Published in final edited form as:

IEEE Trans Nucl Sci. 2008 ; 55(3): 942–952. doi:10.1109/TNS.2008.924064.

Realistic PET Monte Carlo Simulation With Pixelated Block Detectors, Light Sharing, Random Coincidences and Dead-Time Modeling

Bastein Guérin [Student Member, IEEE] and Georges El Fakhri [Member, IEEE]

B. Guérin is with the University of Paris VI, 75006 Paris, France and also with Harvard University, Cambridge, MA 02138 USA. (e-mail: guerin@pet.mgh.harvard.edu).

G. El Fakhri was with Harvard Medical School and Brigham and Women's Hospital, Boston MA 02115 USA. He is now with Harvard Medical School and Massachusetts General Hospital, Boston, MA 02114 USA.

Abstract

We have developed and validated a realistic simulation of random coincidences, pixelated block detectors, light sharing among crystal elements and dead-time in 2D and 3D positron emission tomography (PET) imaging based on the SimSET Monte Carlo simulation software. Our simulation was validated by comparison to a Monte Carlo transport code widely used for PET modeling, GATE, and to measurements made on a PET scanner.

Methods—We have modified the SimSET software to allow independent tracking of single photons in the object and septa while taking advantage of existing voxel based attenuation and activity distributions and validated importance sampling techniques implemented in SimSET. For each single photon interacting in the detector, the energy-weighted average of interaction points was computed, a blurring model applied to account for light sharing and the associated crystal identified. Detector dead-time was modeled in every block as a function of the local single rate using a variance reduction technique. Electronic dead-time was modeled for the whole scanner as a function of the prompt coincidences rate. Energy spectra predicted by our simulation were compared to GATE. NEMA NU-2 2001 performance tests were simulated with the new simulation as well as with SimSET and compared to measurements made on a Discovery ST (DST) camera.

Results—Errors in simulated spatial resolution (full width at half maximum, FWHM) were 5.5% (6.1%) in 2D (3D) with the new simulation, compared with 42.5% (38.2%) with SimSET. Simulated (measured) scatter fractions were 17.8% (21.3%) in 2D and 45.8% (45.2%) in 3D. Simulated and measured sensitivities agreed within 2.3 % in 2D and 3D for all planes and simulated and acquired count rate curves (including NEC) were within 12.7% in 2D in the [0: 80 kBq/cc] range and in 3D in the [0: 35 kBq/cc] range. The new simulation yielded significantly more realistic singles' and coincidences' spectra, spatial resolution, global sensitivity and lesion contrasts than the SimSET software.

Index Terms

Block detectors; dead-time; light sharing; Monte Carlo simulation; positron emission tomography; random coincidences

I. Introduction

Monte Carlo simulation allows accurate modeling of photon interactions in the object and the PET detector, while yielding perfect knowledge of the underlying reference activity and

attenuation distributions as well as the distributions of scattered, unscattered and random coincidences. Simulation System for Emission Tomography (SimSET) [1] is a free software package available from the University of Washington Imaging Research Laboratory that models voxel-based activity and attenuation distributions and continuous PET detectors operating in 2D and 3D modes. Previous studies have validated the modeling of photon transport in non-uniform attenuation distributions with SimSET [2]–[4] but reported a systematic overestimation of the performances of PET scanners based on the block design (spatial resolution [3], [5]; scatter fraction [3]–[6]; sensitivity [3], [5], [6]) because of the absence of modeling of pixelated detectors, light-sharing among crystal elements, random coincidences and detector dead-time in SimSET. In PET scanners utilizing the block-design scheme [7], spatial resolution is fundamentally limited by the size of crystal elements [8] and the light-sharing read-out which makes use of less than one photo-multiplier tube (PMT) per crystal element [9], while the system sensitivity is degraded by the presence of gaps between blocks [10]. Modeling these detector effects would yield more realistic simulations of the detector response at low count rates. Modeling dead-time and the distribution of random coincidences for specific attenuation/activity distributions and activity levels allows accurate modeling of acquisitions at high count rates as well as the noise equivalent count rate (NEC), a metric widely used to assess PET scanner performance [11].

In this work, we modified the SimSET software to include modeling of block detectors and crystals, random coincidences and detector dead-time and used it to simulate the Discovery ST PET scanner (General Electric Medical Systems, Milwaukee, WI). We compared our simulation with the original version of SimSET, the validated Monte-Carlo software GATE [23] and with measurements made on a DST camera.

II. Description of the Monte-Carlo Model

A. Photon Propagation

1) Photon Generation—SimSET was used to generate coincidence events and simulate positron range as well as photon non-collinearity. Photon transport was modeled in two steps. The first step consisted of particle transport through the object and septa and the second step consisted of propagation through the detector, modeling of detector effects and binning.

2) Propagation of Photons in the Object and Septa—We modified the currently available version of SimSET to allow independent tracking of photons originating from the same annihilation event, i.e., single photons were simulated even if their annihilation counterparts were not detected. This is currently not performed in SimSET to reduce the number of events to track, and thus to reduce simulation time. However, this is essential to estimate accurately single events in addition to coincidence events, which is crucial to compute dead-time and random coincidences. Previously validated importance sampling methods implemented in SimSET, i.e., non-absorption, forced detection and stratification [12], were used for tracking photons in the object in order to optimize simulation time. Propagation of photons through the septa was modeled with Monte-Carlo techniques and accounted for septal penetration, photoelectric absorption, Compton and coherent scattering. All single events reaching the detector were stored in a list-mode file organized by coincidences.

