem–Katchalsky equation for diffusive membrane transport [1]. If the solute fluxes which are going through the layer l , the membrane M and the layer h , are designated, respectively, by J_l , J_m and J_h , then, in the stationary state $J_l = J_m = J_h$. The permeability coefficients of the layers l and h can be written as $\omega_l = D_l (RT\delta_l)^{-1}$ and $\omega_h = D_h (RT\delta_h)^{-1}$ [9]. In these equations, D_l and D_h are diffusion coefficients of the substance in layers l and h . Then, we will obtain:

$$D_l(\delta_l)^{-1}(C_e - C_l) = \omega_m RT(C_i - C_e) = D_h(\delta_h)^{-1}(C_h - C_i). \quad (6)$$

Making suitable transformations of the above equations, we obtain:

$$C_e - C_l = \omega_m RT D_h \delta_l [\omega_m RT (D_l \delta_h + D_h \delta_l) + D_h D_l]^{-1} \quad (7)$$

$$C_h - C_i = \omega_m RT D_l \delta_h [\omega_m RT (D_l \delta_h + D_h \delta_l) + D_h D_l]^{-1}. \quad (8)$$

Including formulas (9) and (10) in (4) and (5), and then in (2) and (3), leads to equations describing the thicknesses δ_l and δ_h of layers l and h , respectively, for isothermal membrane transport:

$$\alpha_1 \delta_l^4 - \beta_1 \delta_l - \chi_1 \delta_h - \gamma_1 = 0 \quad (9)$$

$$\alpha_2 \delta_h^4 - \beta_2 \delta_h - \chi_2 \delta_l - \gamma_2 = 0 \quad (10)$$

where $\alpha_1 = \omega_m g RT D_h \left(\frac{\partial \rho}{\partial C} \right) (C_h - C_l)$, $\beta_1 = \omega_m R TR_{Cl} \rho_l \nu_l D_l D_h$, $\chi_1 = \omega_m R TR_{Cl} \rho_l \nu_l D_l^2$, $\gamma_1 = R_{Cl} \rho_l \nu_l D_l^2 D_h$, $\alpha_2 = \omega_m g RT D_l \left(\frac{\partial \rho}{\partial C} \right) (C_h - C_l)$, $\beta_2 = \omega_m R TR_{Ch} \rho_h \nu_h D_l D_h$, $\chi_2 = \omega_m R TR_{Ch} \rho_h \nu_h D_h^2$, $\gamma_2 = R_{Ch} \rho_h \nu_h D_h^2 D_l$.

Solving the systems of (9) and (10), respectively, for δ_l and δ_h , we will obtain:

$$\alpha_2 \left[\chi_1^{-1} (\alpha_1 \delta_l^4 - \beta_1 \delta_l - \gamma_1) \right]^4 - \chi_1^{-1} \beta_2 (\alpha_1 \delta_l^4 - \beta_1 \delta_l - \gamma_1) - \chi_2 \delta_l - \gamma_2 = 0 \quad (11)$$

$$\alpha_1 \left[\chi_2^{-1} (\alpha_2 \delta_h^4 - \beta_2 \delta_h - \gamma_2) \right]^4 - \chi_2^{-1} \beta_1 (\alpha_2 \delta_h^4 - \beta_2 \delta_h - \gamma_2) - \chi_1 \delta_h - \gamma_1 = 0. \quad (12)$$

If we assume that $R_{Cl} = R_{Ch} = R_C$, $D_l = D_h = D$, $\rho_l = \rho_h = \rho$ and $\nu_l = \nu_h = \nu$, we obtain

$$\alpha \delta^4 - 2\beta \delta - \gamma = 0 \quad (13)$$

where $\alpha = \omega_m g RT \left(\frac{\partial \rho}{\partial C} \right) (C_h - C_l)$, $\beta = \chi = \omega_m R TR_C \rho \nu D$, $\gamma = \vartheta = \rho \nu R_C D^2$ and $\delta_l = \delta_h = \delta$.

These equations allow us to calculate numerically the thicknesses δ_l and δ_h of the concentration boundary layers as parameters g , C_h , R_C and ω_m are varied, respectively, with other parameters fixed.

