Published in final edited form as:

IEEE Nucl Sci Symp Conf Rec (1997). 2007 ; 5: 3609–3615. doi:10.1109/NSSMIC.2007.4436905.

The Influence of Photon Attenuation on Tumor-to-Background and Signal-to-Noise Ratios for SPECT Imaging

Edward J. Soares, Ph.D.^{*,†}, Michael A. King, Ph.D.[†], Charles L. Byrne, Ph.D.[‡], Howard C. Gifford[†], and Andre Lehovich, Ph.D.[†]

^{*} Department of Mathematics and Computer Science, College of the Holy Cross, Worcester, MA 01610

[†] Department of Radiology, University of Massachusetts Medical School, Worcester, MA 01655

[‡] Department of Mathematical Sciences, University of Massachusetts at Lowell, Lowell, MA 01854

Abstract

Expanding on the work of Nuyts *et. al* [1], Bai *et. al.* [2], and Bai and Shao [3], who all studied the effects of attenuation and attenuation correction on tumor-to-background ratios and signal detection, we have derived a general expression for the tumor-to-background ratio (TBR) for SPECT attenuated data that have been reconstructed with a linear, non-iterative reconstruction operator O . A special case of this is when O represents discrete filtered back-projection (FBP). The TBR of the reconstructed, uncorrected attenuated data (TBR_{no-AC}) can be written as a weighted sum of the TBR of the FBP-reconstructed unattenuated data (TBR_{FBP}) and the TBR of the FBP-reconstructed “difference” projection data (TBR_{diff}). We evaluated the expression for TBR_{no-AC} for a variety of objects and attenuation conditions. The ideal observer signal-to-noise ratio (SNR_{ideal}) was also computed in projection space, in order to obtain an upper bound on signal detectability for a signal-known-exactly/background-known-exactly (SKE/BKE) detection task. The results generally show that SNR_{ideal} is lower for tumors located deeper within the attenuating medium and increases for tumors nearer the edge of the object. In addition, larger values for the uniform attenuation coefficient μ lead to lower values for SNR_{ideal} . The TBR for FBP-reconstructed, uncorrected attenuated data can both under- and over-estimate the true TBR, depending on several properties of the attenuating medium, including the shape of the attenuator, the uniformity of the attenuator, and the degree to which the data are attenuated.

I. Introduction

Over the last decade, several authors have investigated the usefulness of attenuation correction on lesion detection and localization in single-photon emission computed tomography (SPECT) and positron emission tomography (PET). In 2000, Wells *et. al.* [4] compared ordered-subset expectation-maximization (OS-EM) reconstruction with filtered back-projection (FBP) to determine if attenuation correction helped for detection and localization of thoracic gallium-labeled lesions in SPECT. Using psychophysical studies, they found no difference in observer performance using images derived from FBP reconstruction, with and without attenuation compensation (AC). While OS-EM with AC did improve observer performance relative to the other methods, OS-EM without AC yielded comparable detection to images reconstructed with

E.J. Soares is with the Department of Mathematics and Computer Science, College of the Holy Cross, Worcester, MA 01610 USA (telephone: 508-793-3368, e-mail: esoares@holycross.edu).

M.A. King, H.C. Gifford, and A. Lehovich are with the Department of Radiology, Division of Nuclear Medicine, University of Massachusetts Medical School, Worcester, MA 01655 USA (telephone: 508-856-4255, e-mail: Michael.King@umassmed.edu).

C.L. Byrne is with the Department of Mathematical Sciences, University of Massachusetts at Lowell, Lowell, MA 01854 (telephone: 978-934-2447, e-mail: Charles.Byrne@uml.edu).

FBP and no AC. Narayanan *et. al.* [5] studied compensation strategies for ^{99m}Tc myocardial perfusion imaging. They compared FBP with no AC, with OS-EM reconstruction with AC, with AC and scatter correction, and with AC, scatter correction, and resolution compensation. The results of their psychophysical studies suggested that accurate modeling and compensation for these degrading effects improved detection of coronary artery disease. FBP exhibited the lowest observer performance, while performance increased as image-degrading factors were included in the OS-EM reconstruction process.