3) Propagation in the Detector—The list-mode file obtained after tracking in the object was used as input in the second step of the simulation, which modeled propagation in the blocks. Only one block of the full PET detector was modeled. Incoming photon tracks were rotated into the block frame of reference and, after propagation in the detector, were rotated back into their original frame. Propagation in other blocks was modeled by rotating photons' tracks into the new block's frames each time they entered a new block, thereby allowing photons to scatter in several contiguous blocks. However, backscatter followed by detection in an opposite block

was not modeled because tracking of photons in the object and the detector were performed separately, allowing only one passage through the attenuation distribution. The error associated with the absence of modeling of backscatter events was evaluated using GATE. Corresponding results are shown in Section IV-A-2-a. Defining one block rather than the full ring minimized memory requirements and defined the geometrical characteristics of one block with greater accuracy. Because of the use of forced detection when tracking photons in the object, coincidence events in the input list-mode data could eventually consist of more than two photons [12]. In this case, all photons were propagated independently in the detector and every possible combination of two photons originating from the same annihilation was considered a valid coincidence with an associated statistical weight equal to the product of the two photons' weights, since the two photon histories were independent. The energy deposited in each block was computed as

$$E_{dep} = \sum_i E_{dep}^i \quad (1)$$

where index i runs over interactions inside one block. The position of the interaction within the block was determined as

$$X^j = \left(\sum_i E_{dep}^i X_i^j \right) / E_{dep} \quad (2)$$

where X_i^j is the j th Cartesian coordinate of the i th interaction point. Finite energy resolution of the system was modeled using a Gaussian blurring

$$FWHM = FWHM_{511} \sqrt{511 \times E_{dep}}. \quad (3)$$

Small variations of the energy resolution within a block were neglected and the simulated resolution of the system at 511 keV was set to 20% for the DST as reported in [13].

B. Modeling the Block Intrinsic Spatial Resolution/p

Even when saw cuts are filled with reflective material to reduce optical cross-talk between crystal elements, flood source measurements show imperfect separation of the crystal domains due to light-sharing at the base of the block in real systems [14]. For a fixed number of photons emitted per gamma photon interacting in the detector, the number of low energy photons (scintillation photons) impinging on a PMT is a random variable and the subsequent measure of the deposited energy (E_{dep}) is also random. Uncertainty in the measurement of E_{dep} adds uncertainty to locating the interaction's position inside the block [7], [15]. Let PMT A and B be located on the left ($x < 0$) and the right ($x > 0$) of the X axis, respectively (Fig. 1). Assuming a perfect detector, the probabilities of detecting a low energy photon in PMT A (p_A) or PMT B (p_B) follows:

$$p_A + p_B = 1. \quad (4)$$

Therefore the total number of photons detected in PMT A for a given interaction location follows a binomial distribution. As proposed in [16], we assumed that the interaction of 511 keV photons in the BGO detector created on average 150 scintillation photons, which reflects the ~20% energy resolution of the DST. In this case, the binomial can be approximated by a Gaussian. The position signal, which is proportional to the responses of the PMTs, is thus also a Gaussian function. By forcing the center of the crystal's Gaussian response to be at the center of the crystal in which the interaction occurred, one can derive the values of p_A for every crystal (for the six crystals of the DST the values of p_A are from left to right, 11/12, 9/12, 7/12, 5/12, 3/12 and 1/12). In this calculation, one-dimensional crystals' point spread functions are Gaussian with FWHMs equal to 2.0 mm (left edge crystal), 3.1 mm, 3.6 mm, 3.6 mm, 3.1 mm

and 2.0 mm (right edge crystal), as shown in Fig. 1. This model shows that light-sharing among crystal elements is greatest at the center of the block.

C. Dead-Time Modeling

The following equation, proposed by Moisan *et al.* [17], was used along with dead-time-free photon fluence estimated in a small run to calculate the dead-time fractions in every block

$$\%DT_i = 100 \left[1 - \frac{e^{-\tau_{block} \times n_i}}{1 + \tau_{block} \times n_i} \right] \quad (5)$$

where $\%DT_i$ is the dead-time fraction in block i , n_i is the dead-time free photon flux on block i and τ_{block} is the intrinsic block dead-time. This model accounts for paralyzable losses due to pulse pile-up (paralyzable component) and non-paralyzable losses due to finite integration time of pulses (non-paralyzable component). Instead of randomly rejecting photons according to the dead-time fractions computed with (5), a variance reduction technique was used: all events were detected but a statistical weight smaller than one was attributed to them in order to reflect their probability of not being detected due to dead-time. Because the DST does not have buckets, a single coincidence circuitry processes all detected coincidences. Dead-time in the coincidence processor was modeled with a non-paralyzable model applied to the total prompt coincidences rate [17]

$$\%DT_{elec} = 100 \left[1 - \frac{1}{1 + \tau_{elec} \times n_t} \right] \quad (6)$$

where $\%DT_{elec}$ is the dead-time fraction in the electronic circuitry, n_t is the total prompt coincidence rate and τ_{elec} is the intrinsic electronic dead-time. Dead-time losses were modeled by multiplying dead-time-free coincidence rates by the live-time fraction of the coincidence processor (equal to $1 - \text{dead-time fraction}$), as proposed by Eriksson *et al.* [18]. This model can be easily extended to simulate scanners with buckets by attributing to each bucket the same electronic dead-time parameter. Dead-time in each bucket would then be a variable of the number of coincidences processed by this bucket per second, and would vary from one bucket to the other.

