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Sliding Window SENSE Calibration for Reducing Noise in fMRI

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Abstract

Functional magnetic resonance imaging (fMRI) at high magnetic field with parallel imaging (PI) has become increasingly popular for high resolution imaging. We present a method of self-calibrated PI-fMRI in which sensitivity profiles are calculated using a sliding window of fully-sampled multi-shot imaging data. We show that by updating these sensitivity profiles in a sliding fashion, thermal noise is reduced in the reconstructed image time series. This is accomplished by retaining thermal noise in the sensitivity profiles; no spatial smoothing is performed. These noisy profiles actually provide a closer match to those required for thermal noise-free reconstruction than conventional sensitivity map generation. Our proposed technique is especially applicable in acquiring high spatial resolution images, where thermal noise exceeds physiological noise. With conventional sensitivity calculation, PI-fMRI sensitivity is preserved only when using a voxel size large enough such that physiological noise predominates. With small voxel size, our technique reveals activation from visual stimulation where conventional sensitivity calculation techniques falter. Our technique enhances fMRI detection especially when higher spatial resolution is desired.

Keywords

Functional magnetic resonance imaging; CG-SENSE; noise; parallel imaging

Introduction

As the magnetic field is increased, in principle, blood-oxygen-level-dependence (BOLD) imaging can have improved spatial specificity in functional magnetic resonance imaging (fMRI) because more signal originates from tissue and small capillaries rather than from large veins. (1) The fundamental tradeoffs, between spatial and temporal resolution, present many challenges for high resolution and high field fMRI. Higher spatial resolution requires longer readout duration and postponed echo time (TE) that respectively result in increased sensitivity to off-resonance and non-optimum BOLD contrast. Readout duration and TE can be shortened by multi-shot acquisition, but temporal resolution will consequently decrease. Despite reduction of signal-to-noise ratio (SNR), parallel imaging (PI) methods in fMRI are known to be advantageous for reducing signal loss in susceptibility-induced signal dropout regions of the brain. (2) Parallel imaging allows increase in spatial and temporal resolution with shorter TE and is particularly advantageous at higher-field.

Weiger *et al.* first demonstrated use of a spiral sequence together with parallel imaging. (3) They applied parallel imaging to reduce readout duration in the spiral trajectory, which

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lessens signal dropout in frontal regions. Consequently, they successfully detected activation in a tasting task stimulating the orbito-frontal cortex. To calibrate coil sensitivity, required by the sensitivity encoding (SENSE) reconstruction algorithm, they gathered an extra fully-sampled image prior to their fMRI experiment. We refer to their technique for calculating sensitivity maps as the *reference frame* method.

An alternative method for deriving coil sensitivity, from a time-series without a separate image acquisition, was presented by Kellman *et al.* (4) There, the UNFOLD method, “Unaliasing by Fourier-Encoding the Overlaps Using the Temporal Dimension,” together with SENSE reconstruction was employed to remove spatial and temporal aliasing. This Adaptive Sensitivity Encoding Incorporating Temporal Filtering method (TSENSE) was demonstrated for real-time nonbreath-held cardiac imaging with an *acceleration factor* $R = 2$. (Acceleration is a measure of undersampling in k -space.) They acquire odd and even k -space lines in sequential images. Sensitivity profiles are calculated by temporally filtering the aliased images of the same undersampled data. Kellman shows that, with TSENSE, sensitivity profiles can be calculated adaptively to track signal changes due to breathing or motion. In order to obtain accurate sensitivity profiles, images used to generate them must be alias-free. The amount of residual artifact after temporal filtering is related to a subject’s dynamic nature, number of temporal frames used for filtering, and filter bandwidth. Depending upon the application and subjects (*e.g.*, task design in fMRI and children *vs.* adults), filtering parameters may require monitoring and adjustment.

Preibisch *et al* (5) used an EPI-SENSE sequence to investigate SNR *vs.* fMRI sensitivity at two spatial resolutions. At low spatial resolution, parallel imaging was used to reduce readout duration; surprisingly, SNR and fMRI sensitivity were hardly reduced. At high spatial resolution, parallel imaging was used to increase spatial resolution. Physiological noise was reduced and fMRI sensitivity was similar to that of the regular low-resolution single-shot acquisition. They concluded that parallel imaging has negligible effect on fMRI sensitivity despite decrease in SNR.

The lack of reduction in fMRI sensitivity with decreasing SNR in parallel imaging has been observed and explained by several authors. (5,6) fMRI sensitivity is inversely related to temporal signal instability originating from physiological and thermal noise. Parallel imaging increases thermal noise but not physiological noise. Thus, when physiological noise dominates, fMRI sensitivity is hardly affected. Therefore, as long as physiological noise is much greater than thermal noise, PI fMRI sensitivity is comparable to non-PI techniques. Nevertheless, a major incentive to use parallel imaging at high field is high spatial resolution; *i.e.*, small voxel size. As voxel size decreases, thermal noise dominates. At present, the limiting factor in achieving high spatial resolution with parallel imaging (at high field and with sufficient fMRI sensitivity) is the fact that thermal noise leads to temporal signal instability, thereby degrading fMRI sensitivity.

Herein we present a PI technique for improving fMRI sensitivity at high spatial resolution when thermal noise predominates. We propose measurement of coil sensitivity by combining fully sampled data from a number of adjacent frames in an interleaved acquisition. In a two-shot interleaved acquisition, for example, k -space is fully sampled at every two repetition times (TR). Sensitivity profiles are then calculated from these full-resolution alias-free images and updated synchronously with image acquisition. This technique provides the convenience of self-calibrating sensitivity maps. In addition, we demonstrate that the technique provides inherent noise filtering and significantly enhances fMRI activation measurement. Improvement is especially significant at higher resolution and thinner slices where thermal noise exceeds physiological noise. For example, the technique detects activation at high spatial-resolutions (*e.g.* slice thickness of 2mm) better

than previous methods. The technique results in higher temporal resolution and slightly more or comparable activated volumes compared to a standard two-shot reconstruction in fMRI experiments.

The technique may be incorporated into any interleaved acquisition. We demonstrate an implementation with two-shot interleaved spiral-in/out trajectory using an acceleration factor $R = 2$. The spiral-in/out trajectory is known to be effective in reducing signal drop-out from susceptibility-induced field gradients. (7) We compare the proposed *sliding window* PI technique with conventional (non-accelerated) two-shot reconstruction, as well as with the conventional PI *reference* frame method and an *all-frame* method. In the *all-frame* method, data acquired over the whole time-series is averaged together to calculate coil sensitivity. In all three comparisons, the proposed technique demonstrates enhanced BOLD activation. The improvement is most significant for high resolution, thin slice, and low SNR cases.

Sliding Window method

In the sliding window method, several adjacent two-shot images are selected for calculation of sensitivity profiles. These profiles are then applied exclusively to reconstruct data under that window. The minimum sliding window width that will provide fully-sampled data, with which to calculate sensitivity maps, is the acceleration factor R ; which is taken to be 2 here for all demonstrations and further discussion. The technique eliminates the temporal filtering step required by TSENSE. Since the sensitivity profiles are calculated from two-shot images, they are fully sampled, unaliased, and free of other post-processing artifacts. Moreover, these two-shot images are reconstructed from functional data itself; so no extra scans are needed. We shall show that our proposed coil sensitivity calculations reduce image noise when compared with *all-frame* or *reference frame* methods both in theory and *in vivo*, especially when thermal noise equals or exceeds physiological noise.

