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Estimation of the Rigid-Body Motion from Three-Dimensional Images Using a Generalized Center-of-Mass Points Approach

B. Feng [Member, IEEE], P. P. Bruyant [Member, IEEE], P. H. Pretorius [Associate Member], R. D. Beach [Senior Member, IEEE], H. C. Gifford [Member, IEEE], J. Dey [Member, IEEE], M. Gennert, and M. A. King [Senior Member, IEEE]

B. Feng, P. P. Bruyant, P. H. Pretorius, R. D. Beach, H. C. Gifford, J. Dey, and M. A. King are with the Department of Radiology, University of Massachusetts Medical School, Worcester, MA 01655. M. Gennert is with the Department of Computer Science, Worcester Polytechnic Institute, Worcester, MA.

Abstract

We present an analytical method for the estimation of rigid-body motion in sets of three-dimensional SPECT and PET slices. This method utilizes mathematically defined generalized center-of-mass points in images, requiring no segmentation. It can be applied to compensation of the rigid-body motion in both SPECT and PET, once a series of 3D tomographic images are available. We generalized the formula for the center-of-mass to obtain a family of points co-moving with the object's rigid-body motion. From the family of possible points we chose the best three points which resulted in the minimum root-mean-square difference between images as the generalized center-of-mass points for use in estimating motion. The estimated motion was used to sum the sets of tomographic images, or incorporated in the iterative reconstruction to correct for motion during reconstruction of the combined projection data. For comparison, the principle-axes method was also applied to estimate the rigid-body motion from the same tomographic images. To evaluate our method for different noise levels, we performed simulations with the MCAT phantom. We observed that though noise degraded the motion-detection accuracy, our method helped in reducing the motion artifact both visually and quantitatively. We also acquired four sets of the emission and transmission data of the Data Spectrum Anthropomorphic Phantom positioned at four different locations and/or orientations. From these we generated a composite acquisition simulating periodic phantom movements during acquisition. The simulated motion was calculated from the generalized center-of-mass points calculated from the tomographic images reconstructed from individual acquisitions. We determined that motion-compensation greatly reduced the motion artifact. Finally, in a simulation with the gated MCAT phantom, an exaggerated rigid-body motion was applied to the end-systolic frame. The motion was estimated from the end-diastolic and end-systolic images, and used to sum them into a summed image without obvious artifact. Compared to the principle-axes method, in two of the three comparisons with anthropomorphic phantom data our method estimated the motion in closer agreement to than of the Polaris system than the principal-axes method, while the principle-axes method gave a more accurate estimation of motion in most cases for the MCAT simulations. As an image-driven approach, our method assumes angularly complete data sets for each state of motion. We expect this method to be applied in correction of respiratory motion in respiratory gated SPECT, and respiratory or other rigid-body motion in PET.

I. Introduction

In SPECT and PET, patient motion during imaging causes reconstruction artifacts and degrades diagnostic accuracy if not correctly compensated [1-4]. There are numerous papers published on this subject, and more can be expected. Based on the motion-detection technique employed, these papers can be divided into the categories of data-driven methods [5-12] and external tracking system methods [13-21]. Each category of methods have advantages and disadvantages. The external tracking system methods can employ an IR tracking device such as the Polaris, or a video-tracking system (VTS) to detect the motion of isolated markers. This offers high accuracy when estimating the motion of markers, but in some cases (for example respiratory motion) does not directly reflect the motion of the organ imaged. The data-driven methods estimate the motion from the data and may use iterative approaches based on various objective functions. Converging or robust algorithms are not easy to obtain.

We propose a simple analytical method of detecting rigid-body motion from a series of sets of three-dimensional (3D) tomographic images. Other analytical algorithms for rigid-body registration have been published and successfully applied in the medical imaging field. Examples of such analytical methods include the use of tensor-based moment functions [22], and the principle-axes method [23,24]. Our method is a generalization of the conventional center-of-mass approach which gives only information for rigid-body translation [25]. For estimating rotation in addition to translation we defined mathematically a family of “generalized center-of-mass points”. The motion of these points matches the motion in the images. These generalized points can be used to correct rigid-body motion in image reconstruction, as in the case of using the rigid-body motion detected by an external tracking system [15-17]. One distinction between our method and the methods utilizing the tensor-based moment functions or the principle-axes method is that the generalized center-of-mass points are linear in terms of the coordinates, while the other methods mentioned use tensors which are non-linear in terms of the coordinates. An additional feature of our method is the optimization process, implemented to choose the power functions and pre-smoothing Gaussian functions which lead to the optimal alignment between two sets of images. For comparison, the principle-axes method [23] was also implemented and applied to our test data.