D. Simulation Output

At the end of the simulation, events were either binned in sinograms or stored in a list-mode file. True and scatter coincidences could be binned in separate sinograms. Transaxial binning was performed using the position of detected photons so that sinograms were not affected by curvature effects. The entire 3D dataset was stored without axial compression in ring-ring format. We used the single slice-rebinning algorithm [19] in 2D and the Fourier rebinning algorithms [20] in 3D to collapse the 3D dataset in a 2D dataset. When list-mode was used, the position, energy, statistical weight, detector identification number and number of scatters in the object for every photon were recorded in a file. A time-stamp was not included to detected events. Instead, coincidences were simulated sequentially and photons pertaining to the same coincidence were coupled in the list-mode file. As a result, random coincidences could not be estimated explicitly. Instead, the average number of random coincidences in every LOR in the FOV was calculated from single rates using the following formula [21]

$$R_{ij} = \frac{CTW}{T_{acq}} S_i S_j \quad (7)$$

the result of which was binned in a separate sinogram (CTW is the coincidence time window of the system, T_{acq} is the acquisition length and S_i is the number of single photons detected in the i th crystal element). The distribution of singles was also used to compute dead-time fractions in each block, as explained in Section II-C.

III. Validation of the Model

Validation of our simulation approach was performed in two steps. In the first step, we compared energy spectra obtained with our simulation and the GATE software. In the second step, we compared simulations of the NEMA performance test [22] with the corresponding measurements made on a PET scanner. This allowed us to validate the accuracy of our simulator for predicting the performances of a PET scanner in 2D and 3D mode.

A. Comparison With GATE

1) The GATE Software—GATE is a widely used and validated Monte Carlo software for emission tomography based on Geant4 [23] that models photon and electron transport as well as X-ray and delta-ray production. It models a wide range of acquisition geometries and read out schemes used in PET. The main limitation of GATE is the absence of importance sampling methods that yields very long simulation times when modeling complex activity distributions with a clinically relevant number of counts [24].

2) Simulations

a) Validation of basic tracking capabilities: Independent photon tracking in the object was validated by comparing energy spectra obtained with GATE and our simulation. We modeled a point source in four 70 cm long cylinders of 12, 20, 40 and 60 cm diameters, filled with water. X-ray production was disabled in GATE. Spectra were also computed using SimSET.

b) Validation of tracking in the detector: Propagation of photons in the pixelated detector was validated by comparing energy spectra obtained with GATE and our simulation when modeling the DST. A point source in air and in a 70 cm long/20 cm diameter cylinder filled with water was modeled. X-ray production (from Bismuth and Tungsten) was disabled in GATE. Spectra were also computed using the currently available version of SimSET. We also compared absolute single rates detected with GATE and the new simulation, since these are used to model dead-time and random coincidences.

B. Comparison With Measurements

1) The Discovery ST PET Scanner—All acquisitions were made with the DST PET scanner, the characteristics of which are shown in Table I.

2) Spatial Resolution—Spatial resolution of the DST in 2D and 3D was measured at 1 cm and 10 cm by imaging/simulating two point sources filled with ^{18}F FDG and positioned at 1 cm and 10 cm from the center of field of view, as prescribed in [22]. In measurements, both the random coincidences rate and dead-time losses were smaller than 5% of the total event rate. Spatial resolution was estimated with our simulator (random and dead-time were not modeled) as well as with SimSET.

3) Sensitivity—Absolute sensitivity was measured by imaging a 70 cm long ^{18}F FDG-line source inserted into several aluminum sleeves of decreasing thickness, as prescribed in [22]. Sensitivity profiles were obtained by calculating the sensitivity in each plane. In measurements, random coincidences rate and dead-time losses were kept below 1 % of the total event rate. In simulation, a perfect line source was modeled and randoms and dead-time were not modeled.

4) Count Rates Performance and Scatter Fraction—The 70 cm long/20 cm diameter polyethylene cylindrical test phantom containing a ^{18}F FDG-line source located at a distance of 45 mm from the center of the phantom was scanned over a period of 12 hours and imaged repeatedly in 2D and 3D modes for each activity point, as prescribed in [22]. In simulations, random coincidences were modeled. Dead-time was modeled with and without using the

variance reduction technique described in Section II-C and corresponding count rate curves were compared. Count rates and NEC were not estimated with SimSET since SimSET does not model random coincidences. In simulations and measurements, the system scatter fraction was estimated from sinogram profiles according to the NEMA procedure [22]. In measurements, random coincidences rate and dead-time losses were kept below 1 % of the true coincidence count rate. In simulations, the scatter fraction was also calculated from the perfect knowledge of true and scattered coincidences. Randoms and dead-time were not modeled. The patient bed, which we segmented from a CT scan, was modeled. Scatter fraction was also evaluated using SimSET.