The original TSENSE diagram from Kellman (4) and a diagram of our proposed *sliding window* technique are shown in Figure 1 for comparison. The main difference between our technique and TSENSE is that TSENSE calculates its sensitivity maps by temporally lowpass filtering time frames of undersampled k-space data. Those time frames have alternating k-space undersampling patterns so that aliasing is modulated at the Nyquist frequency and then removed via lowpass filtering. The minimum number of time frames required for lowpass filtering is the number of alternating k-space sampling patterns (same as reduction factor R). In practice, the number of time frames used for lowpass filtering is generally greater than R for better separation of desired and aliasing frequency components. In our *sliding window* technique, sensitivity maps are calculated from multi-shot reconstructed images. The number of time frames used in our multi-shot reconstruction is equal to the number of alternating k-space sampling patterns (exactly R), so that the calibration maps are fully sampled, thus distortion free. The impact of averaging more time frames will be discussed later where we will find that reduction in thermal noise is best when R frames are used.

Effect of noise

We express the acquired k-space data as

$$\mathbf{d} = \mathbf{E}\mathbf{S}(\mathbf{m} + \mathbf{p}) + \boldsymbol{\varepsilon} \quad [1]$$

where $\mathbf{m}$ is the object image vector, $\mathbf{p}$ is a physiological noise vector, $\boldsymbol{\varepsilon}$ is a vector of thermal noise, $\mathbf{S}$ is a matrix of coil sensitivities, and $\mathbf{E}$ is the Fourier kernel matrix. A reconstructed image with estimated sensitivity maps $\hat{\mathbf{S}}$ is expressed

$$\hat{\mathbf{m}} = (\mathbf{E}\mathbf{S})^\dagger \mathbf{d}, \text{ where } \dagger \text{ represents pseudoinverse} \quad [2]$$

We show, in Appendix A, how an image reconstructed under the *sliding window* method can be expressed

$$\hat{\mathbf{m}} \approx \mathbf{Q}^{-1} (\mathbf{m} + \mathbf{p} + (\mathbf{E}\mathbf{S})^\dagger (\boldsymbol{\varepsilon} - \bar{\boldsymbol{\varepsilon}})) \quad [3]$$

where $\bar{\boldsymbol{\varepsilon}}$ is a vector of thermal noise averaged over a number of consecutive temporal frames, and $\mathbf{Q}^{-1}$ is a normalization matrix to the estimate of coil sensitivity.

When $\boldsymbol{\varepsilon} = \bar{\boldsymbol{\varepsilon}}$, the image estimate $\hat{\mathbf{m}}$ is free of thermal noise. But that would occur only when $R=1$, which is not of interest since no readout acceleration is provided. When sliding window width is minimal ($R=2$), then $\bar{\boldsymbol{\varepsilon}}$ is the closest approximation to thermal noise $\boldsymbol{\varepsilon}$ achievable and so $\hat{\mathbf{m}}$ will contain the least amount of noise. As the sliding window width becomes wider, $\bar{\boldsymbol{\varepsilon}}$ resembles $\boldsymbol{\varepsilon}$ to a lesser degree which results in $\hat{\mathbf{m}}$ having more thermal noise. When the *all-frame* method is used, on the other hand, $\bar{\boldsymbol{\varepsilon}}$ is averaged to zero under a Gaussian noise assumption; thermal noise sampled in $\mathbf{k}$ -space is therefore propagated to $\hat{\mathbf{m}}$. Smoothing sensitivity maps would also result in thermal noise propagation to $\hat{\mathbf{m}}$.

The difference of the noise terms in Eq.[3] leads to a reduction of noise because thermal noise in $\boldsymbol{\varepsilon}$ is correlated with that in $\bar{\boldsymbol{\varepsilon}}$; that is, the case of a *sliding window* method with a narrow window width. Otherwise, the subtraction would not necessarily lead to a noise reduction.

Data Acquisition

We demonstrate the technique with a two-shot spiral-in/out trajectory providing an acceleration factor of $R=2$. For all even time frames, the trajectory is rotated by 180 degrees so that $\mathbf{k}$ -space is fully sampled every two TRs. Experiments were performed with four slice thicknesses (2mm, 3mm, 4mm, and 5mm) in order to alter the relative contributions of thermal and physiological noise, and the number of slices gathered was 9, 7, 7, and 5 respectively to cover approximately the same volume of brain tissue. Slices were separated with a gap of 1mm. All experiments were performed on a GE 3T whole-body scanner (GE Signa, GE Healthcare, Waukesha, WI) equipped with a maximum gradient amplitude of 40 mT/m and a slew rate of 150 mT/m/ms. An 8-channel head coil (MRI Devices Corporation, Pewaukee, WI) was used for all image acquisitions. The TE was set to be minimal allowed by the spiral-in trajectory (35.4 ms), $\text{TR}/\alpha/\text{matrix size}/\text{FOV} = 2\text{s}/70^\circ/128 \times 128/20\text{cm}$. The scan time was 248 seconds (data from the first 8 seconds discarded). T2-weighted fast spin-echo (FSE) scans were obtained for anatomic reference ($\text{TR}/\text{TE}/\text{ETL} = 4000\text{ms}/68\text{ms}/12$). Six healthy subjects participated after giving informed consent in accordance with a protocol approved by the Stanford Institutional Review Board.

The fMRI task consisted of 6 cycles of an on/off block design having a period of 40s. During the on-block, subjects saw a checker board flashing at 8Hz. Subjects were told to stare at a fixation-cross to reduce eye movement during the off-block. To keep subjects' attention throughout scans, a red cross appeared at random times; twice for 0.5s during each on-block. Subjects were instructed to press a button as soon as they saw the cross. This experiment was performed once for each slice thickness; the order of the slice thickness scans was randomized among subjects.

Subjects 1–3 also participated in noise measurement scans in separate sessions. Noise measurement scans are similar to functional scans above except that the flip angle is set to 0 so that only thermal noise is gathered. Only 3 slices and 64 time frames were collected. Two scans were performed: flip-angle = 70° and 0°. A fast T1 mapping scan (8) was utilized to obtain images for later gray matter segmentation; one for each slice thickness. Subjects were instructed to relax while no stimulation or task was given.

Data Analysis

Sensitivity profiles are calculated using the functional data itself. No extra calibration scans are needed; this reduces inconsistency between a reference frame and the functional data to be reconstructed. Standard spiral trajectory gridded-reconstruction is used to make two-shot images from each coil's data. (9) For every two TRs, in other words, two fully-sampled two-shot images (one spiral-in and one spiral-out) are reconstructed for each coil. The sensitivity profile of each coil is the ratio of a fully sampled two-shot image from that coil to the square root of pixel-wise sum of squares of all coil images. Separate sensitivity profiles are generated for spiral-in and spiral-out data. For the *all-frame* method, all fully sampled two-shot images of each coil over the entire time series are first averaged for sensitivity profile calculation. The resulting sensitivity profiles are used for conjugate gradient (CG-)SENSE reconstruction (10,11) of the entire time series. The number of iterations in CG-SENSE reconstruction is 7 regardless of sensitivity map calculation method. For the *sliding window* method (window width = 2 TRs), a new fully sampled two-shot image from each coil is formed after every TR. The sensitivity profile for each coil is then updated at every TR and used for CG-SENSE image reconstruction at that TR. The window width (number of TRs included in the average for SENSE map calculation) is assumed to be 2 TRs for the *sliding window* method unless otherwise noted. Concomitant field effects correction and navigator correction are first performed on the raw data. (12) The same CG-SENSE program (written in MATLAB, Mathworks R13, Natick, MA) is used for reconstruction regardless of sensitivity profile calculation methods; it generates one spiral-in image and one spiral-out image for every TR.