Nuyts *et. al* [1] examined the effect of the attenuation artifact on the tumor-to-background ratio (TBR) and tumor detection for PET. They showed the attenuation artifact had scaling and shifting components. The scaling component was the result of loss of counts due to attenuation, while the shifting component is a smooth negative contribution that causes an artificially high TBR in some circumstances. Bai *et. al.* [2] also investigated the effects of attenuation in PET. They noted that images without AC could have locally enhanced contrast in regions of uniform attenuation. In regions of nonuniform attenuation such as the thorax, high count foci can disappear in images without AC, or even appear photopenic. This effect is dependent on the size, location, and density of the foci. Bai and Shao [3] extended their analysis to SPECT oncological imaging. They found similar behavior in tumor detectability as compared with their analysis of PET imaging. Chang's attenuation correction method was shown to not improve estimation of TBR, relative to no-AC.

Motivated by this previous work, we have attempted to analytically quantify the effect of photon attenuation in the projection data and the lack of compensation for the degradation in SPECT reconstructed images. We have derived an analytic expression that relates the TBR of FBP-reconstructed, uncorrected attenuated data to that of FBP-reconstructed, unattenuated data and FBP-reconstructed, "difference" projection data. Using simulation studies, we demonstrate the effect of object size and shape, non-uniformity of attenuator, and degree of attenuation on the TBR and ideal observer signal-to-noise ratio (SNR).

II. Theory

A. Imaging Model

In our analysis, we consider a simple signal-known-exactly/background-known-exactly (SKE/BKE) detection task. Let $\mathbf{f}_0$ denote the discretized signal-absent object and $\mathbf{f}_1$ denote the discretized signal-present object. Thus, the *background* and *tumor* (signal) are given, respectively, by

$$\begin{aligned}\mathbf{f}_B &= \mathbf{f}_0 \\ \mathbf{f}_T &= \mathbf{f}_1 - \mathbf{f}_0.\end{aligned}\tag{1}$$

The discrete Radon transform and the discrete attenuated Radon transform can be expressed as matrices, $\mathbf{H} = (h_{ij})$ and $\mathbf{H}_a = (a_{ij} h_{ij})$, respectively, where h_{ij} is the weight for determining the contribution of object pixel j to measurement i and a_{ij} is the associated attenuation factor. Next, we define the *difference* transform $\Delta\mathbf{H}$, where $\Delta\mathbf{H} = \mathbf{H} - \mathbf{H}_a = ((1 - a_{ij})h_{ij})$. The importance of this operator becomes clear in our subsequent derivations.

Both signal-absent and signal-present objects are then projected incorporating the effects of photon attenuation. Thus, we may write the mean attenuated projections of the signal-absent and signal-present objects as

$$\begin{aligned}\bar{\mathbf{g}}_0 &= \mathbf{H}_a \mathbf{f}_0 \\ \bar{\mathbf{g}}_1 &= \mathbf{H}_a \mathbf{f}_1.\end{aligned}\quad (2)$$

The object-dependent noise $\mathbf{n}$ is defined as the deviation between a measurement and its mean

$$\begin{aligned}\mathbf{n}_0 &= \mathbf{g}_0 - \bar{\mathbf{g}}_0 \\ \mathbf{n}_1 &= \mathbf{g}_1 - \bar{\mathbf{g}}_1,\end{aligned}\quad (3)$$

which, when incorporated with (2), yields the imaging model

$$\begin{aligned}\mathbf{g}_0 &= \mathbf{H}_a \mathbf{f}_0 + \mathbf{n}_0 \\ \mathbf{g}_1 &= \mathbf{H}_a \mathbf{f}_1 + \mathbf{n}_1.\end{aligned}\quad (4)$$

Next, the data are reconstructed with a linear, non-iterative operator $\mathbf{O}$. An example of $\mathbf{O}$ would be discrete filtered back-projection (FBP). Thus, we may write the reconstructed signal-absent and signal-present images as

$$\begin{aligned}\widehat{\mathbf{f}}_0 &= \mathbf{O} \mathbf{H}_a \mathbf{f}_0 + \mathbf{O} \mathbf{n}_0 \\ \widehat{\mathbf{f}}_1 &= \mathbf{O} \mathbf{H}_a \mathbf{f}_1 + \mathbf{O} \mathbf{n}_1,\end{aligned}\quad (5)$$

with mean reconstructed signal-absent and signal-present images given by

$$\begin{aligned}\bar{\widehat{\mathbf{f}}}_0 &= \mathbf{O} \mathbf{H}_a \mathbf{f}_0 \\ \bar{\widehat{\mathbf{f}}}_1 &= \mathbf{O} \mathbf{H}_a \mathbf{f}_1,\end{aligned}\quad (6)$$

since the noise in (3) is defined tacitly as having zero mean. Finally, using equations (1) and (6), we see that the mean reconstructed background and tumor images may be written as

$$\bar{\widehat{\mathbf{f}}}_B = \bar{\widehat{\mathbf{f}}}_0 = \mathbf{O} \mathbf{H}_a \mathbf{f}_B, \quad (7)$$

and

$$\bar{\widehat{\mathbf{f}}}_T = \bar{\widehat{\mathbf{f}}}_1 - \bar{\widehat{\mathbf{f}}}_0 = \mathbf{O} \mathbf{H}_a \mathbf{f}_T, \quad (8)$$

respectively.