5) Image Quality—Image quality in 2D and 3D modes was assessed using the IEC torso phantom. The background of the phantom was filled with 0.14/ μ Ci/cc (5.3 kBq/cc) of 18 FDG, hot spheres were filled with an activity equal to 4 times the background activity and cold spheres were filled with no activity, as prescribed in [22]. Contrasts were measured on simulated and acquired images following the NEMA procedure [22]. Signal to noise ratio (SNR) was calculated using twelve 17 mm ROIs placed on the background of the central slice of the reconstructed volume

$$SNR = \frac{1}{12} \sum_{i=1}^{12} \frac{m_i}{\sigma_i} \quad (8)$$

where m_i and σ_i are the mean and standard deviation of ROI i , respectively. Images acquired on two DST cameras were scatter corrected using the Bergstrom convolution method in 2D mode [25] and an analytic model-based technique in 3D mode [26] available on the scanner. Simulated images contained only true coincidences since we assumed perfect scatter rejection. Sinograms were corrected for attenuation and reconstructed using the ordered subsets expectation maximization (OSEM) algorithm [27] with 20 subsets and 2 iterations. In 2D, low noise sinograms were generated using variance reduction techniques in order to reduce simulation time. Poisson deviates were generated from very low noise sinograms before reconstruction yielding images with similar noise level (as measured by the SNR) than in measured images. In 3D, variance reduction techniques were not used in order to properly account for propagation of Poisson noise in the FORE algorithm (FORE uses linear combinations of the sinogram projections and since $Poisson(\lambda a) \neq \lambda Poisson(a)$, it is not equivalent to add noise before and after rebinning). Therefore, the noise level in 3D simulations was directly related to the number of photons simulated.

C. Simulations With SimSET

When possible, simulations performed with the new simulation were also performed with SimSET. The scanner modeled with SimSET was as close as possible to the real DST scanner, containing four continuous annular BGO detectors corresponding to the four detector rings of the DST and with energy resolution of 20% at 511 keV.

D. Errors

The errors between simulated data $\{S_i\}_{i=1\dots N}$ and measurements $\{M_i\}_{i=1\dots N}$ reported in this work were calculated as follows:

$$err = \frac{100}{N} \sum_{i=1}^N \frac{|S_i - M_i|}{M_i}. \quad (9)$$

IV. Results

A. Comparison With GATE

1) Validation of Tracking in the Object—Fig. 2 shows energy spectra obtained with the new simulation, SimSET and GATE (reference) when simulating a point source in a 12 cm (a), 20 cm (b), 40 cm (c) and 60 cm (d) diameter cylinder filled with water. The photopeaks of spectra obtained with the new simulation and SimSET, defined as $E_{\text{photo}} = 511 \pm 3.5$ keV, were scaled to GATE. Since this validation step was dedicated to propagation in the object, no detector was modeled and all photons were detected with perfect energy resolution. Discrepancies between the new simulation and GATE (SimSET and GATE), averaged over all energy bins, were **a**: $1.5 \pm 2.9\%$ ($2.6 \pm 2.9\%$), **b**: $0.3 \pm 2.4\%$ ($1.7 \pm 2.4\%$), **c**: $0.4 \pm 2.5\%$ ($0.1 \pm 2.5\%$) and **d**: $1.2 \pm 2.9\%$ ($3.3 \pm 2.9\%$), respectively. These errors are not statistically significant for SimSET nor for the new simulation.

2) Validation of Tracking in the Detector

a) Estimation of backscatter events: We assessed the error associated with the absence of modeling of events that backscattered in one block and scattered in an opposite block (Section II-A-3) by estimating the proportion of these events using the GATE software. The object modeled was a point source in air imaged in 2D and 3D mode. Among all detected events, the proportions of backscatter events in the [375: 650 keV] energy range were 0.12% in 3D and 0.04% in 2D. Because of attenuation of backscattered events, we expect the error introduced by neglecting this effect to be smaller than 0.12% in 2D and 3D when modeling an attenuating medium.

b) Energy spectra: Fig. 3 shows single photon energy spectra obtained with the new simulation and GATE (reference) when modeling the DST with perfect energy resolution. Spectra are decomposed in their zeroth, first, second and third order components, corresponding to primary photons and photons that interacted once, twice and three times in the object, respectively (unlike other spectra, orders in spectrum **a** are the numbers of scatters in the detector, not in the object). Errors of the new simulation, shown in Fig. 3, are smaller than 5%. Features marked with arrows were reproduced identically by GATE and the new simulation. Spectra were also computed with SimSET. However, discrepancies between GATE and SimSET were significant (**a**: 55.8%, **b**: 41.9%, **c**: 47.1%, **d**: 45.6%), which is due to the fact that SimSET does not model single photons but only coincidences.

Fig. 4 shows coincidence spectra simulated with the new simulation, SimSET and GATE when modeling the DST with energy resolution equal to 20% at 511 keV and a point source in water. Whereas Fig. 3 contains all detected photons, only photons detected in coincidence are plotted in Fig. 4. Coincidence spectra are decomposed into their zeroth (primary coincidences), first (single scatter distribution, SSD) and multiple scatter component (multiple scatter distribution, MSD). Errors associated with the new simulation (SimSET) are 4.4% (41.3%), 2.9% (11.0%) and 4.4% (6.2%), for the primary distribution, SSD and MSD, respectively. Modeling block detectors had a greater effect on the primary distribution than on the SSD and MSD since it characterizes the detector, whereas the SSD and MSD characterize the object imaged.

c) Single rates: It is crucial to estimate accurately single events since these are used to compute dead-time (Section II-C) and random coincidences (Section II-D). We compared single rates obtained in the DST PET scanner with GATE and the new simulation while simulating a point source in air and in water (the same numbers of photons were simulated in all simulations and distributions obtained with GATE and the new simulation were not scaled to each other). Discrepancies between distributions, averaged over single rates obtained in every block, were

2.3% (2D, air), 3.1% (2D, water), 1.9% (3D, air) and 2.5% (3D, water). Single distributions were not calculated with SimSET since it does not model blocks.