One limitation of our method is that a series of three-dimensional tomographic images, one for each state of motion of the object, must be available. Usually a SPECT acquisition cannot provide such a series of tomographic images. However, in a respiratory gated acquisition, one set of images can be reconstructed for each stage of respiration. In PET, it is possible to divide acquisition data into independent time bins and thereby reconstruct a series of images [26]. Our method might be applied to the above cases. In reality respiratory motion has been shown to be complex and non-linear [27], we expect our method could be employed to compensate for the rigid-body part of the respiratory motion affecting the heart.

II. Methods

A. Detection of the Rigid-Body Motion from the Generalized Center-of-Mass Points and by the Principle-Axes Method

We assume a series of tomographic reconstructions of a moving object are available, from which the rigid-body motion of the object can be determined. Each of the reconstructions describes the 3D intensity distribution of the object at a certain time. The i^{th} voxel ($i = 1, 2, \dots, N_p$, where N_p is the number of voxels) has intensity m_i . From time t to $t+dt$, we assume that the center of the i^{th} voxel m_i moves from coordinates (x_i, y_i, z_i) to $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$, due to a rigid-body

motion. The coordinate transformation for this is $\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \\ \tilde{z}_i \end{bmatrix} = [M] \begin{bmatrix} x_i - x^o \\ y_i - y^o \\ z_i - z^o \end{bmatrix} + \begin{bmatrix} x^o + \Delta x \\ y^o + \Delta y \\ z^o + \Delta z \end{bmatrix}$, where

$[M]$ is the rotation matrix, (x^o, y^o, z^o) is the center of rotation, and $\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$ is the translation vector. We define generalized center-of-mass points at time t as

$$\begin{bmatrix} x_c^f \\ y_c^f \\ z_c^f \end{bmatrix} = \frac{\sum_{i=1}^{N_p} \left\{ f(m_i) \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \right\}}{\sum_{i=1}^{N_p} f(m_i)}, \quad (1)$$

where function $f(m_i)$ gives a cost to m_i , and a set of functions gives a set of points. The conventional center-of-mass point uses $f(m_i) = m_i$. Higher order moments of m_i can be used to provide sets of functions $f(m_i)$. Note that the definition is linear with coordinates (x_i, y_i, z_i) . At time $t+dt$, the generalized center-of-mass points are

$$\begin{bmatrix} \tilde{x}_c^f \\ \tilde{y}_c^f \\ \tilde{z}_c^f \end{bmatrix} = \frac{\sum_{i=1}^{N_p} \left\{ f(m_i) \begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \\ \tilde{z}_i \end{bmatrix} \right\}}{\sum_{i=1}^{N_p} f(m_i)}. \quad (2)$$

Keeping in mind that the rotation matrix and translation vector consist of unknown constants,

it is straightforward to prove $\begin{bmatrix} \tilde{x}_c^f \\ \tilde{y}_c^f \\ \tilde{z}_c^f \end{bmatrix} = [M] \begin{bmatrix} x_c^f - x^o \\ y_c^f - y^o \\ z_c^f - z^o \end{bmatrix} + \begin{bmatrix} x^o + \Delta x \\ y^o + \Delta y \\ z^o + \Delta z \end{bmatrix}$, which means that the generalized center-of-mass points have the same rigid-body motion as the object. Function $f(m_i)$ should be chosen under the constraint of $f(0) = 0$, otherwise it leads to change of the image support and truncation of the transformed image $f(m_i)$. One simple choice of $f(m_i)$ that satisfies $f(0) = 0$ is a power function as employed herein.

We need at least three generalized center-of-mass points in each image to track 6-degree-of-freedom rigid-body motion. This is an analog to the case of fixing an object in 3D where you need at least three pins. That is, one point can determine translation, and a second can fix two rotational degrees of freedom about the first. However a third point is needed to establish the third rotational degree-of-freedom. By choosing the function $f(m)$ in Equation (1) as power functions $f_j(m) = m^j$ ($j=1, 2, 3, \dots, N$), N generalized center-of-mass points are obtained. Generally different points will be obtained for different power functions, since the higher the power function index, the closer the generalized center-of-mass point is to the hottest point in the image. Since more than three points are available, we optimize the method by using the “best” three points. The criterion employed is the root-mean-square difference (RMSD) between the image at time $t+dt$ and the image at time t with its location moved by the rigid-body motion that is determined from three of N points. Once any three points (characterized by the indexes of power functions) are chosen, we employ a least-squares algorithm [28] that uses singular value decomposition (SVD) to estimate the rigid-body motion. The RMSD is calculated by