Data gathered for noise measurement (flip angle 70° and 0° with no task performed) is reconstructed with *all-frame* and *sliding window* methods described above. A gray matter mask is segmented manually from a T1 map for each slice and each slice thickness. This mask is used to extract gray matter pixel time-series from data at flip-angle=70°. The temporal standard deviation of each gray matter pixel is recorded and then averaged over three slices; this gives the total noise σ . The thermal noise σ_0 is calculated by the same procedure using images with flip-angle = 0°, and the resulting standard deviation is multiplied by 1.527 to correct for the Rayleigh distribution of Gaussian noise in magnitude images. (13,14) Physiological noise σ_p is then calculated from (15)

$$\sigma = \sqrt{\sigma_p^2 + \sigma_0^2}. \quad [4]$$

The method for calculating t-score is adapted from Lee *et al.* (16) (also see 17–19). The steps involved are summarized here: each fMRI time series is first fitted to a second order polynomial for detrending. It is then correlated with sine and cosine functions at the fundamental task frequency (higher harmonics are negligible because the task blocks are short). The temporal phase of those voxels, with correlation coefficients exceeding a threshold of 0.2, is computed from the arctangent of sine and cosine correlations and averaged across each slice. The correlation is then projected onto this phase, per voxel, to provide a maximally in-phase correlation map. This model-free (Fourier) method eliminates bias from inaccuracy in assumed hemodynamic response function. Finally, a Fisher

transform converts the correlation coefficient to t-score according to each voxel's degrees of freedom. A sigma filter (12,20) is applied to these maps to cluster pixels in a 3×3 region, thereby reducing single-voxel false positives.

Results

Figure 2 shows gray matter signal magnitude, physiological noise, and thermal noise averaged over three subjects using the *all-frame* and *sliding window* methods of sensitivity map calculation at each slice thickness. As expected, signal magnitude increases with slice thickness. Physiological noise, being proportional to signal magnitude, also increases with slice thickness while thermal noise is fairly constant. It is evident that the *sliding window* method effectively reduces thermal noise while signal magnitude and physiological noise are independent of sensitivity map calculation methods.

Figure 3 shows typical images obtained using *all-frame* and *sliding window* methods of sensitivity map calculation at 2mm and 5mm thickness. Images using the *sliding window* method are less noisy than those using *all-frame*. Standard deviation (SD) images, signal-to-fluctuation noise (SFNR) images, and activation maps all benefit when *sliding window* is used. This is more evident in thin slices (2mm) than in thick slices (5mm). These results are consistent for both spiral-in and spiral-out trajectories. The *sliding window* width default is 2 TRs. However, noise measurement data from subjects 1–3 were further reconstructed using the *sliding window* method with window widths of 4, 8, and 16 TRs. Figure 4 shows the standard deviation for the thermal noise σ_0 vs. *sliding window* width. The noise reduction is only apparent for window width of 2 TRs. Beyond a window width of 2 TRs, *sliding window* reconstruction becomes the *all-frame* method in the limit. For 2mm to 5mm spiral-in and spiral-out images, Figure 5 shows ratios of activation volumes using the *sliding window* method to those using the *all-frame* method. The fat bars are averaged ratios of all six subjects. Standard deviations among subjects are indicated by the skinny error bars. For comparison, ratios of activation volumes using 2-shot reconstruction to those using *all-frame* are also shown. The activation threshold is set to $p < 0.005$ in all cases.

Denoising with *Sliding Window*

We hypothesize that the *sliding window* reconstruction removes image noise because the noise information is retained in the corresponding sensitivity maps. To demonstrate that the denoising effect (shown in Figure 3–Figure 5) derives from incorporation of temporally localized fluctuations in sensitivity maps, 2mm slice thickness data from subject 2 was again reconstructed with *sliding window* methods (width = 2 TRs) but with a window positioned 2 TRs in advance of its optimal position (concurrent with the time frame being reconstructed). When using sensitivity maps from temporally advanced data, reconstructed images are visibly noisier; as illustrated in Figure 6.

As further demonstration of the hypothesis that *sliding window* sensitivity maps reduce image noise after CG-SENSE reconstruction, the sensitivity maps used in *sliding window* reconstruction were spatially smoothed by spline interpolation for Subject 4. Figure 7 demonstrates unsmoothed and smoothed sensitivity maps from Coil 1 as well as images reconstructed using both types of sensitivity maps. We expect that smoothing sensitivity maps would result in noisier images than without smoothing. It is apparent that the observed increase in image noise, particularly in Figure 7(c–d), is due to sensitivity map smoothing. Hence, smoothing the sensitivity maps is not desirable for the *sliding window* method. But that is not generally true for sensitivity maps corrupted by noise less correlated with images under reconstruction, as occurs with the conventional *all-frame* or *reference frame* methods.

Comparison with Reference Frame Method

As mentioned earlier, a widely used technique for gathering sensitivity profiles is to obtain a full resolution scan prior to engagement of the subsampling methods. We define our *reference frame* method as averaging four image time frames from the beginning of a time series and then smoothing with spline interpolation. Figure 8 shows ratios of numbers of activated pixels using the *reference frame* method to those using the *all-frame* technique for 2mm and 5mm spiral-in and spiral-out images. The bars are the averaged ratios of all six subjects. Standard deviations among them are indicated with lines drawn above each bar. For comparison, ratios of numbers of activated pixels using *sliding window* to those using *all-frame* are also shown. Activation threshold is set to $p < 0.005$ in all cases. As shown in Figure 8, the *sliding window* technique performs significantly better than both *reference frame* and *all-frame* techniques, while the latter two perform comparably. The improvement was especially more significant at the 2mm thickness.

The SNR for each subject's reconstructions using *sliding window*, *all-frame*, and *reference frame* methods was calculated. Signal measurements were made by averaging magnitude intensities within a rectangular ROI (region of interest) inside the brain at each time frame. For a given slice, the same rectangular ROI, which included gray matter, white matter, and CSF were used in *sliding window*, *all-frame*, and *reference frame* reconstructed data. A similar rectangular region drawn outside the brain was used to estimate the noise from the standard deviation among those pixels at each time frame. Time series SNR was calculated by dividing the signal mean by noise standard deviation. Averaged mean and standard deviation of this time series SNR over all slices are listed in Table 1. *Sliding window* gives 15% to 70% increase in SNR over *all-frame* or *reference frame* methods. SNR is generally higher at thinner slice thickness for which thermal noise predominates over physiological noise.

Sensitivity Maps from Off Blocks and On-Off Blocks

To assess impact of signal intensity change (due to the task paradigm) on sensitivity maps and consequent image quality, two additional methods of generating sensitivity maps were studied:

In the first method, sensitivity maps were generated by averaging full resolution images only during the *off blocks* (no task) of task paradigm. These sensitivity maps are used for CG-SENSE reconstruction of the entire time series data. This is referred to as the *off blocks* method.