B. Comparison With NEMA Measurements

1) Spatial Resolution—Table II shows the measured and simulated spatial resolutions of the DST. Average errors associated with the new simulation (SimSET), calculated using (9), were 5.5% (42.5%) in FWHM values and 8.7% (40.9%) in full width at tenth maximum (FWTM) values in 2D and 6.1% (38.2%) FWHM and 7.5% (33.9%) FWTM in 3D. Modeling pixelated detectors and light sharing reduced the error on spatial resolution from more than 38% in FWHM values (SimSET) to less than 7% (new simulation), allowing determination of the point spread function with accuracy better than 0.8 mm FWHM and 2.0 mm FWTM in every direction, at 1 and 10 cm and in 2D and 3D mode.

2) Scatter Fraction—Measured and simulated scatter fractions (SF) shown in Table in were estimated from sinogram profiles according to the NEMA procedure [22]. The Monte Carlo scatter fraction, estimated using the primary (un-scattered) events and scattered events, was 21.2% (respectively 20.3%) in 2D and 46.7% (respectively 45.7%) in 3D with the new simulation (SimSET). In 2D, the NEMA procedure underestimated the actual scatter fraction because scatter profiles did not vary linearly between -2 cm and $+2$ cm as assumed in [22]. Instead, it increased and then decreased, peaking at 0 cm, due to coincidences that interacted only once in the collimator and were thus only slightly deviated from their original trajectory (septa are close to the detector). The 3D scatter fraction was accurately estimated by the NEMA procedure. SimSET underestimated both the NEMA and Monte Carlo scatter fractions because of the absence of modeling of blocks and gaps [5]. Underestimation of the 2D NEMA scatter fraction by the new simulation is most likely due to inaccuracies in our modeling of the patient bed (segmented from a CT scan) and absence of modeling of PMTs and some of the moving parts of the septa.

3) Sensitivity—Measured and simulated axial sensitivity profiles are shown in Fig. 5.

Total sensitivity in 2D (3D) was 2.10 cps/kBq (9.5 cps/kBq) in measurements; 2.98 cps/kBq (12.12 cps/kBq) with the new simulation and 3.31 cps/kBq (14.17 cps/kBq) with SimSET, respectively. Modeling blocks and air gaps allowed reproducing the total sensitivity more accurately than SimSET, which models the PET detector as four uniform cylinders of BGO (detector rings). Our simulation overestimated the total sensitivity because of the absence of modeling of PMTs and imperfect sensitivity of individual crystal elements that further degrade the performance of the real system. Error on axial profiles averaged over planes were, after scaling the simulated total sensitivity to the measured one, 2.3% (1.8%) in 2D and 1.8% (1.4%) in 3D mode for the new simulation (SimSET).

4) Count Rates Performance—A direct estimation of τ_{block} obtained by fitting the single rates with the dead-time model presented in Section II-C was not feasible because we only disposed of the coincidence rates, which are indirectly affected by block dead-time. Hence, we performed several simulations with values of τ_{block} varying from 100 ns to 500 ns with increments of 50 ns (errors did not change significantly for increments smaller than 50 ns). The value yielding the best agreement between simulated and measured count rates was 300 ns. A major constraint in this estimation procedure was that τ_{block} had to be the same in 2D and 3D mode. For τ_{block} equal to 300 ns, an electronic dead-time constant of 50 ns was used to match measured and simulated count rates curves at very high count rates.

Fig. 6 shows coincidence and NEC rates measured on a DST camera (lines) and simulated (symbols). Simulated count rates were scaled to measurements by matching true count rates

at low activity (in 2D, scatter and random rates were scaled using a different factor). Errors associated with the new simulation, averaged over activity points in a low (clinical range) and a high activity window, calculated using (9), are shown in Table IV. Discrepancies between simulated and measured count rates are due to the fact that dead-time in the real system does not exactly follow (5) and (6) but an unknown combination of paralyzable and non-paralyzable equations [28]. Moreover, absence of modeling of pulse pile-up (degrading the energy and spatial resolutions) and saturation of the electronic system cause further deviation of the model from measurements at very high count rates.

The accuracy of our modeling for activities in the clinical range and up to 80 kBq/cc in 2D (35 kBq/cc in 3D) is better than 13%. Discrepancies between count rates obtained with and without using the variance reduction technique for modeling dead-time, described in Section II-C, were less than 1%.

5) Image Quality—Fig. 7 shows measured and simulated reconstructed central slices of the NEMA quality phantom and Fig. 8 shows associated hot and cold sphere contrasts.

Both the new simulation and SimSET overestimate hot and cold sphere contrasts because we assumed perfect scatter rejection (simulated images do not contain scatters). However in 3D, the large proportion of scatters and randoms makes acquisitions very sensitive to correction errors. 3D measurements were over-corrected for scatters and randoms, causing hot (cold) sphere contrasts to be lower (higher) than in 2D, whereas 3D and 2D contrasts are comparable in simulated images. Accurate modeling of spatial resolution with our simulation allows for reproduction of the partial volume effect, which causes hot sphere contrast to decrease with decreasing sphere diameter, more realistically than SimSET.

