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Air-coupled ultrasound stimulated optical vibrometry for resonance analysis of rubber tubes

Xiaoming Zhang^{a)}, Randall R. Kinnick, and James F. Greenleaf

Department of Physiology and Biomedical Engineering, Mayo Clinic College of Medicine, 200 First Street SW, Rochester, MN 55905

Abstract

Air-coupled ultrasound stimulated optical vibrometry is proposed to generate and detect the resonances of a rubber tube in air. Amplitude-modulated (AM) focused ultrasound radiation force from a broadband air-coupled ultrasound transducer with center frequency of 500 kHz is used to generate a low frequency vibration in the tube. The resonances of several modes of the tube are measured with a laser vibrometer of 633 nm wavelength. A wave propagation approach is used to calculate the resonances of the tube from its known material properties. Theoretical and experimental resonance frequencies agree within 5%. This method may be useful in measuring the in vitro elastic properties of arteries from the resonance measurements in air. It may also be helpful in better understanding the coupling effects of surrounding tissue and interior blood on the vessel wall by measuring the resonance of the vessel in vitro and in vivo.

Keywords

Noncontact; resonance; tube; air-coupled ultrasound; vibrometry

1. Introduction

Cardiovascular disease (CVD) has been the number one killer in the United States [1]. It has long been recognized that a high percentage of all cardiovascular disease is associated with the hardening of arteries or arteriosclerosis [2]. Arteriosclerosis involves the buildup of plaque within the artery walls, which causes the thickening and hardening of arteries. Several imaging techniques, including x-ray angiography, ultrasound, magnetic resonance imaging (MRI), etc., are available for imaging atherosclerotic plaques [3]. Ultrasound sonography is unique in quick and efficient identification of stenosis in carotid or peripheral arteries. B-mode gray scale sonography allows for imaging of atherosclerotic plaques and intima-media thickness (IMT). Color Doppler sonography allows simultaneous real-time visualization of vascular lesions, associated flow abnormalities and spectral Doppler hemodynamic analysis [4]. Most imaging techniques measure luminal diameter, wall thickness and plaque volume. However, no current imaging methods can measure the material properties of the vessel wall.

Estimation of arterial stiffness can be derived from pulse contour analysis, pulse wave velocity (PWV) measurements, or analysis of arterial diameter and blood pressure (relating change in

a)corresponding author: zhang.xiaoming@mayo.edu.

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diameter of an artery to distending pressure) [5]. PWV is a simple and robust index for application in clinical settings. Usually, PWV is measured by the time of travel of the “foot” of the cardiac induced pulse pressure wave over a known distance [6]. The “foot” is defined as the point, at the end of diastole, when the steep rise of the pressure begins. The measured pulse wave velocity is an average parameter between the two measurement points. The measured length, for example the aorta pulse wave velocity measurement from carotid – femoral arteries, is typically a few hundred millimeters, therefore, it is difficult to evaluate the local arterial stiffness and its variation.

The long-term goal of our research is to non-invasively measure and image local artery wall material properties with high accuracy using our novel ultrasound stimulated vibro-acoustic methods. Towards to this goal, we have developed several novel methods for non-invasively estimating elastic properties of arteries. One of the methods is to generate a bending wave on the arterial vessel and then to calculate the elastic modulus from the wave speed [7-9]. We have also developed a noncontact ultrasound stimulated optical vibrometry method for studying the coupled vibration of arterial tubes embedded in a tissue-mimicking gelatin phantom [10]. However, in this case the measured resonance depends on the characteristics of the tube, the surrounding gelatin, and the fluid inside the tube. It is found that the tissue-mimicking gelatin significantly affects the estimation of stiffness of rubber tubes and arteries [11]. A tube's vibration can also be tested in air with a mechanical shaker [12]. However, the mechanical shaker may introduce significant effects in the case of a vessel. Because the vessel is much softer than a rubber tube, the physical connection between a shaker and a vessel may change the dynamics of the vessel. Here we studied the use of the air-coupled ultrasound transducer to generate the resonance of a rubber tube. Both the generation and detection of the tube vibration are noncontact, so that the dynamic behavior of the tube can be measured without any coupling effects. This method can be used to measure the resonance of ex vivo arteries. It can also be used to study the coupling effects of in vivo arteries with surrounding tissue and blood by comparing the ex vivo and in vivo resonance results.