B. Signal-to-Noise Ratio (SNR)

In order to objectively assess the effect of photon attenuation on tumor detectability, we computed the ideal observer signal-to-noise ratio ($\text{SNR}_{\text{ideal}}$) using the projection data. This is

slightly unconventional, as it is the reconstructed images that are typically used for diagnostic purposes and thus image quality tends to be assessed in reconstruction space. But evaluating $\text{SNR}_{\text{ideal}}$ in projection space serves two purposes. First, it sets an upper bound on detectability, as $\text{SNR}_{\text{ideal}}$ is invariant to invertible reconstruction operators $\mathbf{O}$ and may only decrease after image reconstruction. One explanation for such a decrease would be if the reconstruction operator $\mathbf{O}$ contains null components. Second, it will allow us to determine which signal locations within the background produce data with higher detectability measures. In some sense, it's a measure of favorability of tumor location under our SKE/BKE paradigm.

The ideal observer uses the full joint probability distributions of both signal-present and signal-absent data to compute a test statistic based on maximizing the likelihood ratio. For simplicity, we used the normal approximation to the Poisson-distributed data

$$\begin{aligned}\mathbf{g}_0 &\sim N(\bar{\mathbf{g}}_0, \Sigma_{\mathbf{g}_0}) \\ \mathbf{g}_1 &\sim N(\bar{\mathbf{g}}_1, \Sigma_{\mathbf{g}_1}),\end{aligned}\tag{9}$$

where the signal-absent and signal-present class covariance matrices are diagonal and given by

$$\begin{aligned}\Sigma_{\mathbf{g}_0} &= \text{diag}\{\bar{\mathbf{g}}_0\} \\ \Sigma_{\mathbf{g}_1} &= \text{diag}\{\bar{\mathbf{g}}_1\}.\end{aligned}\tag{10}$$

Assuming the probabilities of occurrence of each class are P_0 and P_1 , respectively, then $\text{SNR}_{\text{ideal}}$ can be calculated as

$$\begin{aligned}\text{SNR}_{\text{ideal}}^2 &= (\bar{\mathbf{g}}_1 - \bar{\mathbf{g}}_0)' (P_0 \Sigma_{\mathbf{g}_0} + P_1 \Sigma_{\mathbf{g}_1})^{-1} (\bar{\mathbf{g}}_1 - \bar{\mathbf{g}}_0) \\ &= \bar{\mathbf{g}}_T' (P_0 \Sigma_{\mathbf{g}_0} + P_1 \Sigma_{\mathbf{g}_1})^{-1} \bar{\mathbf{g}}_T,\end{aligned}\tag{11}$$

where $\bar{\mathbf{g}}_T$ are the mean attenuated projections of the tumor. The reader will note that the inverse of the average class covariance matrix $(P_0 \Sigma_{\mathbf{g}_0} + P_1 \Sigma_{\mathbf{g}_1})$ is easily computed, as this matrix is diagonal.

C. Tumor-to-Background Ratio (TBR)

The tumor-to-background ratio (TBR) is a local measure of contrast between the tumor activity and that of the surrounding background. Intuitively, we might hypothesize that a higher TBR would correlate with higher detectability, as the tumor “stands out” more against the local background. However, tumor detection also depends on the noise properties of image, particularly the noise correlations. It is well known that FBP reconstruction introduces noise correlations that inhibit human observer performance in SKE/BKE detection tasks. Thus, we wished to compare this contrast measure with our detectability figure-of-merit $\text{SNR}_{\text{ideal}}$ to better understand any relationships that may exist.

In the following, we use the mean reconstructed background and tumor (eqns. (7) and (8)) rather than those containing the noise terms. This is due to the fact that the inclusion of the noise terms would introduce an additional level of complexity in our derivation. Also, the average activity in both the reconstructed background and tumor is computed within a region-

of-interest (ROI), and this averaging process would most likely minimize the influence of the noise terms on the overall TBR.