Measured and simulated SNR values shown in Table V are in good agreement, indicating that noise levels in simulated and measured images are identical. The slight difference between noise structures in simulated and measured images, especially in 3D mode, is most likely due to normalization and corrections for dead-time, scatters and randoms that cause noise in measurements to depart from the Poisson assumption.

Simulation time was 53 (44) hours in 3D and 400 (333) hours in 2D with the new simulation (SimSET) on a 3.4 GHz dual processor desktop computer. Low noise 2D sinograms obtained with SimSET and the new simulation contained on average 3110 coincidences per bin and 3D sinograms, which were not low noise as explained in Section III-B.5, contained on average 109 coincidences per bin.

V. Discussion

We present in this work a simulation approach improving upon SimSET by modeling block detectors, light sharing among crystal elements, random coincidences and detector dead-time. Our simulation allows for the generation of realistic PET acquisitions in 2D and 3D modes while keeping computation time reasonable when using a single desktop computer by using importance sampling methods implemented in SimSET and a variance reduction technique for simulating dead-time. On a 3.4 GHz dual processor desktop computer, simulation time is typically ~2–3 days for acquisitions with a number of counts comparable to clinical studies and ~2–3 weeks for low noise acquisitions (exact simulation time depends on the activity and attenuation distributions and total number of annihilations simulated). Realistic clinical imaging situations can be modeled using a voxelized anthropomorphic phantom.

Our modeling of energy spectra is within 5% as compared with GATE. Modeling block detectors affected energy spectra since one photon interacting in different blocks was seen as

a single event in a simulation with continuous detectors (SimSET) whereas it was seen as several independent events in GATE and the new simulation. This is due to the fact that the detection process in a block is independent from one block to another. In our experience, small changes in the geometry of the detector modeled drastically affected energy spectra. The concordance of features present in GATE and the new simulation spectra, for all scatter orders, indicates that propagation of photons in the pixelated detectors with the new simulation is accurate. Comparison with SimSET spectra also indicates that our modeling of the detector is more realistic for single and coincidence events.

The error of ~40% in spatial resolution values (FWHM) with SimSET is due to the absence of modeling of individual crystals and light sharing. Modeling these effects decreased the error to less than 7% in FWHM values. Sensitivity profiles, scaled by the ratio of measured to simulated global sensitivity, agreed with measurements within 3%. Modeling dead-time in the blocks and the electronic circuitry allowed reproducing measured count rates up to 80 kBq/cc in 2D and 35 kBq/cc in 3D with an accuracy better than 13%. Our modeling of dead-time allows accounting for non-uniform dead-time in blocks when simulating objects without a cylindrical geometry (in which case different blocks see different activity levels). In agreement with [5], we found that the scatter fraction increases when modeling blocks with air gaps because the sensitivity for true coincidences decreases faster than the sensitivity for scattered coincidences. We also found that reproducing the scatter fraction requires modeling the patient bed.

The model presented in this work was specifically validated for the DST PET scanner but can be used to simulate any cylindrical PET scanners using uniform block detectors (detectors with layers made of different scintillation materials can not be modeled at present). Future work includes modeling PET scanners with time-of-flight capability. This can be implemented easily since the distance traveled by every photon is recorded so the TOP difference can be computed.

VI. Conclusion

We have presented and validated a Monte Carlo PET simulation allowing accurate modeling of single photons, block detectors, random coincidences and dead-time. Our simulation uses validated importance sampling techniques implemented in SimSET as well as a variance reduction technique for simulating dead-time, allowing generation of PET studies with clinically relevant number of counts in reasonable time using a 3.4 GHz dual processor desktop computer. Our modeling can be extended easily to other PET scanners using block detectors. It will be useful to assess new scanner design and to evaluate quantitation techniques for PET in clinically relevant imaging situations.

Acknowledgements

The authors would like to thank R. Harrison for his help with SimSET, R. D. Badawi for useful information about the modeling of random coincidences, M. Defrise for providing them with the FORE software, and S. C. Moore, S. D. Wollenweber, and A. Sitek for useful discussions.

This work was supported in part by grants from the Rotary and Arthur-Sachs foundations and in part by NIH R01-EB005876.

References

1. Harrison RL, Haynor DR, Gillispie SB, Vannoy SD, Kaplan MS, Lewellen TK. A public-domain simulation system for emission tomography: Photon tracking through heterogeneous attenuation using importance sampling. *J Nucl Med* 1993;34(5):60P.