$$RMSD = \sqrt{\frac{\sum_{i=1}^{N_p} (m_i^{t+dt} - \widehat{m}_i^t)^2}{N_p}}, \quad (3)$$

where N_p is the number of voxels in each image, m_i^{t+dt} is the image intensity at i^{th} voxel measured at time $t+dt$, and $\widehat{m}_i^t$ is the image intensity at the same voxel on the image that is measured at time t and shifted with the estimated rigid-body motion. Note that both m_i^{t+dt} and $\widehat{m}_i^t$ represent the original unsmoothed images. RMSD is dependent on the estimated rigid-body motion, therefore it is dependent on the power functions chosen to generate the generalized center-of-mass points. The time for optimization was approximately proportional to

$\binom{3}{N} = N(N-1)(N-2)/6$. N was chosen as 5 in our study since a higher N would slow down the optimization significantly.

Generally images are degraded by noise which is caused by the random nature in the imaging process. Therefore we investigated whether it was preferable to apply pre-smoothing to the images before calculation of the generalized center-of-mass points. The pre-smoothing function we chose is a three-dimensional Gaussian function, whose sigma is chosen from a few discrete values (from 0 up to 5.5 pixels with a step size of 0.5 pixel). The optimal pre-smoothing Gaussian function is chosen as the one that gives the minimum RMSD.

By minimizing RMSD, the rigid-body motion is estimated from images at times t and $t+dt$. In the case of more than two time points, a baseline configuration is chosen (for example, the first time point), and the motion from the baseline can be estimated for other time points. The estimated motion can be used post-reconstruction to generate a single summed set of slices from the series of slices for different states of motion. Once the motion is estimated from the set of 3D tomographic slices, it can also be used to model the motion in reconstruction of all of the projection data as a whole dataset [20]. In this case, the reconstruction goes through all the motion states at each projection angle modeling the motion relative to some reference position. These two motion compensation strategies (intelligent summing and modeling motion in reconstruction) will both be evaluated through the MCAT simulations and experimental measurements.

The principle-axes [23] method was also implemented to provide a comparison of the generalized center-of-mass approach to an existing method. Based on the classical theory of the rigid-body motion, the principle-axes were defined as the eigenvectors of the inertia tensor. Defined in Equation (9) in Faber et al [23], the inertia tensor is symmetric and its eigenvectors are orthogonal to each other if the three eigenvalues are non-degenerate. These orthogonal principle-axes rotate with the object in the case of the rigid-body rotation. The rotation matrix $[M]$ is equal to the sum of the tensor products of the unit eigenvectors (denoted by v_1, v_2, v_3) at time t (before motion) and their counterparts (denoted by $\tilde{v}_1, \tilde{v}_2, \tilde{v}_3$) at time $t+dt$. To write explicitly,

$$[M] = \tilde{v}_1 \otimes v_1 + \tilde{v}_2 \otimes v_2 + \tilde{v}_3 \otimes v_3, \quad (4)$$

where $\otimes$ denotes the tensor product of two vectors. For each eigenvector, there is freedom of going positive or negative direction. Eigenvectors v_1, v_2, v_3 and $\tilde{v}_1, \tilde{v}_2, \tilde{v}_3$ are chosen to form right-handed sets. In addition, the angles between v_1 and $\tilde{v}_1$, between v_2 and $\tilde{v}_2$ are forced to be no greater than 90 degrees to avoid ambiguity. Since for any vectors a, b , and c , we have

$$(a \otimes b) \bullet c = (b \bullet c) a, \quad (5)$$

where $\bullet$ denotes the dot product. It is simple to show that

$$[M] \bullet v_1 = \tilde{v}_1, [M] \bullet v_2 = \tilde{v}_2, [M] \bullet v_3 = \tilde{v}_3, \quad (6)$$

which means that $[M]$ rotates v_1, v_2, v_3 to $\tilde{v}_1, \tilde{v}_2, \tilde{v}_3$, respectively.

The translation was calculated from the center-of-masses at time t and time $t+dt$.