The second method is referred to the *on-off blocks* method; it makes two distinct sets of sensitivity maps: one for *on blocks* (task) and another for *off blocks*. The *on blocks* sensitivity maps are made from full resolution images gathered during task, and they are subsequently applied to CG-SENSE reconstruction of *on blocks* images only. The *off blocks* sensitivity maps calculated are the same as those generated from the *off blocks* method but are applied only to reconstruct *off blocks* images. (One image immediately adjacent to block boundaries is excluded in the sensitivity map calculation to avoid reaction delays and to ensure proper comparison.)

Figure 9–Figure 11 show SD images, SFNR images, and correlation coefficient maps using *off blocks* and *on-off blocks* methods as well as those from *all-frame*, *reference frame*, and *sliding window* methods. From these figures, it is clear that neither the *off blocks* or the *on-off blocks* method provide lower image noise or higher SFNR values than the *sliding window* method.

Comparison with Theory

Equation 3 implies that reduction of thermal noise in the reconstructed time frame occurs when thermal noise ε in a given time frame is close to that in the sensitivity map, $\bar{\varepsilon}$ (Appendix A). Intuitively, peak correlation between thermal noise in sensitivity map and thermal noise of the raw data is inversely proportional to the width of sliding window, *i.e.*, narrower window width corresponds to higher peak correlation. It is shown in Appendix B that the correlation coefficient of raw data thermal noise and sensitivity map thermal noise is approximately $1 / \sqrt{2W}$, where W is width of the sliding window (in number of TRs).

To quantitatively verify our theory, we gathered data containing only thermal noise in a separate experiment with echo planar imaging and gradient recalled echo (GRE EPI) (matrix size = 64×64 , FOV=50cm, 8-channel head coil, no ramp sampling). The reason for using EPI instead of spiral trajectories is: with EPI we were able to acquire the fully sampled noise at each time frame and conveniently generate undersampled data by throwing away odd or even k -space lines. Although we were also able to collect a fully sampled k -space with a single-shot spiral, we were not able to generate undersampled data easily on a two-interleaf spiral without using data interpolation (which might further complicate the noise statistics).

In this GRE EPI experiment, a total of 60 time frames were acquired with the RF transmitter turned off. The first four time frames were discarded. Data from each coil at each time frame was reconstructed by Fourier transform. For each time frame, one thermal noise image was calculated by combining each coil image using square root of sum of squares. These images contain raw data noise ε . Sensitivity maps with window width of 2 TRs were formed using data from odd k -space lines of time frame 1 and data from even k -space lines of time frame 2. Sensitivity maps with window width of 4 TRs were formed using averaged data from odd k -space lines of time frame 1 and 3 and averaged data from even k -space line of time frame 2 and 4. Twenty eight sets of sensitivity maps (widths = 2, 4, 6, ..., 56 TRs) were formed. Temporal correlation coefficients were calculated between each set of sensitivity map and thermal noise images, as illustrated in Figure 12a. Correlation coefficients between the sensitivity map with a width of 2 (calculated using time frames 1 and 2) and the noise images at time frames 1 and 2 are approximately 0.5, whereas correlation coefficients between this sensitivity map and later time frames are close to 0. Correlation coefficients between the sensitivity map with a width of 4 (using time frames 1–4) and the noise images at time frames 1–4 are about 0.35. Figure 12b plots the peak correlation coefficients as a function of window width. The peak correlation coefficients for each window width are calculated as the mean coefficient over the entire plateau region of each corresponding curve shown in Figure 12a. The number of points used for calculating the peak coefficient is the same as sliding-window width. The theoretically predicted peak correlation coefficients are also plotted in Figure 12b, which agrees well with the experimentally measured values.

Figure 12b also plots the noise standard deviation of CG-SENSE reconstructed images as a function of window width. A total of 28 images are reconstructed using the same data set (odd k -space lines of the first time frame) but with sensitivity maps of various sliding-window widths. Since k -space data is acquired on a Cartesian grid, reconstruction is carried out using simple least-squares matrix inversion. Standard deviation of each image is calculated and normalized by that using the widest window width. As expected, standard deviation decreases with decreasing window width. This behavior agrees with observations from the fMRI study shown in Figure 4.

Discussion and Conclusions

We have shown, both experimentally and theoretically, that thermal noise is significantly reduced by using *sliding window* sensitivity maps for CG-SENSE reconstruction in fMRI.

This is especially important when imaging at high resolution where thermal noise predominates. With small voxel size, the technique reveals activation from visual stimulation where other conventional sensitivity calculation techniques falter. With a slice thickness of 2mm, for example, the proposed technique results in improvement by a factor of 4 when compared to conventional PI techniques (Figure 5 and Figure 8). Further, activation volume is statistically equivalent to conventional 2-shot reconstruction with half the temporal resolution; which provides an incentive to perform fMRI with parallel imaging (CG-SENSE). Use of parallel imaging for fMRI has been debated in the literature with regard to loss of SNR and activation detectability. (3,21–23) Utilizing the sliding window CG-SENSE approach, temporal resolution is enhanced and acquisition readout time is reduced (less off-resonance effects) with little loss in activation detectability.

Admitting noise into sensitivity maps appears to contradict conventional wisdom that sensitivity maps should be smooth since receiver coil sensitivity is slowly varying in space. It is true that coil sensitivity is inherently smooth and noiseless. That coil sensitivity which is measured by the *sliding window* technique not only provides an estimation of intrinsic coil sensitivity, but it also provides an estimate of thermal noise. Once this thermal noise estimate is obtained, the noise in the final image can be reduced by using these noisy sensitivity profiles in the image reconstruction process. By reducing thermal noise while maintaining physiological fluctuation induced by functional activation, we are able to improve the fMRI detection. In other words, receiving data using multiple coils simultaneously provides a means to separate physiological noise and thermal noise. The ability to separate these two noise types should increase with increasing number of coils. Further, by improving temporal resolution with parallel imaging, our technique also improves activation detection when compared to standard multi-shot technique.

Since the *sliding window* method uses the current and adjacent time frames for calculating sensitivity maps, which are then applied to CG-SENSE reconstruction, the temporal features of the time series can be slightly smeared from the increased autocorrelation. In particular, degrees of freedom among time frames are reduced. We calculated degrees of freedom for each pixel using a first order autoregressive model, proposed by Worsley *et al.* (24) and Kruggel *et al.* (17). Degrees of freedom are compared in Figure 13 using data (subject 1, 2mm spiral-out, slices 1–9) reconstructed via *sliding window*, *all-frame*, and *reference frame* methods. Histograms show distribution of degrees of freedom using these reconstruction methods. Increase in correlation and reduction in degrees of freedom using the *sliding window* method is small, and cause little adverse effects in activation statistics. That is because the t-distribution approaches a normal distribution, when degrees of freedom are greater than 30, for which degrees of freedom are irrelevant to the critical value for a p-statistic. (25,26) For example, for probability of error $p < 0.005$, degrees of freedom 115 corresponds to critical t-score of 2.619, while degrees of freedom 90 corresponds to critical t-score of 2.632. Applying those t-scores to volunteer 3, for example (2mm spiral-out *sliding window*), the number of activated pixels is 136 and 133, respectively. For *reference frame* method, the number of activated pixels is 31 and 30 respectively. For *all-frame* method, the number of activated pixels is 31 in both cases.

Though demonstrated with fMRI, we expect this proposed *sliding window* technique to directly benefit other dynamic SENSE applications such as cardiac and flow imaging. It is also likely to reduce motion artifacts when scanning young children if motion is slow relative to sliding window widths, since sensitivity maps are updated over time and follow the motion.