3 Numerical Solutions of Equations (11), (12) and (13)

Numerical solutions of (11), (12) and (13) were performed for the system presented in Fig. 1, in which an artificial membrane mounted horizontally separates two aqueous ethanol solutions with concentrations $C_l=0 \text{ mol m}^{-3}$ (pure water) and $C_h=n\Delta C$, where $\Delta C=31.25 \text{ mol m}^{-3}$, $n=1, 2, \dots, 24$. In such configuration of the membrane system, stable concentration boundary layers (CBLs) are formed. For the numerical calculations, the following dependences of the density and kinematic viscosity on the ethanol solution concentration were used: $\rho_h=(998.2-0.009C_h) \text{ kg m}^{-3}$ and $\nu_h=(1,006.80+0.26C_h)\times 10^{-9} \text{ m}^2\text{s}^{-1}$ [15]. The density variation with the solute concentration, calculated on the basis of the data, is equal to $(\partial\rho/\partial C)=0.009 \text{ kg(mol)}^{-1}$. Due to the fact that the diffusion coefficient of the ethanol in the aqueous ethanol solution within the tested concentrations slightly depends on the concentration, the constant value $D_l=D_h=D=1.074\times 10^{-9} \text{ m}^2\text{s}^{-1}$ [15] was assumed for calculations. For the calculations the flat neutral membrane was used. The transport properties of this membrane were characterised by the solute permeability coefficient (ω_m). For calculations, values of this coefficient ranging from $\omega_m=10^{-12} \text{ mol N}^{-1}\text{s}^{-1}$ to $\omega_m=10^{-9} \text{ mol N}^{-1}\text{s}^{-1}$ were used. All calculations were made for isothermal conditions at the temperature $T=295 \text{ K}$. The thickness δ_h for the membrane system presented in Fig. 1 was calculated by *Mathematica* computer programs. The results of calculations of the thicknesses δ_l and δ_h as functions of parameters g , C_h , R_C and ω_m are presented in Figs. 2, 3 and 4.

In the studied range of physicochemical parameters of solutions, the membrane, and the gravitational conditions, (11) and (12) have six real numerical solutions. Two of them are in the first quadrant of the co-ordinate axis and only they have physical meaning. Therefore, we will confine ourselves to study these solutions only. Omitting superscripts l and h in (11) and (12) gives (13), that is it reduces the order of (11) and (12) from the sixteen to four, and reduces the number of solutions from the first quadrant of the co-ordinate axis from two to one. It should be pointed out that the curves of fourth order, from the first quadrant of the co-ordinate axis, illustrating the solutions of (13), have the same shape as the curves of sixteenth range, from the first quadrant of the co-ordinate axis, illustrating the first solution of (12). It should be pointed out that the curves calculated on the basis of (11) are exactly the same as the curves calculated on the basis of (12). Therefore, we will study the curves calculated on the basis of (12).

4 Results and Discussion

Figure 2 shows $\delta_h = f(g)_{C_h, R_C, \omega_m}$ calculated on the basis of (12) and (13) for $C_h=250 \text{ mol m}^{-3}$. The solid curves were calculated on the basis of (12), and the dotted curves on the basis of (13). The solid curves 1 and 1', and the dotted curve 1 were obtained for $\omega_m=1.47\times 10^{-8} \text{ mol N}^{-1}\text{s}^{-1}$ and $R_{Ch}=5\times 10^2$. The solid curves 2, and 2', and the dotted curve 2, were obtained for $\omega_m=1.4\times 10^{-9} \text{ mol N}^{-1}\text{s}^{-1}$ and $R_{Ch}=10^4$. The solid curves 3, and 3', and the dotted curve 3, were obtained for $\omega_m=1.47\times 10^{-9} \text{ mol N}^{-1}\text{s}^{-1}$ and $R_{Ch}=5\times 10^4$. The dotted curves numbered 1, 2 and 3 have a similar shape as the corresponding solid curves.

Fig. 2 The figure shows $\delta_h = f(g)_{C_h, \omega_m, R_{Ch}}$ calculated on the basis of (12) for $C_h = 250 \text{ mol m}^{-3}$, $R_{Ch} = 5 \times 10^4$, $\omega_m = 1.47 \times 10^{-10} \text{ mol N}^{-1} \text{ s}^{-1}$ (curves 1, 1'); for $C_h = 250 \text{ mol m}^{-3}$, $R_{Ch} = 10^4$, $\omega_m = 1.47 \times 10^{-9} \text{ mol N}^{-1} \text{ s}^{-1}$ (curves 2, 2'); for $C_h = 250 \text{ mol m}^{-3}$, $R_{Ch} = 500$, $\omega_m = 1.47 \times 10^{-8} \text{ mol N}^{-1} \text{ s}^{-1}$ (3, 3'). Dotted curves 1, 2 and 3 were obtained on the basis of (13)

The shapes of the curves presented in the Fig. 2 illustrate that, the linear increase of the value of g causes a nonlinear decrease of the value of δ_h . It means that the decrease of the value of δ_h differs in different ranges of the value g . In the case of curve 1, the drop is highest for $g < 10^{-2} g_z$ (where g_z is the gravitational (terrestrial) acceleration), and, in the case of curve 3, for $g < 10^{-1} g_z$. It should be pointed out that when g approaches zero, then δ_h approaches $+\infty$, and when g approaches $+\infty$, δ_h approaches zero.