B. MCAT Phantom Simulations with Different Noise Levels

To experimentally investigate our method under controlled conditions we employed an MCAT phantom which contained only the heart and liver as sources. We created two configurations: (1) before the motion (at time t) and (2) after the motion (at time $t+dt$). The motion consisted of a 15-degree rotation around the vertical (Y) axis and a translation ($\Delta x = 2cm$, $\Delta y = 3cm$ and $\Delta z = 2.5cm$). The projection data were generated for the two configurations by using a ray-driven projector which modeled attenuation and distance-dependent blurring. Each set of projection data contained 60 frames, simulating a 180-degree acquisition on a SPECT system of a Tc-99m perfusion imaging-agent. Four noise-levels were simulated: noise-free, 1 million (M), 0.5 M and 0.1 M counts in the heart, with Poisson noise added. This resulted in 4 motion-free datasets for each configuration. After reconstructing these datasets with compensations for attenuation and distance-dependent blurring, we calculated the generalized center-of-mass points on the reconstructed images, with optimization of the pre-smoothing Gaussian function and the three power functions in terms of the minimum RMSD. The estimated motion was used to move the image at time $t+dt$ back and sum it into the image at time t . Three-dimensional Gaussian interpolation [20] was employed to move images. To test usefulness of derived motion in an iterative reconstruction with motion compensation, 4 other datasets (for 4 noise levels, respectively) were created by combining data of the two configurations, simulating an acquisition of a periodic motion which consisted of only two motion states (at each angle the phantom moved between configurations 1 and 2 back and forth). Using the OSEM algorithm which models the known rigid-body motion with the 3D Gaussian interpolation [20], we reconstruct the motion-present datasets with the motion derived from the generalized center-of-mass points. To evaluate the quantitative accuracy of the reconstruction, we calculated the root-mean-square error (RMSE) which has an expression similar to Equation (3). In other words, the RMSE was defined as the RMSD between a certain reconstruction and the truth. For each noise-level, the truth was assumed to be the image reconstructed with compensation for the true motion.

The principle-axes method was also applied to the images at t and $t+dt$ which had been reconstructed from the motion-free datasets to allow comparison to this established method. The rigid-body motion obtained from the principle-axes method was substituted into Equation (3) to calculate the RMSD.

C. Experimental Study with an Anthropomorphic Phantom

We also acquired four datasets of the Data Spectrum anthropomorphic phantom with cardiac insert on an IRIX system. One mCi, 10.2 mCi, and 6.3 mCi of Tc-99m was added into heart, liver, and body of the phantom, respectively. This resulted in approximate concentrations of 1.0, 0.1, and 1.0 in these three structures. A Polaris (Northern Digital Inc.) tool consisting of four spherical markers was attached tightly to the top of the anthropomorphic phantom. Between each acquisition the phantom was translated and or rotated. The orientation was recorded by Polaris. In each acquisition, two heads of the IRIX acquired a total of 204 degree data with 102 degree gantry rotation at 34 angular steps. A scatter window (5% width centered at 120 keV) was also acquired. After reconstructing each individual acquisition, we calculated the generalized center-of-mass points and the rigid-body motion, by optimizing the pre-smoothing Gaussian function and determining which three power functions to be employed based on minimizing the RMSD. Summed images were created with and without taking into

account the motions. A mixed dataset was generated to simulate an acquisition in which the phantom moved periodically at each angle in an exaggerated “respiratory motion” manner. We reconstructed the mixed dataset with and without modeling the motion in the iterative reconstruction.

The principle-axes method was again utilized to estimate the rigid-body motions in the images reconstructed from the four acquisitions to provide a comparison to our method.

D. Estimation of the Rigid-Body Motion in Presence of Non-Rigid-Body Motion

The end-diastolic (ED) and end-systolic (ES) frames of a gated MCAT phantom [29] were chosen for this study. Only the heart (the left and right ventricles) of the phantom was used in our test. The cardiac motion between the ED and ES frames consisted of shortening of the left ventricle (LV) and thickening of the LV wall. The shape of the heart changed from the end-diastole to the end-systole, thus the cardiac motion in the gated MCAT phantom was a non-rigid-body motion. We applied an exaggerated translation ($\Delta x = 2\text{cm}$, $\Delta y = 3\text{cm}$ and $\Delta z = 2.5\text{cm}$) and rotation (15 degrees around Y-axis) to the ES image. The motion between the ED and moved ES was estimated with the generalized center-of-mass points approach. The estimated motion was compensated in summing the ES image into ED image. It should be noted that this will not result in a full correction as only a rigid-body correction is being made of a motion which is in part non-rigid. However, this serves as a test of correction of the rigid-body component of motion when there is a non-rigid component also present. For comparison, a directly summed image was also generated without compensating for the rigid-body motion. Also to serve as a comparison, the principle-axes method was applied to estimate the rigid-body motion between the ED image and the moved ES image.