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Appendix A

Data received by the i^{th} coil at a $\mathbf{k}$ -space location is

$$d_i(\mathbf{k}) = \int (m(\mathbf{r}) + p(\mathbf{r})) s_i(\mathbf{r}) e^{-j\mathbf{k} \cdot \mathbf{r}} d\mathbf{r} + \varepsilon_i(\mathbf{k}) \quad [\text{A1}]$$

where $\mathbf{r}$ represents image index, and

- $d_i(\mathbf{k})$: Sampled $\mathbf{k}$ -space data of i^{th} coil
- $M(\mathbf{r})$: Original image
- $p(\mathbf{r})$: Physiologic noise
- $\varepsilon_i(\mathbf{k})$: Thermal noise
- $s_i(\mathbf{r})$: Sensitivity of i^{th} coil

With K $\mathbf{k}$ -space samples and N object samples, define the following vectors and matrices with corresponding sizes:

$$\mathbf{d}_i(\mathbf{k}) = \begin{bmatrix} d_i(\mathbf{k}_1) \\ d_i(\mathbf{k}_2) \\ \vdots \\ d_i(\mathbf{k}_K) \end{bmatrix} \quad K \times 1 \quad [\text{A2}]$$

$$\mathbf{m}(\mathbf{r}) = \begin{bmatrix} m(\mathbf{r}_1) \\ m(\mathbf{r}_2) \\ \vdots \\ m(\mathbf{r}_N) \end{bmatrix} \quad N \times 1 \quad [\text{A3}]$$

$$\mathbf{p}(\mathbf{r}) = \begin{bmatrix} p(\mathbf{r}_1) \\ p(\mathbf{r}_2) \\ \vdots \\ p(\mathbf{r}_N) \end{bmatrix} \quad N \times 1 \quad [\text{A4}]$$

$$\boldsymbol{\varepsilon}_i(\mathbf{k}) = \begin{bmatrix} \varepsilon_i(\mathbf{k}_1) \\ \varepsilon_i(\mathbf{k}_2) \\ \vdots \\ \varepsilon_i(\mathbf{k}_K) \end{bmatrix} \quad K \times 1 \quad [\text{A5}]$$

$$\mathbf{E}_0(\mathbf{k}, \mathbf{r}) = \begin{bmatrix} e^{-j\mathbf{k}_1 \bullet \mathbf{r}_1} & e^{-j\mathbf{k}_1 \bullet \mathbf{r}_2} & \dots & e^{-j\mathbf{k}_1 \bullet \mathbf{r}_N} \\ e^{-j\mathbf{k}_2 \bullet \mathbf{r}_1} & e^{-j\mathbf{k}_2 \bullet \mathbf{r}_2} & \dots & e^{-j\mathbf{k}_2 \bullet \mathbf{r}_N} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\mathbf{k}_K \bullet \mathbf{r}_1} & e^{-j\mathbf{k}_K \bullet \mathbf{r}_2} & \dots & e^{-j\mathbf{k}_K \bullet \mathbf{r}_N} \end{bmatrix} \quad K \times N, \quad K < N \leq cK \quad [\text{A6}]$$

Matrix $\mathbf{E}_0$ is a subsampled (fat) 2D DFT matrix constituted by entries $e^{-j\mathbf{K}_1 \bullet \mathbf{r}_1} = e^{-j(k_{x1}x_1 + k_{y1}y_1)}$; it is some phase of a complete DFT matrix. In the case of reduction factor R , the modulo frame index is defined $l \in \{1, 2, \dots, R\}$. For reconstruction of any particular frame, one of R phases of the DFT matrix is used depending on l . Matrix $\mathbf{E}_0$ may therefore be considered constant in all the following equations.

$$\mathbf{S}_i = \begin{bmatrix} s_i(\mathbf{r}_1) & & & 0 \\ & s_i(\mathbf{r}_2) & & \\ & & \ddots & \\ 0 & & & s_i(\mathbf{r}_N) \end{bmatrix} \quad N \times N \quad [\text{A7}]$$

For a number c of coils ($c \geq 1$), we have

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_c \end{bmatrix} \quad cK \times 1 \quad [\text{A8}]$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \\ \vdots \\ \boldsymbol{\varepsilon}_c \end{bmatrix} \quad cK \times 1 \quad [\text{A9}]$$

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}_0 & & & 0 \\ & \mathbf{E}_0 & & \\ & & \ddots & \\ 0 & & & \mathbf{E}_0 \end{bmatrix} \quad cK \times cN \quad [\text{A10}]$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \\ \vdots \\ \mathbf{S}_N \end{bmatrix} \quad cN \times N \quad [\text{A11}]$$

The acquired data can be written as

$$\mathbf{d} = \mathbf{E}\mathbf{S}(\mathbf{m} + \mathbf{p}) + \boldsymbol{\varepsilon} \quad [\text{A12}]$$

Estimated coil sensitivity $\hat{s}_i(\mathbf{r})$ is the ratio of the 2-shot reconstructed image from the i^{th} coil to the square root of sum of squares of 2-shot images from all coils:

$$\begin{aligned} \hat{s}_i(\mathbf{r}) &= \frac{(m(\mathbf{r}) + \bar{p}(\mathbf{r})) s_i(\mathbf{r}) + \bar{e}_i(\mathbf{r})}{\sqrt{\sum_i |(m(\mathbf{r}) + \bar{p}(\mathbf{r})) s_i(\mathbf{r}) + \bar{e}_i(\mathbf{r})|^2}} \\ &= \frac{s_i(\mathbf{r}) + \frac{\bar{e}_i(\mathbf{r})}{m(\mathbf{r}) + \bar{p}(\mathbf{r})}}{\frac{|m(\mathbf{r}) + \bar{p}(\mathbf{r})|}{m(\mathbf{r}) + \bar{p}(\mathbf{r})} \sqrt{\sum_i \left| s_i(\mathbf{r}) + \frac{\bar{e}_i(\mathbf{r})}{m(\mathbf{r}) + \bar{p}(\mathbf{r})} \right|^2}} \end{aligned} \quad [\text{A13}]$$

$$\text{where } \bar{p}(\mathbf{r}) = \frac{1}{M} \sum_{t=1}^M p(\mathbf{r}, t) \quad [\text{A14}]$$

$$\text{and } \bar{e}_i(\mathbf{r}) = \frac{1}{M} \sum_{t=1}^M \mathbf{E}_0^H(\mathbf{k}, \mathbf{r}) \varepsilon_i(\mathbf{k}, t) \quad [\text{A15}]$$

Here, $\bar{p}(\mathbf{r})$ and $\bar{e}_i(\mathbf{r})$ are the noise averaged over a number of M consecutive temporal frames. Without frame averaging the corresponding variables are denoted

$$e_i(\mathbf{r}) = \mathbf{E}_0^H(\mathbf{k}, \mathbf{r}) \varepsilon_i(\mathbf{k}) \quad [\text{A15a}]$$

When the estimated sensitivity maps are used in the image reconstruction, the sensitivity matrix becomes,

$$\widehat{\mathbf{S}} = (\mathbf{S} + \mathbf{X})\mathbf{Q} \quad [\text{A16}]$$

where $\mathbf{X} = \bar{\mathbf{e}}(\mathbf{M} + \mathbf{P})^{-1}$ is assumed relatively small because $\bar{\mathbf{e}}$ is small compared to $\mathbf{M} + \mathbf{P}$.