The decreased gravitation favours the formation of stable boundary layers with large thickness. This has been confirmed by a great number of experiments made in the state of weightless and artificial microgravitation [3, 29, 30]. Microgravitation conditions favour the formation of boundary layers around the cells in the form of stationary small membranes. In these cells, the uptake rate of oxygen and nutrients is changed, and therefore the metabolism is significantly affected. The gravitational effects at the cellular level cause changes in the molecular organisation, genetics, growth, cell division and differentiation, and morphological characteristics of single cells and the whole organism (the changes in a human body during long space flights are particularly important) [3].

Figure 3 shows $\delta_h = f(\omega_m)_{C_h, R_{Ch}, g}$ calculated on the basis of (12) and (13) for $C_h = 250 \text{ mol m}^{-3}$. The solid curves illustrate the results of calculations made on the basis of (12), and the dotted curves on the basis of (13). The solid curves 1 and 1', and the dotted curve 1, were obtained for $g = 10^{-3} g_z$ and $R_{Ch} = 10^4$. The solid curves 2 and 2', and the dotted curve 2, were obtained for $g = 10^{-3} g_z$ and $R_{Ch} = 10^3$. The solid curves 3 and 3', and the dotted curve 3, were obtained for $g = 10^{-1} g_z$ and $R_{Ch} = 10^4$. The dotted curves numbered 1, 2 and 3 have similar shapes as the corresponding solid curves. From the shape of the curves presented in Fig. 3 we see that the linear increase of the value of ω_m causes a nonlinear decrease of the value of δ_h . Thus the decrease of the value of δ_h differs in different ranges of the value ω_m . In the case of the curve 1 the drop is the highest for $\omega_m < 0.16 \times 10^{-10} \text{ mol N}^{-1} \text{ s}^{-1}$, and, in the case of the curve 3, for $\omega_m < 0.4 \times 10^{-10} \text{ mol N}^{-1} \text{ s}^{-1}$. It should be pointed out that when ω_m approaches zero, then δ_h approaches $+\infty$, and, when ω_m approaches $+\infty$, δ_h approaches zero.

Figure 4 shows $\delta_h = f(C_h)_{\omega_m, R_{Ch}, g}$ calculated on the basis of (12) and (13) for $g = 10^{-3} g_z$. The solid curves illustrate the results of calculations made on the basis of (12), and the dotted

Fig. 3 The figure shows $\delta_h = f(\omega_m)_{C_h, R_{Ch}, g}$ calculated on the basis of (12). Solid curves 1 and 1' were obtained for $C_h = 250 \text{ mol m}^{-3}$, $R_{Ch} = 10^4$ and $g = 10^{-3} \text{ g}_z$ (1 and 1'), solid curves 2 and 2' were obtained for $C_h = 250 \text{ mol m}^{-3}$, $R_{Ch} = 1,000$ and $g = 10^{-3} \text{ g}_z$, and curves 3 and 3' for $C_h = 250 \text{ mol m}^{-3}$, $R_{Ch} = 10^4$ and $g = 10^{-1} \text{ g}_z$. Dotted curves were obtained on the basis of (13)

curves show results based on (13). The solid curves 1 and 1', and the dotted curve 1, were obtained for $\omega_m = 1.47 \times 10^{-8} \text{ mol N}^{-1} \text{ s}^{-1}$ and $R_{Ch} = 10^3$. The solid curves 2 and 2', and the dotted curve 2, were obtained for $\omega_m = 1.47 \times 10^{-11} \text{ mol N}^{-1} \text{ s}^{-1}$ and $R_{Ch} = 10^4$. The solid curves 3 and 3', and the dotted curve 3, were obtained for $\omega_m = 1.47 \times 10^{-11} \text{ mol N}^{-1} \text{ s}^{-1}$ and $R_{Ch} = 2 \times 10^5$. The dotted curves numbered 1, 2 and 3 have similar shapes as the corresponding solid curves. The shape of the curves presented in the Fig. 4 shows that the linear increase of the value of C_h causes the nonlinear decrease of the value of δ_h . Thus the decrease of the value of δ_h differs in different ranges of the value of C_h . In the case of the curve 1, the decrease is sharpest for $C_h < 8 \text{ mol m}^{-3}$, and, in the case of the curve 3, for $\omega_m < 30 \text{ mol m}^{-3}$. It should be pointed out that when C_h approaches zero, then δ_h approaches $+\infty$, and, when C_h approaches $+\infty$, δ_h approaches zero.