To estimate the average tumor activity and average background activity, we must first define a ROI-observer template $\mathbf{w}$ that is non-zero for pixels within the ROI and zero otherwise. Often, the template values are designed to sum to unity. We can now compute the estimated average tumor activity ($\hat{T}$) in the ROI as

$$\begin{aligned}\hat{T} &= \mathbf{w}' \bar{\mathbf{f}}_T \\ &= \mathbf{w}' \mathbf{O} \mathbf{H}_a \mathbf{f}_T \\ &= \mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_T - \mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_T,\end{aligned}\tag{12}$$

as well as the estimated average background activity ($\hat{B}$) in the ROI as

$$\begin{aligned}\hat{B} &= \mathbf{w}' \bar{\mathbf{f}}_B \\ &= \mathbf{w}' \mathbf{O} \mathbf{H}_a \mathbf{f}_B \\ &= \mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_B - \mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_B.\end{aligned}\tag{13}$$

Finally, we define the TBR of the reconstructed, uncorrected attenuated data as

$$\text{TBR}_{\text{no-AC}} = \frac{\hat{T}}{\hat{B}} = \frac{\mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_T - \mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_T}{\mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_B - \mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_B}.\tag{14}$$

The quotient above can be simplified using component-operation long division as discussed in [6] to yield

$$\begin{aligned}\text{TBR}_{\text{no-AC}} &= \frac{\mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_T}{\mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_B} \left(1 + \frac{\mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_B}{\mathbf{w}' \mathbf{O} \mathbf{H}_a \mathbf{f}_B} \right) \\ &+ \frac{\mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_T}{\mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_B} \left(1 - \frac{\mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_B}{\mathbf{w}' \mathbf{O} \mathbf{H}_a \mathbf{f}_B} \right).\end{aligned}\tag{15}$$

We note that

$$\begin{aligned}\Delta \mathbf{H} \mathbf{f}_B &= \mathbf{H} \mathbf{f}_B - \mathbf{H}_a \mathbf{f}_B \\ \Rightarrow \mathbf{O} \Delta \mathbf{H} \mathbf{f}_B &= \mathbf{O} \mathbf{H} \mathbf{f}_B - \mathbf{O} \mathbf{H}_a \mathbf{f}_B \\ \Rightarrow \mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_B &= \mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_B - \mathbf{w}' \mathbf{O} \mathbf{H}_a \mathbf{f}_B.\end{aligned}\tag{16}$$

Dividing both sides by $\mathbf{w}' \mathbf{O} \mathbf{H}_a \mathbf{f}_B$ (assuming it is non-zero), we yield the identity

$$\frac{\mathbf{w}' \mathbf{O} \Delta \mathbf{H} \mathbf{f}_B}{\mathbf{w}' \mathbf{O} \mathbf{H}_a \mathbf{f}_B} = \frac{\mathbf{w}' \mathbf{O} \mathbf{H} \mathbf{f}_B}{\mathbf{w}' \mathbf{O} \mathbf{H}_a \mathbf{f}_B} - 1.\tag{17}$$

Substituting (17) into (15), we are able to write

$$\begin{aligned} \text{TBR}_{\text{no-AC}} &= \frac{\mathbf{w}^T \mathbf{O} \mathbf{h}_B}{\mathbf{w}^T \mathbf{O} \mathbf{h}_B} \left(\frac{\mathbf{w}^T \mathbf{O} \mathbf{h}_B}{\mathbf{w}^T \mathbf{O} \mathbf{h}_a \mathbf{f}_B} \right) \\ &+ \frac{\mathbf{w}^T \mathbf{O} \Delta \mathbf{h}_B}{\mathbf{w}^T \mathbf{O} \Delta \mathbf{h}_B} \left(1 - \frac{\mathbf{w}^T \mathbf{O} \mathbf{h}_B}{\mathbf{w}^T \mathbf{O} \mathbf{h}_a \mathbf{f}_B} \right). \end{aligned} \quad (18)$$

Let us now assume that the attenuated data are reconstructed with FBP. Thus, $\mathbf{O}$ is the discrete FBP matrix and so (18) becomes

$$\begin{aligned} \text{TBR}_{\text{no-AC}} &= \text{TBR}_{\text{FBP}} \left(\frac{\mathbf{w}^T \mathbf{O} \mathbf{h}_B}{\mathbf{w}^T \mathbf{O} \mathbf{h}_a \mathbf{f}_B} \right) \\ &+ \text{TBR}_{\text{diff}} \left(1 - \frac{\mathbf{w}^T \mathbf{O} \mathbf{h}_B}{\mathbf{w}^T \mathbf{O} \mathbf{h}_a \mathbf{f}_B} \right), \end{aligned} \quad (19)$$

where TBR_{FBP} is the TBR of the FBP-reconstructed un-attenuated data and TBR_{diff} is the TBR of the images derived from the FBP-reconstructed *difference* projection data (difference between the un-attenuated and attenuated data).