$$\mathbf{Q}^{-1} = \begin{bmatrix} \frac{m(\mathbf{r}_1) + \bar{p}(\mathbf{r}_1)}{|m(\mathbf{r}_1) + \bar{p}(\mathbf{r}_1)|} \sqrt{\sum_i \left| s_i(\mathbf{r}_1) + \frac{\bar{e}_i(\mathbf{r}_1)}{m(\mathbf{r}_1) + \bar{p}(\mathbf{r}_1)} \right|^2} & & 0 \\ & \ddots & \\ 0 & & \frac{m(\mathbf{r}_N) + \bar{p}(\mathbf{r}_N)}{|m(\mathbf{r}_N) + \bar{p}(\mathbf{r}_N)|} \sqrt{\sum_i \left| s_i(\mathbf{r}_N) + \frac{\bar{e}_i(\mathbf{r}_N)}{m(\mathbf{r}_N) + \bar{p}(\mathbf{r}_N)} \right|^2} \end{bmatrix}$$

$N \times N$

[A17]

$$\bar{\mathbf{P}} = \begin{bmatrix} \bar{p}(\mathbf{r}_1) & & 0 \\ & \bar{p}(\mathbf{r}_2) & \\ & & \ddots \\ 0 & & & \bar{p}(\mathbf{r}_N) \end{bmatrix}$$

$N \times N$

[A18]

$$\bar{\mathbf{e}}_i = \begin{bmatrix} \bar{e}_i(\mathbf{r}_1) \\ \bar{e}_i(\mathbf{r}_2) \\ \vdots \\ \bar{e}_i(\mathbf{r}_N) \end{bmatrix}$$

$N \times 1$

[A19]

$$\bar{\mathbf{e}} = \begin{bmatrix} \text{diag}(\bar{\mathbf{e}}_1) \\ \text{diag}(\bar{\mathbf{e}}_2) \\ \vdots \\ \text{diag}(\bar{\mathbf{e}}_c) \end{bmatrix}$$

$cN \times N$

[A20]

$$\mathbf{M} = \begin{bmatrix} m(\mathbf{r}_1) & & 0 \\ & m(\mathbf{r}_2) & \\ & & \ddots \\ 0 & & & m(\mathbf{r}_N) \end{bmatrix}$$

$N \times N$

[A21]

Using the estimated coil sensitivity, image is reconstructed as

$$\begin{aligned} \hat{\mathbf{m}} &= (\widehat{\mathbf{E}}\mathbf{S})^\dagger \mathbf{d} \\ &= ((\widehat{\mathbf{E}}\mathbf{S})^H (\widehat{\mathbf{E}}\mathbf{S}))^{-1} (\widehat{\mathbf{E}}\mathbf{S})^H \mathbf{d} \end{aligned}$$

where $\dagger$ represents pseudoinverse

[A22]

Under the assumptions that $\mathbf{X}$ is small and using first-order Taylor series approximation for an inverse matrix, for

$$\mathbf{A} = \mathbf{E}^H \mathbf{E} \quad cN \times cN$$
[A23]

$$\begin{aligned}
\widehat{\mathbf{m}} &= (\widehat{\mathbf{S}^H \mathbf{A} \mathbf{S}})^{-1} \widehat{\mathbf{S}^H \mathbf{E}^H \mathbf{d}} \\
&= (\mathbf{Q}(\mathbf{S}+\mathbf{X})^H \mathbf{A}(\mathbf{S}+\mathbf{X}) \mathbf{Q})^{-1} \mathbf{Q}(\mathbf{S}+\mathbf{X})^H \mathbf{E}^H \mathbf{d} \\
&= \mathbf{Q}^{-1} (\mathbf{S}^H \mathbf{A} \mathbf{S} + \mathbf{S}^H \mathbf{A} \mathbf{X} + \mathbf{X}^H \mathbf{A} \mathbf{S} + \mathbf{X}^H \mathbf{A} \mathbf{X})^{-1} (\mathbf{S}+\mathbf{X})^H \mathbf{E}^H \mathbf{d} \\
&= \mathbf{Q}^{-1} \left((\mathbf{S}^H \mathbf{A} \mathbf{S}) \left(\mathbf{I} + (\mathbf{S}^H \mathbf{A} \mathbf{S})^{-1} (\mathbf{S}^H \mathbf{A} \mathbf{X} + \mathbf{X}^H \mathbf{A} \mathbf{S} + \mathbf{X}^H \mathbf{A} \mathbf{X}) \right) \right)^{-1} (\mathbf{S}+\mathbf{X})^H \mathbf{E}^H \mathbf{d} \\
&= \mathbf{Q}^{-1} \left(\mathbf{I} + (\mathbf{S}^H \mathbf{A} \mathbf{S})^{-1} (\mathbf{S}^H \mathbf{A} \mathbf{X} + \mathbf{X}^H \mathbf{A} \mathbf{S} + \mathbf{X}^H \mathbf{A} \mathbf{X}) \right)^{-1} (\mathbf{S}^H \mathbf{A} \mathbf{S})^{-1} (\mathbf{S}+\mathbf{X})^H \mathbf{E}^H \mathbf{d}
\end{aligned} \tag{A24}$$

Using first-order Taylor series approximation for matrix inversion and assuming $\mathbf{X}$ is small,

$$\widehat{\mathbf{m}} \approx \mathbf{Q}^{-1} (\mathbf{I} - (\mathbf{S}^H \mathbf{A} \mathbf{S})^{-1} (\mathbf{S}^H \mathbf{A} \mathbf{X} + \mathbf{X}^H \mathbf{A} \mathbf{S})) (\mathbf{S}^H \mathbf{A} \mathbf{S})^{-1} (\mathbf{S}+\mathbf{X})^H \mathbf{E}^H \mathbf{d} \tag{A25}$$

$$\text{Let } \mathbf{B} = (\mathbf{S}^H \mathbf{A} \mathbf{S})^{-1} \tag{A26}$$

$$\begin{aligned}
\widehat{\mathbf{m}} &= \mathbf{Q}^{-1} (\mathbf{B} - \mathbf{B} \mathbf{S}^H \mathbf{A} \mathbf{X} \mathbf{B} - \mathbf{B} \mathbf{X}^H \mathbf{A} \mathbf{S} \mathbf{B}) (\mathbf{S}+\mathbf{X})^H \mathbf{E}^H \mathbf{d} \\
&= \mathbf{Q}^{-1} (\mathbf{B} - \mathbf{B} \mathbf{S}^H \mathbf{A} \mathbf{X} \mathbf{B} - \mathbf{B} \mathbf{X}^H \mathbf{A} \mathbf{S} \mathbf{B}) (\mathbf{S}+\mathbf{X})^H \mathbf{E}^H (\mathbf{E} \mathbf{S}(\mathbf{m}+\mathbf{p}) + \boldsymbol{\varepsilon})
\end{aligned} \tag{A27}$$