2. Du Y, Frey EC, Wang WT, Tocharoenchai C, Baird WH, Tsui BMW. Combination of MCNP and SimSET for Monte-Carlo simulation of SPECT with medium- and high-energy photons. *IEEE Trans Nucl Sci* 2002;49(3):668–674.
3. LeMeunier L, Mathy F, Fagret D. Validation of a PET Monte-Carlo simulator with random events and dead-time modeling. *IEEE Trans Nucl Sci* 2003;50(5):1462–1468.
4. Schmitz, RE.; Kinahan, PE.; Harrison, RL.; Karp, JS.; Surti, S.; Stearns, CW., et al. Combined simulations for evaluation of 2D and 3D wholebody PET imaging protocols. presented at the IEEE NSS/MIC Conf.; Rome, Italy. 2004.
5. Barret O, Carpenter TA, Clark JC, Ansorge RE, Fryer TD. Monte Carlo simulation and scatter correction of the GE Advance PET scanner with SimSET and Geant4. *Phys Med Biol* 2005;50:4823–40. [PubMed: 16204875]
6. Badawi RD, Lewellen TK, Harrison RL, Kohlmyer SG, Vannoy SD. The effect of camera geometry on singles flux, scatter fraction and trues and randoms sensitivity for cylindrical 3D PET—A simulation study. *IEEE Trans Nucl Sci* 2000;47(3):1228–1232.
7. Casey ME, Nutt R. A multicrystal two-dimensional BGO detector system for positron emission tomography. *IEEE Trans Nucl Sci* 1986;33(1):460–463.
8. Stickel JR, Cherry SR. High-resolution PET detector design: Modeling components of intrinsic spatial resolution. *Phys Med Biol* 2005;50:179–195. [PubMed: 15742938]
9. Moses WW, Derenzo SE. Empirical observation of resolution degradation in positron emission tomographs utilizing block detectors. *J Nucl Med* 1993;34(5):101P.
10. Levin CS. Design of a high-resolution and high-sensitivity scintillation crystal array for PET with nearly complete light collection. *IEEE Trans Nucl Sci* 2002;49(5):2236–2243.
11. Strother SC, Casey ME, Hoffman EJ. Measuring PET scanner sensitivity: Relating count rates to image signal-to-noise ratios using noise equivalent counts. *IEEE Trans Nucl Sci* 1990;37(2):783–787.
12. Haynor DR, Harrison RL, Lewellen TK. The use of importance sampling techniques to improve the efficiency of photon tracking in emission tomography simulations. *Med Phys* 1991;18(5):990–1001. [PubMed: 1961165]
13. Williams, JJ.; McDaniel, DL.; Kim, CL.; West, LJ. Detector characterization of Discovery ST whole-body PET scanner. presented at the IEEE NSS/MIC Conf.; Portland, OR. 2003.
14. Rogers JG, Taylor AJ, Rahimi MF, Nutt R, Andreaco M, Williams CW. An improved multicrystal 2-D BGO detector for PET. *IEEE Trans Nucl Sci* 1992;39(4):1063–1068.
15. Tornai MP, Hoffman EJ, Cherry SR. Effect of refraction index and light sharing on detector element identification for 2D detector modules in Positron Emission Tomography. *Nucl Instrum Methods Phys Res, Sect A* 1994;348:618–622.
16. Dahlbom M, Hoffman EJ. An evaluation of a two-dimensional array detector for high resolution PET. *IEEE Trans Med Imaging* 1988;7(4):264–272. [PubMed: 18230478]
17. Moisan C, Rogers JG, Douglas JL. A count rate model for PET and its application to an LSO HR PLUS scanner. *IEEE Trans Nucl Sci* 1997;44(3):1219–1224.
18. Eriksson L, Wienhardt K, Dahlbom M. A simple data loss model for positron camera systems. *IEEE Trans Nucl Sci* 1994;41(4):1566–1570.
19. Daube-Witherspoon ME, Muehllehner G. Treatment of axial data in three-dimensional PET. *J Nucl Med* 1987;28:1717–1724. [PubMed: 3499493]
20. Defrise M, Kinahan PE, Townsend DW, Michel C, Sibomana M, Newport DF. Exact and approximate rebinning algorithms for 3D-PET data. *IEEE Trans Med Imaging* 1997;16(2):145–158. [PubMed: 9101324]
21. Stearns, CW.; McDaniel, DL.; Kohlmyer, SG.; Arul, PR.; Geiser, BP.; Shanmugam, V. Random coincidence estimation from single event rates on the Discovery ST PET/CT scanner. presented at the IEEE NSS/MIC Conf.; Portland, OR. 2003.
22. *Performance Measurements of Positron Emission Tomographs*, NEMA Standards Publication NU 2-2001, National Electrical Manufacturers Association, 2001, spatial resolution pp. 7–9, sensitivity pp. 17–20, count rates and scatter fraction pp. 11–16, image quality pp. 25–32.
23. Jan S, Santin G, Strul D, Staelens S, Assié K, Autret D, et al. GATE: A simulation toolkit for PET and SPECT. *Phys Med Biol* 2004;49:4543–4561. [PubMed: 15552416]

24. De Beenhouwer, J.; Kruecker, D.; Staelens, SG.; Ferrer, L.; Chatziioannou, AF.; Rannou, FR. Distributed computing platform for PET and SPECT simulations with GATE. presented at the IEEE NSS/MIC Conf.; Puerto Rico. 2005.
25. Bergstrom M, Eriksson L, Bohm C, Blomqvist G, Litton J. Correction for scattered radiation in a ring detector positron camera by integral transformation of the projections. *J Comput Assist Tomo* 1983;7 (1):42–50.
26. Wollenweber S. Parametrization of a model-based 3-D PET scatter correction. *IEEE Trans Nucl Sci* 2002;49:722–727.
27. Hudson HM, Larkin RS. Accelerated image reconstruction using ordered subsets of projection data. *IEEE Trans Med Imaging* 1994;13:601–609. [PubMed: 18218538]
28. Knoll, GF. *Radiation Detection and Measurement*. 3. New York: Wiley; 2000. Dead-time; p. 119-128.