The reader's attention should be directed to the weighting factor in (19)

$$\alpha = \frac{\mathbf{w}^T \mathbf{O} \mathbf{h}_B}{\mathbf{w}^T \mathbf{O} \mathbf{h}_a \mathbf{f}_B}. \quad (20)$$

The numerator of α is simply the average activity within the ROI of the FBP-reconstructed background from unattenuated data. We would assume that this value would be close to that in the original object. However, the denominator plays a crucial role in determining the overall $\text{TBR}_{\text{no-AC}}$. As discussed in [1]–[3], the FBP-reconstructed background from uncorrected, attenuated data may contain regions of zero or negative activity. If the average background activity in the ROI is small, this will produce an extremely large weighting factor and thus a high $\text{TBR}_{\text{no-AC}}$.

III. Methods

We studied uniformly emitting objects and uniform and nonuniform attenuators in our experiments. Examples of these can be seen in Figure 1. Each object was of dimension 64×64 pixels. The emitters included both circular (radius = 24 pixels) and elliptical (major axis = 28 pixels, minor axis = 20 pixels) objects, so that we could see the effect of object shape on both $\text{SNR}_{\text{ideal}}$ and $\text{TBR}_{\text{no-AC}}$. We simulated a background activity level of 100 counts per object pixel in our uniform emitters. Tumors were modeled as 10% positive contrast uniform disks (radius = 3 pixels). The attenuators also included both circular and elliptical shapes and were simulated to model attenuating mediums for photon energies 73 keV (^{201}Tl), 140 keV ($^{99\text{m}}\text{Tc}$), and 247 keV (^{111}In), in order to see the effect of degree of attenuation on both $\text{SNR}_{\text{ideal}}$ and $\text{TBR}_{\text{no-AC}}$. The uniform attenuation coefficients for each of these energies are given, respectively, by $\mu = .129 \text{ cm}^{-1}$, $\mu = .15 \text{ cm}^{-1}$, and $\mu = .192 \text{ cm}^{-1}$. Finally we studied a non-uniformly attenuating medium using the MCAT simulation modeling $^{99\text{m}}\text{Tc}$ emission and attenuation.

For each object and attenuator pair, we created 2D maps of $\text{SNR}_{\text{ideal}}$ and $\text{TBR}_{\text{no-AC}}$ as a function of tumor location within the object. For each object pixel, a tumor was placed within the object, centered at that object pixel. If the tumor fell completely within the object's support, we then computed $\text{SNR}_{\text{ideal}}$ (using (11)) and $\text{TBR}_{\text{no-AC}}$ (using (19)) and set the value of the SNR and TBR maps at that object pixel accordingly. Otherwise, these values were set to zero. As a result, brighter pixel values in these maps correspond to higher $\text{SNR}_{\text{ideal}}$ and $\text{TBR}_{\text{no-AC}}$. In this way, one can clearly determine which locations within the object correspond to higher detectability and local contrast. We used the support of the tumor as our ROI template $\mathbf{w}$, although it is clear that the way with which one defines the ROI will affect the value of $\text{TBR}_{\text{no-AC}}$.

IV. Results

A. Circular Objects

In Figure 2, we show the results for the circularly-shaped emitter and attenuator. The top row contains a 1D plot of $\text{SNR}_{\text{ideal}}$ vs. radial tumor location for the cases of no attenuation ($\mu = 0$) and the aforementioned levels of uniform photon attenuation. The bottom row contains a 1D plot of $\text{TBR}_{\text{no-AC}}$ vs. radial tumor location for the same cases. The reader will note that due to the isotropic nature of the object, computing 2D SNR and TBR maps was unnecessary.