Since $\mathbf{X}$ and $\boldsymbol{\varepsilon}$ are small and $\mathbf{E} \mathbf{e} = \boldsymbol{\varepsilon}$ (or $\mathbf{e} = \mathbf{E}^H \boldsymbol{\varepsilon}$),

$$\begin{aligned}
\widehat{\mathbf{m}} &\approx \mathbf{Q}^{-1} \mathbf{B} ((\mathbf{E} \mathbf{S})^H \mathbf{E} \mathbf{S}(\mathbf{m}+\mathbf{p}) + (\mathbf{E} \mathbf{X})^H \mathbf{E} \mathbf{S}(\mathbf{m}+\mathbf{p}) + (\mathbf{E} \mathbf{S})^H \boldsymbol{\varepsilon} - (\mathbf{S}^H \mathbf{A} \mathbf{X} + \mathbf{X}^H \mathbf{A} \mathbf{S}) \mathbf{B} (\mathbf{E} \mathbf{S})^H \mathbf{E} \mathbf{S}(\mathbf{m}+\mathbf{p})) \\
&= \mathbf{Q}^{-1} (\mathbf{m}+\mathbf{p}) + \mathbf{Q}^{-1} (\mathbf{E} \mathbf{S})^\dagger \boldsymbol{\varepsilon} - \mathbf{Q}^{-1} (\mathbf{E} \mathbf{S})^\dagger \mathbf{E} \mathbf{X} (\mathbf{m}+\mathbf{p}) \\
&= \mathbf{Q}^{-1} (\mathbf{m}+\mathbf{p}) + \mathbf{Q}^{-1} (\mathbf{E} \mathbf{S})^\dagger \left(\boldsymbol{\varepsilon} - \mathbf{E} \bar{\mathbf{e}} (\mathbf{M} + \bar{\mathbf{P}})^{-1} (\mathbf{m}+\mathbf{p}) \right)
\end{aligned} \tag{A28}$$

Assuming physiological noise is slowly varying under width of sliding window, $\bar{\mathbf{P}} \approx \mathbf{p}$. (In the limit of window width = 1 frame, $\bar{\mathbf{P}} = \mathbf{p}$ and exact cancellation occurs.)

$$\begin{aligned}
\widehat{\mathbf{m}} &\approx \mathbf{Q}^{-1} (\mathbf{m}+\mathbf{p}) + \mathbf{Q}^{-1} (\mathbf{E} \mathbf{S})^\dagger \left(\boldsymbol{\varepsilon} - \mathbf{E} \begin{bmatrix} \bar{\mathbf{e}}_1 \\ \bar{\mathbf{e}}_2 \\ \vdots \\ \bar{\mathbf{e}}_c \end{bmatrix} \right) \\
&= \mathbf{Q}^{-1} (\mathbf{m}+\mathbf{p}) + \mathbf{Q}^{-1} (\mathbf{E} \mathbf{S})^\dagger (\boldsymbol{\varepsilon} - \bar{\boldsymbol{\varepsilon}})
\end{aligned} \tag{A29}$$

Clearly, resulting noise $\boldsymbol{\varepsilon} - \bar{\boldsymbol{\varepsilon}}$ in the reconstructed image is reduced if thermal noise $\boldsymbol{\varepsilon}$ at a given frame is similar to the noise averaged over a number of consecutive frames $\bar{\boldsymbol{\varepsilon}}$.

Appendix B

Thermal noise in EPI raw data of a time series can be expressed by independent random variables $O_{1i}, E_{1i}, O_{2i}, E_{2i} \dots O_{Ni}, E_{Ni}$, where O and E denote odd and even $\mathbf{k}$ -space lines of time frames 1 through N of the i^{th} coil.

The sensitivity map formed using time frames 1 and 2 is

$$\widehat{S}_{i,w=2} = F \{ [O_{1i}, E_{2i}] \} / \sqrt{\sum_i |F \{ [O_{1i}, E_{2i}] \}|^2}.$$

The bracket notation represents filling in odd and even $\mathbf{k}$ -space lines by its contents, and F denotes Fourier Transform. Since raw data is entirely made up with thermal noise, then $\widehat{S}_{i,w=2} = \bar{e}$ where $\bar{e}$ is thermal noise in the image domain, and $\bar{\varepsilon}$ is thermal noise in $\mathbf{k}$ -space.

For sliding window width = 4 (using time frames 1–4), the sensitivity map is

$$\widehat{S}_{i,w=4} = F \{ ([O_{1i}, E_{2i}] + [O_{3i}, E_{4i}]) \} / 2 \sqrt{\sum_i |F \{ ([O_{1i}, E_{2i}] + [O_{3i}, E_{4i}]) / 2 \}|^2}.$$

Thermal noise ε in equation [A29] of the first time frame is O_{1i} or E_{1i} depending upon which half of $\mathbf{k}$ -space is sampled. Problems arise in obtaining thermal noise e , since applying a Fourier Transform to O_{1i} or E_{1i} directly will result in aliasing. This leads to the reason for fully sampling $\mathbf{k}$ -space in this experiment: the Fourier Transform can be applied to data of each time frame without aliasing; e.g., first image $im = F \{ [O_{1i}, E_{1i}] \}$ with thermal noise e embedded. We then calculate correlation between im_1 and $\widehat{S}_{i,w}$. With eight coils, sensitivity $\widehat{S}_{i,w}$ has Rician distribution, and im_1 is Rayleigh distributed.

For sliding window width = 2, the correlation coefficient is $\rho_{\widehat{S}_{i,w=2}, im_1} = \frac{\text{cov}(\widehat{S}_{i,w=2}, im_1)}{\sigma_{\widehat{S}_{i,w=2}} \sigma_{im_1}} = \frac{0.5}{1}$

For window width = 4, $\rho_{\widehat{S}_{i,w=4}, im_1} = \frac{\text{cov}(\widehat{S}_{i,w=4}, im_1)}{\sigma_{\widehat{S}_{i,w=4}} \sigma_{im_1}} = \frac{0.25}{\sqrt{0.5}} = 0.3536$

In general, for window width = W , $\rho_{\widehat{S}_{i,w=W}, im_1} = \frac{1}{\sqrt{2W}}$

Figure 1.

(a) TSENSE diagram adapted from Kellman (4). (b) Proposed *sliding window* technique (R=2) as comparison.

Figure 2. (top) Mean and standard deviation of signal magnitude, (middle) thermal noise, and (bottom) physiological noise in gray matter from three subjects. Solid lines represent results using the *all-frame* method. Dash lines are results using *sliding window*.

Figure 3.

Typical Spiral-in and Spiral-out images of 2mm and 5mm thickness. Four types of images are shown in each case: (a) single time frame images, (b) standard deviation (SD) images, (c) signal-to-fluctuation (SFNR) images, and (d) activation maps ($p < 0.005$). In all cases, the first row uses the *all-frame* method for sensitivity map calculations while the second row uses the *sliding window* method with a width of 2 TRs.

Figure 4. Thermal noise using various widths of sliding window from Subjects 1–3 over 3 slices each with 64 frames.

Figure 5. Ratios of numbers of activated pixels from using *sliding window* to those from using *all-frame*. Also shown are the ratios using 2-shot reconstruction to all-frame ($p < 0.005$). Advantage of using *sliding window* over *all-frame* is more apparent in thinner slices. *Sliding window* gives results comparable to 2-shot reconstructed images.

Figure 6. (top) Spiral-in and (bottom) Spiral-out images of 2mm thickness from volunteer 2. Images on the left use correct (current) *sliding window* sensitivity maps. Those on the right use sensitivity maps generated from images 2TRs ahead. The increase in noise is evident from using the advanced sensitivity maps.

Figure 7. (top) Spiral-in and (bottom) Spiral-out images with 2mm thickness from subject 4. (a) Images reconstructed with *sliding window* sensitivity maps of which those corresponding to coil 1 are shown in (e). (f) *Sliding window* sensitivity maps are spatially smoothed with spline interpolation. Corresponding images are shown in (b). (c) and (d) show zoomed-in portions of (a) and (b).