2. Theory

The rubber tube is considered as a three-dimensional elastic cylindrical shell with thickness h , median radius R , and length L . The three-dimensional displacements of the tube are defined by u , v , w in the x , θ , z directions, respectively. The reference surface of the shell is taken to be at its middle surface. The x coordinate is taken in the longitudinal direction, where the θ and z coordinates are respectively in the circumferential and radial directions of the tube as shown in Figure 1. The equations of motion for the cylindrical shell can be written by Love's theory as [13,14]

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (1)$$

where

$$\begin{aligned} L_{11} &= R^2 \frac{\partial^2}{\partial x^2} + \frac{1-\nu}{2} \frac{\partial^2}{\partial \theta^2} - \gamma \frac{\partial^2}{\partial r^2}, L_{12} = R \frac{1+\nu}{2} \frac{\partial^2}{\partial x \partial \theta}, L_{13} = R \nu \frac{\partial}{\partial x}, L_{21} = R \frac{1+\nu}{2} \frac{\partial^2}{\partial x \partial \theta}, \\ L_{22} &= R^2 \frac{1-\nu}{2} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \theta^2} + \beta \{ (1-\nu) R^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \theta^2} \} - \gamma \frac{\partial^2}{\partial r^2}, L_{23} = \frac{\partial}{\partial \theta} - \beta (R^2 \frac{\partial^3}{\partial x^2 \partial \theta} + \frac{\partial^3}{\partial \theta^3}), \\ L_{31} &= -R \nu \frac{\partial}{\partial x}, L_{32} = -\frac{\partial}{\partial \theta} - \beta \{ (2-\nu) R^2 \frac{\partial^3}{\partial x^2 \partial \theta} + \frac{\partial^3}{\partial \theta^3} \}, \\ L_{33} &= -1 - \beta (R^4 \frac{\partial^4}{\partial x^4} + 2R^2 \frac{\partial^4}{\partial x^2 \partial \theta^2} + \frac{\partial^4}{\partial \theta^4}) - \gamma \frac{\partial^2}{\partial r^2}, \end{aligned}$$

where $\beta = \frac{h^2}{12R^2}$, $\gamma = \frac{\rho R^2(1-\nu^2)}{E}$, E is Young's modulus, ρ is the mass density, and ν is Poisson's ratio of the tube material.

The resonance of the cylindrical tube can be solved by the wave propagation approach [13, 14]. In this approach, the displacements of the tube can be expressed in the format of wave propagation, associated with an axial wavenumber k_x and circumferential mode parameter n , and defined by

$$\begin{aligned} u &= U_m \cos(n\theta) e^{(i\omega t - ik_x x)} \\ v &= V_m \sin(n\theta) e^{(i\omega t - ik_x x)} \\ w &= W_m \cos(n\theta) e^{(i\omega t - ik_x x)} \end{aligned} \quad (2)$$

where U_m , V_m and W_m are respectively the wave amplitudes in the axial x , circumferential θ , and radial z directions, and ω is the circular driving frequency.

Substituting equation (2) into equation (1) gives

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} U_m \\ V_m \\ W_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

where C_{ij} ($i, j=1, 2, 3$) are the parameters from the L_{ij} after they are operated with the x and θ .

Expansion of the determinant of above equation provides the system characteristic equation

$$F(k_x, \omega) = 0 \quad (4)$$

where $F(k_x, \omega)$ is a polynomial function. This characteristic function can be used to investigate the wave propagation in the shell as well as the resonance of the shell.