It can be seen that tumors placed closer to the center of the emitter yield lower values for $\text{SNR}_{\text{ideal}}$. As the tumor is moved closer to the edge, $\text{SNR}_{\text{ideal}}$ increases. This is due to the fact that the effect of attenuation decreases as we move toward the edge and corrupts the data less. Also, we can see that there is a clear ordering to the SNRs, with $\text{SNR}_{\text{ideal}}$ consistently lower for $\mu = .192 \text{ cm}^{-1}$, followed by $\mu = .15 \text{ cm}^{-1}$, and then finally $\mu = .129 \text{ cm}^{-1}$. Again, lower attenuation coefficients lead to less corrupt data and thus higher SNRs.

Finally, we note that $\text{SNR}_{\text{ideal}}$ for $\mu = 0$ is mostly flat throughout the object, but with a slight increase at the edge. This is not an artifact of the reconstruction process. This slight increase exists because when the tumor is placed near the edge of the object, the noise variance in the support of the projected tumor is proportional to the projected background. The value of this projected background decreases at the edge of the projection because of the object shape. This yields lower values for the noise variance, which slightly increases $\text{SNR}_{\text{ideal}}$.

Examining the $\text{TBR}_{\text{no-AC}}$ plots, we see that when no attenuation is present ($\mu = 0$), our contrast measure is invariant to tumor location with a value of approximately 7.4%. The reader will note that this is less than our simulated value of 10% contrast. This is due to the fact that the ROI template was defined as the support of the tumor and since some of the tumor activity is represented as partial volumes, averaging all of the tumor activity would lead to a slight decrease in the simulated contrast.

As we might predict, $\text{TBR}_{\text{no-AC}}$ is lowest in the center of the object because photon attenuation decreases local contrast, and if not corrected, this decrease will be retained in the FBP-reconstructed image. However, as the tumor is moved closer to the edge, $\text{TBR}_{\text{no-AC}}$ increases, as attenuation corrupts the data less. Throughout most of the background, the values of $\text{TBR}_{\text{no-AC}}$ for the three levels of attenuation exhibited the same ordering as was seen with $\text{SNR}_{\text{ideal}}$. Again, higher levels of photon attenuation lead to decreased contrast, relative to lower levels of attenuation. However, as we move toward the edge, $\text{TBR}_{\text{no-AC}}$ exceeds the correct value and this ordering

The data contained in Figure 3 provides the explanation for this effect. In this figure, we show FBP reconstructions of the uncorrected, attenuated projections of the uniform background for the three levels of attenuation. As we can see from all three images, there is a hot ring at the

periphery of each of the reconstructions, relative to the cooler interior. However, the ring is hotter for lower attenuation coefficients, which can be seen by examining the maximum values on the respective colorbars. Since the average activity in these images within the support of the tumor is used to compute the denominator of the scaling factor α (eqn. (20)), this value will be larger for lower attenuation coefficients. Thus, TBR_{no-AC} (eqn. (19)) will be higher for larger attenuation coefficients ($\mu = 0.192 \text{ cm}^{-1}$) vs. smaller ones ($\mu = 0.129 \text{ cm}^{-1}$).

B. Elliptical Objects

In Figure 4, we show the results for the elliptically-shaped emitter and attenuator. The top row contains 2D images of SNR_{ideal} (left) and TBR_{no-AC} (right), both vs. radial tumor location for the cases of no attenuation ($\mu = 0$). The middle row contains analogous images for the case where $\mu = 0.129 \text{ cm}^{-1}$, while the bottom row contains analogous images for the case where $\mu = 0.15 \text{ cm}^{-1}$. The reader will note that due to the anisotropic nature of the object, computing 2D SNR and TBR maps was necessary.

When no attenuation is present, both SNR_{ideal} and TBR_{no-AC} are approximately invariant to tumor location, as was also seen in our study of circular objects and attenuators.

When attenuation is present, it can be seen that tumors placed closer to the center of the emitter yield lower values for SNR_{ideal} . As the tumor is moved closer to the edge, SNR_{ideal} increases. Again, this is due to the fact that the effect of attenuation decreases as we move toward the edge and corrupts the data less. Also, we note that SNR_{ideal} is higher at the edge along the minor axis of the ellipse, relative to the edge along the major axis. Tumors located along the minor axis, especially those near the edge, undergo less attenuation producing data that are less corrupted and thus have higher SNRs.