Figure 8. Ratios of numbers of activated pixels from using *sliding window* to those from using *all-frame*. Also shown are the ratios from using *reference frame* to *all-frame* ($p < 0.005$). *Reference frame* performance is very similar to that of the *all-frame* technique and both are inferior to the *sliding window* method.

Figure 9. Standard deviation images from subject 3 reconstructed using sensitivity maps from (a) *off-blocks*, (b) *on-off blocks*, (c) *all-frame*, (d) *reference frame*, and (e) *sliding window*. All images are scaled to the same level for display. *Sliding window* sensitivity maps result in lowest image noise.

Figure 10.

SFNR images corresponding to those in Figure 9. (a) *Off-blocks*, (b) *on-off blocks*, (c) *all-frame*, (d) *reference frame*, and (e) *sliding window* sensitivity maps are used. All images are scaled to the same level for display. *Sliding window* sensitivity maps result in highest SFNR values.

Figure 11.

Activated pixels ($p < 0.005$) overlaid on anatomic images corresponding to those in Figure 9. Number of activated pixels are indicated next to each image. (a) *Off-blocks*, (b) *on-off blocks*, (c) *all-frame*, (d) *reference frame*, and (e) *sliding window* sensitivity maps are used.

Figure 12.

(a) Correlation coefficient, over time, between thermal noise data and sensitivity maps of different sliding window widths. (b) Averages of the plateau of each curve in (a) are plotted as circles. These results agree well with theory (square). First time frame was SENSE reconstructed using sensitivity maps with various sliding window widths. Standard deviation of each image was calculated and normalized by that from using the widest sliding window (indicated by crosses and corresponded to the right y-axis).

Figure 13.

Histograms showing degrees of freedom in a pixel by pixel basis using same data reconstructed by *all-frame* and *sliding window* (width = 2 TRs) methods. *Sliding window* method slightly reduces the degrees of freedom in time series than the *all-frame* method.

Table 1

SNR measurements of each subject using *sliding window* (width = 2 TRs), *all-frame*, and *reference scan* methods. Numbers inside parentheses indicate standard deviations over all slices and time frames.

	Subject number	1	2	3	4	5	6	Average(SD) among subjects
2mm spiral-in	<i>Sliding window</i>	14.3 (0.9)	18.9 (2.0)	21.5 (2.4)	16.7 (1.1)	17.7 (1.0)	19.7 (1.3)	18.1 (1.5)
	<i>All-frame</i>	10.1 (0.6)	13.8 (1.6)	14.4 (1.5)	11.6 (0.5)	12.1 (0.9)	13.6 (1.0)	12.6 (1.0)
	<i>Reference frame</i>	10.7 (0.7)	14.3 (1.5)	15.5 (1.6)	12.6 (0.5)	11.8 (1.4)	14.2 (1.0)	13.2 (1.1)
2mm spiral-out	<i>Sliding window</i>	19.3 (1.2)	19.5 (2.2)	22.9 (1.3)	20.3 (1.0)	21.0 (1.1)	22.7 (1.1)	21.0 (1.3)
	<i>All-frame</i>	12.8 (0.7)	12.2 (1.4)	15.2 (0.7)	12.8 (0.5)	13.8 (0.6)	13.1 (0.7)	13.3 (0.8)
	<i>Reference frame</i>	13.3 (0.6)	15.0 (1.7)	17.0 (0.8)	14.8 (0.5)	14.4 (1.1)	14.0 (1.6)	14.7 (1.0)
3mm spiral-in	<i>Sliding window</i>	18.5 (2.4)	23.3 (2.2)	27.3 (1.7)	21.6 (1.9)	21.4 (2.2)	22.1 (3.1)	22.4 (2.3)
	<i>All-frame</i>	13.2 (1.9)	16.5 (1.6)	18.9 (1.1)	16.2 (1.0)	16.1 (1.3)	13.8 (2.3)	15.8 (1.5)
	<i>Reference frame</i>	12.1 (2.3)	17.4 (1.9)	21.5 (1.2)	17.6 (0.8)	17.3 (1.4)	16.4 (2.3)	17.1 (1.7)
3mm spiral-out	<i>Sliding window</i>	24.3 (1.7)	25.3 (3.1)	33.4 (1.5)	28.1 (1.4)	21.7 (1.7)	31.9 (1.9)	28.8 (1.9)
	<i>All-frame</i>	17.3 (1.1)	18.3 (1.8)	21.0 (0.7)	18.0 (0.7)	20.4 (0.7)	19.9 (1.6)	19.2 (1.1)
	<i>Reference frame</i>	16.0 (1.7)	19.5 (2.2)	24.3 (0.9)	20.2 (0.9)	22.2 (0.7)	22.2 (2.0)	20.7 (1.4)
4mm spiral-in	<i>Sliding window</i>	20.7 (2.9)	27.2 (3.4)	31.3 (2.4)	23.3 (3.1)	25.2 (2.8)	25.7 (4.1)	25.6 (3.1)
	<i>All-frame</i>	17.2 (2.0)	22.6 (3.3)	21.7 (1.8)	17.8 (1.0)	19.0 (2.0)	15.5 (3.3)	18.9 (2.2)
	<i>Reference frame</i>	16.5 (1.7)	19.7 (3.0)	24.8 (1.7)	20.5 (0.9)	21.0 (1.4)	17.1 (4.3)	19.3 (2.2)
4mm spiral-out	<i>Sliding window</i>	34.9 (3.0)	28.9 (3.1)	40.4 (2.3)	34.1 (2.0)	37.2 (1.7)	39.4 (2.8)	35.8 (2.5)
	<i>All-frame</i>	24.8 (0.9)	19.9 (1.8)	25.1 (1.4)	21.9 (1.1)	25.6 (0.8)	23.3 (2.1)	23.4 (1.4)
	<i>Reference frame</i>	26.6 (1.2)	22.0 (2.2)	29.4 (1.1)	25.5 (1.3)	28.0 (0.9)	25.1 (4.0)	26.1 (1.8)
5mm spiral-in	<i>Sliding window</i>	27.9 (4.1)	30.8 (4.3)	33.3 (2.6)	24.8 (2.8)	24.7 (3.0)	25.4 (4.5)	26.7 (3.5)
	<i>All-frame</i>	23.7 (3.3)	26.3 (4.1)	25.1 (1.4)	20.9 (1.6)	17.0 (2.8)	15.7 (2.1)	21.5 (2.5) *
	<i>Reference frame</i>	23.6 (2.2)	24.1 (3.8)	26.8 (1.9)	21.7 (1.4)	18.5 (3.2)	16.6 (2.7)	21.9 (2.5)
5mm spiral-out	<i>Sliding window</i>	37.2 (4.6)	34.4 (4.7)	46.9 (2.3)	38.4 (2.8)	39.8 (2.3)	44.5 (2.5)	40.2 (3.2)
	<i>All-frame</i>	23.5 (1.6)	25.2 (2.9)	29.3 (1.6)	25.8 (1.3)	24.1 (2.3)	27.4 (2.1)	25.9 (2.0)
	<i>Reference frame</i>	27.2 (1.8)	27.2 (3.4)	34.6 (1.3)	28.7 (2.1)	23.4 (3.7)	28.5 (3.7)	28.3 (2.7)

* Paired t-tests between *sliding window* and *all-frame* and *sliding window* and *reference scan* show the increases in SNR in *sliding window* reconstruction are significant when compared to *all-frame* and *reference scan* methods except for data denoted with.