In this paper, equation (4) is used to calculate the resonance frequencies of the finite shell. In the wave propagation approach, the axial wavenumber k_x is chosen to satisfy the required boundary conditions at the two ends of the cylindrical shell. For example, $k_x = m\pi / L$ is used for the simply supported-simply supported (SS-SS) boundary conditions at the ends. Recommended wavenumbers can be found in [13,14] for different boundary conditions. One advantage of the wave propagation approach is that complex boundary conditions can be easily handled which may be difficult for other analytical methods.

With known k_x for specific boundary conditions, equation (4) can be written as

$$\omega^6 + \beta_1 \omega^4 + \beta_2 \omega^2 + \beta_3 = 0 \quad (5)$$

where β_i ($i=1, 2, 3$) are the coefficients of equation (5). Solving equation (5), the resonance frequencies of the cylindrical shell are obtained.

3. Experiment

The air-coupled ultrasound stimulated optical vibrometry system uses the radiation force of ultrasound in air to vibrate an object. The vibration of the object is measured with a laser vibrometer. A schematic of the system is shown in Fig. 2. A custom-built commercial broadband air-coupled transducer or BATtm (MicroAcoustic Instruments Inc., Ottawa, Ontario, Canada) was used to generate a localized force in the tube. The transducer has a focal length of 70 mm (or ~200 μ sec in air), and an effective aperture of 30 mm. The center of the tube was placed in the focal plane of the transducer. The diameter of the ultrasound beam was about 0.7 mm in the focal plane which generated almost a point force on the tube. The center frequency of the transducer was 500 kHz. To generate the low frequency vibration of the tube, an amplitude-modulated (AM) signal was generated by an HP 33120A function generator (Agilent Technologies, Inc., Santa Clare, CA). The AM signal was then amplified by a RF amplifier (ENI 240L) and sent to the air-coupled ultrasound transducer. Transducer voltage

was 150 Vp-p. The vibration response of the tube was measured with a Polytec VibraScan Laser Vibrometer system (Polytec, Inc., Tustin, CA). This laser vibrometer uses the helium neon laser of 633 nm wavelength and the Doppler effect to measure the vibration of the tube. The laser was synchronized with the modulation.

Experimental studies were carried out on a rubber tube (Kent Elastomer Products, Inc., Kent, OH). The tube was clamped to rigid connectors at each end and mounted in a three-sided aluminum/acrylic frame as shown in Fig. 3. The length, outer diameter and thickness of the tube were respectively 310 mm, 5 mm and 1 mm. The tube material had a mass density $\rho = 960 \text{ kg/m}^3$, Poisson's ratio $\nu = 0.44$ and Young Modulus $E = 1.1 \times 10^6 \text{ N/m}^2$. The frequency spectrum of the tube was measured by sweeping the AM signal over the frequency range of interest from which the resonance frequencies were determined.

4. Results and discussion

The resonance frequencies of the rubber tube are calculated and shown in Fig. 4 for the circumferential mode n ranging from 1 to 3 and the axial mode m ranging from 1 and 7. For a given n , the frequency increases with the mode parameter m . In our experiments, we are interested in the frequency range between 50 Hz and 200 Hz. Higher resonance modes may not be generated with sufficient signal to noise ratios. We do not test frequencies below 50 Hz, because there might be strong electronic and mechanical noise. It is shown in Fig. 4 that the frequencies associated with $n=2$ are higher than 1000 Hz. Therefore, in the frequency range of interest the measured frequencies of the tube are only associated with mode parameter $n=1$.

Fig. 5 shows the calculated resonance frequencies of the tube for $n=1$ and different m between 0 Hz and 300 Hz. Eight resonance frequencies are found between 50 Hz and 200 Hz. These resonance frequencies are associated with $n = 1$ for $m = 8$ to 15.

One advantage of the wave propagation approach is that the boundary conditions can be easily handled for calculating the resonance. Table 1 shows the wave numbers for five different boundary conditions, ranging from simply supported-simply supported (SS-SS) to clamped-clamped (C-C) according to different stiffness of the torsion springs [14]. In this model, a non-dimensional stiffness parameter kL / EI is used, where k is the rotational stiffness of a spring in Nm/rad and I is the area moment of inertia of the tube. $(k_1L / EI, k_2L / EI) = (0, 0)$ corresponds to the SS-SS while $(k_1L / EI, k_2L / EI) = (\infty, \infty)$ corresponds to C-C boundary conditions.