The 2D maps of TBR_{no-AC} exhibit an unusual behavior. At the center of the object, we see artificially high values for the TBR. These values then decrease as we move outward radially, and then increase again as we move outward in a direction close to the major axis of the ellipse. The peak is larger for the case where $\mu = 0.15 \text{ cm}^{-1}$ vs. $\mu = 0.129 \text{ cm}^{-1}$, suggesting that the average activity in the FBP reconstructions of the uncorrected, attenuated projections of the uniform elliptical background is smaller for the case where $\mu = 0.15 \text{ cm}^{-1}$. This would also be the case for those regions along the direction of the major axis which have artificially high TBRs.

The data contained in Figure 5 provides the explanation for this effect. In this figure, we show FBP reconstructions of the uncorrected, attenuated projections of the uniform elliptical background for levels of attenuation $\mu = 0.129 \text{ cm}^{-1}$ and $\mu = 0.15 \text{ cm}^{-1}$. As we can see from the two images, there is a hot ring at the periphery of each of the reconstructions, relative to the cooler interior. However, the ring is hotter for lower attenuation coefficients, which can be seen by examining the maximum values on the respective colorbars. It is more difficult to see which regions are cooler for the different cases. However, we hypothesize that artificially high TBRs would correlate with regions of low activity in these images, while low TBRs would correlate with regions of high activity. Again, this is due to the fact that the average activity in these images within the support of the tumor is used to compute the denominator of the scaling factor α (eqn. (20)).

C. MCAT

In Figure 6, we show results for our study of the effect of uniform emission and non-uniform attenuation on SNR_{ideal} and TBR_{no-AC} using the MCAT simulation. In this case, our tumor was a 10% contrast disk of radius 6 pixels placed within the uniform MCAT object. The

background activity level remained at 100 counts per pixel. The MCAT attenuation map reflected that for ^{99m}Tc attenuation.

First, because of the larger tumor size (radius = 6 pixels instead of radius = 3 pixels), we note that the value of $\text{SNR}_{\text{ideal}}$ with no attenuation present has increased. The larger tumor leads to greater tumor conspicuity from the perspective of the ideal observer, and thus a larger overall $\text{SNR}_{\text{ideal}}$. Further, when no attenuation is present, both $\text{SNR}_{\text{ideal}}$ and $\text{TBR}_{\text{no-AC}}$ are approximately invariant to tumor location, as was also seen in our previous studies.

When no attenuation is present, both $\text{SNR}_{\text{ideal}}$ and $\text{TBR}_{\text{no-AC}}$ become location dependent. The $\text{SNR}_{\text{ideal}}$ map is consistent with the observation that tumors located deeper within the attenuating medium lead to data that is more corrupt and thus produce lower values for $\text{SNR}_{\text{ideal}}$. The $\text{TBR}_{\text{no-AC}}$ map has a striking similarity to a blurred version of the attenuation map (see Fig. 1, bottom right image). The lung areas have lower values for $\text{TBR}_{\text{no-AC}}$, while the heart and mediastinum areas have higher values.

The data contained in Figure 7 provides the explanation for this effect. In this figure, we show the FBP reconstruction of the uncorrected, non-uniformly attenuated projections of the uniform elliptical background. The reader will note that the lung areas possess higher counts, while the heart and mediastinum areas exhibit lower counts. Again, since the average activity in this image within the support of the tumor is used to compute the denominator of the scaling factor α (eqn. (20)), this value will be larger for areas with a lower attenuation coefficient. Thus, $\text{TBR}_{\text{no-AC}}$ (eqn. (19)) will be higher for areas with higher levels of attenuation vs. lower ones.

V. Conclusion

We have derived a general expression for the TBR of SPECT attenuated data that have been reconstructed with a linear, non-iterative reconstruction operator $\mathbf{O}$. A special case of this is when $\mathbf{O}$ represents discrete FBP. We showed that the TBR of the reconstructed, uncorrected attenuated data can be written as a weighted sum of the TBR of the FBP-reconstructed unattenuated data and the TBR of the FBP-reconstructed difference projection data. We evaluated our expression for a variety of objects and attenuation conditions. The ideal observer SNR was also computed to obtain an upper bound on signal detectability for a signal-known-exactly/background-known-exactly detection task. The results generally showed that the ideal SNR is lower for tumors located deeper within the attenuating medium and increases for tumors nearer the edge of the object. In addition, larger values for the uniform attenuation coefficient μ lead to lower values for the ideal SNR. The TBR for FBP-reconstructed, uncorrected attenuated data can both under- and over-estimate the true TBR, depending on several properties of the attenuating medium, including the shape of the attenuator, the uniformity of the attenuator, and the degree to which the data are attenuated. We intend to extend this work by incorporating the contributions of object-dependent noise, in order to more accurately assess the effects of these imaging factors on SNR and TBR.