Figure 5 shows $\delta_h = f(R_C)_{C_h, \omega_m, g}$ calculated on the basis of (12) and (13) for $C_h = 250 \text{ mol m}^{-3}$. The solid curves illustrate the results of calculations made on the basis of (12), and the dotted curves show results obtained using (13). The solid curves 1 and 1', and the dotted

Fig. 4 The figure shows $\delta_h = f(C_h)_{g, \omega_m, R_C}$ calculated on the basis of (12). Solid curves 1 and 1' were obtained for $R_{Ch} = 2 \times 10^5$, $g = 10^{-3} \text{ g}_z$ and $\omega_m = 1.47 \times 10^{-11} \text{ mol N}^{-1} \text{ s}^{-1}$. Curves 2 and 2' were obtained for $R_{Ch} = 10^4$, $g = 10^{-3} \text{ g}_z$ and $\omega_m = 1.47 \times 10^{-11} \text{ mol N}^{-1} \text{ s}^{-1}$, and curves 3 and 3' for $R_{Ch} = 10^3$, $g = 10^{-3} \text{ g}_z$ and $\omega_m = 1.47 \times 10^{-8} \text{ mol N}^{-1} \text{ s}^{-1}$. Dotted curves 1, 2 and 3 were obtained on the basis of (13)

curve 1, were obtained for $g=10^{-3}g_z$ and $\omega_m=1.47\times 10^{-10}\text{ mol N}^{-1}\text{ s}^{-1}$. The solid curves 2 and 2', and the dotted curve 2, were obtained for $g=10^{-3}g_z$ and $\omega_m=1.47\times 10^{-11}\text{ mol N}^{-1}\text{ s}^{-1}$. The solid curves 3 and 3', and the dotted curve 3, were obtained for $g=10^{-2}g_z$ and $\omega_m=1.47\times 10^{-9}\text{ mol N}^{-1}\text{ s}^{-1}$. The dotted curves numbered 1, 2 and 3 have similar shapes as the corresponding solid curves. From the shape of the curves presented in the Fig. 5 we see that the linear increase of the value of R_{Ch} causes a nonlinear increase of the value of δ_h . This means that the increase of the value of δ_h differs in different ranges of the value R_{Ch} . It should be pointed out that when R_{Ch} approaches zero, then δ_h also approaches zero, and, when R_{Ch} approaches $+\infty$, then δ_h approaches asymptotically the maximal value.

The results shown in Figs. 2, 3, 4 and 5 show that under the non-convective conditions the thickness of concentration boundary layers strongly depends on the gravitational acceleration, the membrane permeability coefficient and the concentration Rayleigh number. Such situations can occur in biological systems under microgravitational conditions, such as during long space flights. Under microgravitational conditions, membrane transport as well as transport in areas near the membrane demonstrate a diffusive character. Elimination of convection and sedimentation limits, among other effects, such as changes in the amount of nutrient and oxygen uptake into the cell interior leading to disorders of cell metabolism [3, 29].

Appendix: Symbols

C_h, C_l	Higher (superscript h) and lower (superscript l) concentrations of the bulk solution $\text{mol}\cdot\text{m}^{-3}$
D_l, D_h	Coefficients of the solute diffusion (D) in the layers l_l and l_h , m^2s^{-1}
g	The gravitational acceleration, $\text{m}\cdot\text{s}^{-2}$
g_z	The gravitational (terrestrial) acceleration, $\text{m}\cdot\text{s}^{-2}$
RT	Product of the gas constant and thermodynamic temperature, $\text{J}\cdot(\text{mol})^{-1}$
R_{Cl}, R_{Ch}	Concentration Rayleigh number (R_C) for layers l_l and l_h
δ_l, δ_h	Thickness of the layers l_l and l_h , respectively, m
$\partial\rho/\partial C$	Variation of the solution density with concentration, $\text{kg}\cdot\text{mol}^{-1}$

Fig. 5 The figure shows $\delta_h = f(R_C)_{C_h, g, \omega_m}$ calculated on the basis of (12). Solid curves 1 and 1' were obtained for $C_h=125\text{ mol m}^{-3}$, $g=10^{-3}g_z$ and $\omega_m=1.47\times 10^{-10}\text{ mol N}^{-1}\text{ s}^{-1}$. Curves 2 and 2' were obtained for $C_h=250\text{ mol m}^{-3}$, $g=10^{-2}g_z$ and $\omega_m=1.47\times 10^{-9}\text{ mol N}^{-1}\text{ s}^{-1}$, and curves 3 and 3' for $C_h=500\text{ mol m}^{-3}$, $g=10^{-3}g_z$ and $\omega_m=1.47\times 10^{-11}\text{ mol N}^{-1}\text{ s}^{-1}$. Dotted curves 1, 2 and 3 were obtained on the basis of (13)

ρ_h, ρ_l	higher (superscript h) and lower (superscript l) density of solutions, $\text{kg}\cdot\text{m}^{-3}$
ν_h, ν_l	higher (superscript h) and lower (superscript l) kinematic viscosity coefficients, m^2s^{-1}
ω_m	the solute permeability coefficient of the membrane, $\text{mol N}^{-1}\text{s}^{-1}$

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