Table 2 shows the comparison of the eight resonance frequencies between the C-C boundary condition and the SS-SS boundary condition. It is obvious that the resonance frequency of the C-C boundary condition is higher than the resonance frequency of the SS-SS boundary condition for the same order. Since the results of C-C boundary condition are closer to the experimental setup, we compare the calculated C-C results with the experiments.

Fig. 6 shows the experimental frequency spectrum at a point of the tube under the excitation of the air-coupled ultrasound transducer. The frequency range of interest was 50 Hz to 200 Hz. The frequency resolution can be adjusted in the vibrometer system by choosing the sampling frequency and number of samples of the fast Fourier transform. The frequency resolution was 1.56 Hz in this experiment. In the frequency range of interest seven resonance peaks can be identified which are the resonance frequencies of the tube. They are measured as, respectively, 60 Hz, 88 Hz, 110 Hz, 120 Hz, 133 Hz, 158 Hz, and 181 Hz. There is an anti-resonance mode at 74.3 Hz. This may be because either the force or the measurement point being at the node of the resonance mode.

The measured seven resonance frequencies are compared to the calculated frequencies in Table 3. The relative differences between calculation and experiment are expressed in percentage in Table 3. The experimental results agree within 5% of the calculated values.

5. Conclusion

Resonances of rubber tubes can be generated by the radiation force from an air-coupled ultrasound transducer and measured by a laser vibrometer. Both the generation and detection are noncontact, therefore the elastic property of tubes can be evaluated without any physical coupling effects. Theoretical and experimental results agree within 5%. This method may be useful in measuring the in vitro elastic properties of arteries from the resonance measurements in air. It may also be helpful in better understanding the coupling effects of surrounding tissue and interior blood on a vessel by measuring the resonance of the vessel in vitro and in vivo.

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Fig. 1.
Co-ordinate system and circumferential modal shapes.

Fig. 2. Schema of the air-coupled ultrasound stimulated optical vibrometry system for measuring the resonance of a tube.

Fig. 3. Experimental setup in which a rubber tube is excited by an air-coupled ultrasound transducer and the vibration of the tube is measured by a laser vibrometer system.

Fig. 4. Calculated resonance frequencies of the tube with $n = 1$ to 3 and $m = 1$ to 7.

Fig. 5.
Calculated resonance frequencies of the tube with $n = 1$ and $m = 1$ to 20.

Fig. 6.
Experimentally measured frequency spectrum of the tube under the excitation of the air-coupled ultrasound transducer.

Table 1
Wave number for a tube with torsion springs at both ends

$k_1 \frac{L}{EI}$	$k_2 \frac{L}{EI}$	m=1	m=2	m=3
Wave number $k_x L$				
0	0	3.142	6.283	9.425
1	1	3.398	6.427	9.524
10	10	4.155	7.068	10.065
100	100	4.641	7.710	10.801
∞	∞	4.730	7.854	10.995
				m=4
				12.566
				12.642
				13.105
				13.894
				14.137

Table 2
Comparison of calculated resonance frequencies of the tube between the clamped-clamped (C-C) boundary conditions and simply supported-simply supported (SS-SS) boundary conditions

	Frequency (Hz)							
	1	2	3	4	5	6	7	8
C-C	61.2	74.3	88.5	103.9	120.2	137.5	155.7	174.8
SS-SS	55.1	67.6	81.3	96.1	111.9	128.8	146.5	165.1

Table 3
Comparison of calculated and experimentally measured resonance frequencies of the tube

	Frequency (Hz)						
	1	2	3	4	5	6	7
Calculation	61.2	88.5	103.9	120.2	137.5	155.7	174.8
Experiment	59.4	87.5	109.3	120.3	132.8	157.8	181.2
Difference (%)	3.0	1.1	4.9	0.0	3.5	1.3	3.5