Acknowledgements

The authors would like to thank P. Hendrik Pretorius, Ph.D. of the University of Massachusetts Medical School for his help in creating the MCAT simulations and Johan Nuyts, Ph.D. of Katholieke Universiteit Leuven for insights regarding attenuation and tumor-to-background ratios. This work was supported in part by the National Institute for Biomedical Imaging and Bioengineering under grant R01-EB002798. The contents are solely the responsibility of the authors and do not necessarily represent the official views of the National Institutes of Health.

This work was supported in part by the National Institute for Biomedical Imaging and Bioengineering under grant R01-EB002798. The contents are solely the responsibility of the authors and do not necessarily represent the official views of the National Institutes of Health.

References

1. Nuyts J, Stroobants S, Vleugels S, Flamen P, Mortelmans L. Reducing Loss of Image Quality Because of the Attenuation Artifact in Uncorrected PET Whole-Body Images. *J Nucl Med* August;2002 43(8): 1054–1062. [PubMed: 12163632]
2. Bai C, Kinihan P, Brasse D, Comtat C, Townsend D, Meltzer C, Villemagne V, Charron M, Defrise M. An Analytic Study of the Effects of Attenuation on Tumor Detection in Whole-Body PET Oncology Imaging. *J Nucl Med* November;2003 44(11):1855–1861. [PubMed: 14602870]
3. Bai, C.; Shao, L. A Study of the Effects of Attenuation Correction on Tumor Detection in SPECT Oncology. 2006 IEEE Medical Imaging Conference Proceedings;
4. Wells R, King M, Simkin P, Judy P, Brill A, Gifford H, Licho R, Pretorius P, Schneider P, Seldin D. Comparing Filtered Backprojection and Ordered-Subsets Expectation Maximization for Small-Lesion Detection and Localization in ^{67}Ga SPECT. *J Nucl Med* August;2000 41(8):1391–1399. [PubMed: 10945533]
5. Narayanan M, King M, Pretorius P, Dahlberg S, Spencer F, Simon E, Ewald E, Healy E, MacNaught K, Leppo J. Human-Observed Receiver-Operating-Characteristic Evaluation of Attenuation, Scatter, and Resolution Compensation Strategies for ^{99m}Tc Myocardial Perfusion Imaging. *J Nucl Med* November;2003 44(11):1725–1734. [PubMed: 14602852]
6. Barrett H, Wilson D, Tsui B. Noise properties of the EM algorithm: I. Theory. *Phys Med Biol* 1994;39:833–846. [PubMed: 15552088]

Fig. 1. Simulated emitters (top row) and attenuators (bottom row) used in our study. Top row: uniform disk (left), uniform ellipse (center), uniform MCAT (right). Bottom row: uniform disk (left), uniform ellipse (center), non-uniform MCAT (right).

Fig. 2.

Plots of SNR_{ideal} (top) and TBR_{no-AC} (bottom) as a function of radial tumor location for a circularly-shaped uniform emitter and uniform attenuator.

Fig. 3.

FBP reconstruction of the uncorrected, attenuated projections of the uniform disk background:

$\mu = 0.129 \text{ cm}^{-1}$ (left), $\mu = 0.15 \text{ cm}^{-1}$ (center), $\mu = 0.192 \text{ cm}^{-1}$ (right).

Fig. 4. Plots of $\text{SNR}_{\text{ideal}}$ (left) and $\text{TBR}_{\text{no-AC}}$ (right) as a function of radial tumor location for an elliptically-shaped uniform emitter and uniform attenuator. Top row: No attenuation; Middle row: $\mu = 0.129 \text{ cm}^{-1}$; Bottom row: $\mu = 0.15 \text{ cm}^{-1}$.

Fig. 5.

FBP reconstruction of the uncorrected, attenuated projections of the uniform elliptical background: $\mu = 0.129 \text{ cm}^{-1}$ (left), $\mu = 0.15 \text{ cm}^{-1}$ (right).

Fig. 6. Maps of SNR_{ideal} (top row) and TBR_{no-AC} (bottom row) as a function of radial tumor location for a uniform MCAT emitter and nonuniform MCAT attenuator. Top and bottom rows: No attenuation (left), nonuniform attenuation (right).

Fig. 7.
FBP reconstruction of the uncorrected, non-uniformly attenuated projections of the uniform MCAT background (see Fig. 1, top right image)