Fig. 1. Modeled Gaussian one dimensional point spread function (PSF) of crystal elements in the DST block detector.

Fig. 2. Energy distributions of single photons that propagated in the 12 cm (a), 20 cm (b), 40 cm (c) and 60 cm (d) water cylinders, obtained with GATE (line), SimSET (crosses) and the new simulation (circles).

Fig. 3.

Energy distribution of single events, obtained with GATE and the new simulation while modeling a point source in air and in the 20 cm diameter water filled cylinder. Finite energy resolution was not modeled so as not to confound potential discrepancies between spectra. Spectra are decomposed in their zeroth, first, second and third scatter orders (spectrum a shows scatters in the detector, spectra b shows scatters in the collimator and c and d show scatters in the object). Arrow 1 on the first order of spectrum a and the zeroth order of spectra b, c and d show a Compton edge at 341 keV due to primary photons that scattered once at 180° in the detector. Arrow 2 on spectra b and d show a Compton edge at 170 keV due to photons that backscattered once in the water filled phantom and deposited all their energy in the detector.

Fig. 4. Coincidence spectra of a point source in water simulated in 3D mode with GATE, the new simulation and SimSET. The total spectrum (tot, panel a) is decomposed into its primary distribution (trues, panel a), single scatter distribution (SSD, panel b) and multiple scatter distribution (MSD, panel b).

Fig. 5. Measured and simulated axial sensitivity profiles in 2D (upper panel) and 3D (lower panel).

Fig. 6.

Simulated and measured count rates in 2D and 3D mode (trues, scatters and randoms rates are shown in the upper row; NEC rates in the lower row). The maximum total activities modeled are 48 mCi (80 kBq/cc) and 21 mCi (35 kBq/cc).

Fig. 7.
Central slice of the measured and simulated NEMA NU-2 2001 image quality phantom acquisitions.

Fig. 8. Measured and simulated contrasts in hot (1.0 cm to 2.2 cm) and cold spheres (2.8 cm and 3.7 cm) of the NEMA quality phantom.

TABLE I
Technical Characteristics of the Discovery ST PET Scanner

Collimator:	
Number of septa	23
Septa width	0.8 mm
Septa length	5.4 mm
Collimator material	Tungsten
Detector:	
Detector ring diameter	88.62 cm
Scintillator material	BGO
Number of block rings	4
Number of blocks per ring	70
Block element size	3.8 cm × 3.8 cm × 3 cm [†]
Number crystals/block	6 × 6 arrays
Crystal element size	6.34 mm × 6.34 mm × 30 mm [†]
Number of PMTs/block	1 (four channels per PMT)
Max. ring difference in 2D	±5 crystal rings
Max. ring difference in 3D	±23 crystal rings
Energy window in 2D	375–650 keV
Energy window in 3D	375–650 keV
Coincidence time window	11.7ns

[†]
axial × transaxial × radial

TABLE II

Simulated and Measured Spatial Resolution in mm (First Line: FWHM, Second Line: FWTM)

2D		SimSET	New model	DST
Ax.	@ 1 cm	3.5	4.4	4.6
		6.9	10.8	9.5
	@ 10 cm	4.5	5.5	6.1
		9.8	12.5	12.5
Tang.	@ 1 cm	3.0	6.2	6.4
		6.0	12.9	11.7
	@ 10cm	3.3	6.7	7.0
		6.8	13.9	15.9
Rad.	@ 1 cm	3.0	6.2	6.4
		6.0	12.9	11.7
	@ 10 cm	2.8	5.9	6.3
		5.9	11.4	11.9
3D		SimSET	New model	DST
Ax.	@ 1 cm	4.5	5.9	5.2
		9.4	12.3	10.7
	@ 10 cm	5.2	6.3	6.1
		10.6	12.6	11.7
Tang.	@ 1 cm	3.2	6.3	6.5
		6.4	12.6	11.6
	@ 10 cm	3.0	6.5	6.7
		5.9	12.5	12.1
Rad.	@ 1 cm	3.2	6.3	6.5
		6.4	12.6	11.6
	@ 10 cm	2.8	5.9	6.4
		5.5	11.4	11.7

TABLE III

Simulated and Measured Average Scatter Fractions (Percent)

	NEMA SimSET	NEMA NEW SIM.	NEMA MEAS.
SF 2D	16.9	17.8	21.3
SF 3D	44.9	45.8	45.2

TABLE IV
Simulated Count Rates Errors, Averaged Over Activity Points in Two Activity Windows

<u>2D</u>	[0: 30 kBq/cc] (clinical range)	[30: 80 kBq/cc]
Trues	1.3%	3.6%
Scatters	2.5%	2.2%
Randoms	6.6%	4.2%
NEC	2.8%	8.7%
<u>3D</u>	[0: 15 kBq/cc] (clinical range)	[15: 35 kBq/cc]
Trues	1.0%	3.2%
Scatters	3.0%	5.6%
Randoms	12.7%	5.9%
NEC	4.8%	9.3%

TABLE V
Signal to Noise Ratio (SNR) in Measured and Simulated Reconstructed Images

	Measurement		New simulation		SimSET	
	2D	3D	2D	3D	2D	3D
SNR	11.3	17.2	11.8	17.1	11.8	17.2