III. Results

A. MCAT Phantom Simulations with Different Noise Levels

In the MCAT simulations, the rigid-body motion was estimated from the images before and after the motion. In calculation of the generalized center-of-mass points and estimation of the rigid-body motion, the optimal pre-smoothing Gaussian function and the optimal three power functions were chosen based on the minimum RMSD defined in Equation (3). The results for the optimal parameters and estimated motion are listed in Table 1. As the counts decreased, the accuracy in motion estimation also decreased. This was also reflected as an increase in the minimum relative RMSD for the image with lower counts. The relative RMSD is the RMSD scaled by the counts.

Applied to the same images, the principle-axes method gave more accurate estimates of the rigid-body motion for all the four noise-levels (Table 2). The motion-corrected summed images using the principle-axes method show no visual difference from those images using our method. However, the RMSD from the principle-axes method was significantly larger for each noise-level than the RMSD from the generalized center-of-mass points method. In simulations of the projection data, attenuation and distance-dependent blurring were modeled in the numerical projector. These degradation factors might have affected the accurate reconstruction of the images and thus the determination of the motion. The smaller RMSD from our method could also be the result of the optimization process. The generalized center-of-mass points method being selected on the basis of minimizing the RMSD as opposed to minimizing the error in the estimate of the motion. The smaller the RMSD, the better the overall alignment of the two sets of images, and the less severe motion artifact in the summed image. Thus this was selected as our criterion for optimization of the parameters of the generalized center-of-mass points method.

Due to the optimization process involved, our method was slower than the principle-axes method. On a 2.8-GHZ Xeon computer running Linux, the time to align two sets of $128 \times 128 \times 128$ images was 14 seconds for the principle-axes method, and 50 seconds for the generalized center-of-mass points approach.

The motion estimated from the generalized center-of-mass points was used to move the image at time $t+dt$ back and sum it with the image at time t (Fig. 1). As shown in Figure 1, the motion compensated summing generated artifact-free images. To better differentiate different noise-levels, no post-smoothing was applied to the summed images.

The motions estimated from the generalized center-of-mass points were also compensated in the iterative reconstruction of the motion-present datasets. Modeling the motion in reconstruction greatly reduced the motion artifact, compared with reconstruction with no motion compensation (Fig. 2). Using the true-motion compensated images (top row in Fig. 2) as the truth, the relative (scaled by counts) RMSE is listed in Table 3.

B. Experimental Study with an Anthropomorphic Phantom

The four sets of data were reconstructed using 6 iterations of the OSEM algorithm. The first measurement was chosen as the baseline. Motion from the baseline was estimated by the generalized center-of-mass points method and by the principle-axes method in the reconstructed image for other three cases (Table 4). In Table 4 it can be seen that the generalized center-of-mass points method more closely approximated the motion as estimated by Polaris than the principle-axes method in datasets 2 and 4. Note that there were sub-pixel translations unreported in Table 4 for datasets 3 and 4. The motions estimated from the generalized center-of-mass points method were used to generate a summed image (Fig. 3) or compensate the motion in the reconstruction of the motion-present data (Fig. 4). In each approach, compensation for the rigid-body motion greatly reduced the motion artifact, as shown on the polar maps in Figure 5.

C. Estimation of the Rigid-Body Motion in Presence of Non-Rigid-Body Motion

We applied our method and the principle-axes method to the ED and ES (with an exaggerated rigid-body motion) images. The motion estimated from the generalized center-of-mass points was 2.47, 2.93, and 1.99 cm translations along X, Y, and Z direction, and 11.6 degree rotation by Y-axis. The motion estimated by the principle-axis method was 2.47, 3.44, and 3.14 cm translations along X, Y, and Z direction, and 15.8 degree rotation by Y-axis. The difference between the estimated motion and the true motion ($\Delta x = 2\text{cm}$, $\Delta y = 3\text{cm}$, $\Delta z = 2.5\text{cm}$, and a 15 degree rotation by Y-axis) might have been caused partially by the original displacement between the ED and ES images. If the original displacement between the ED and ES images was ignored, our method estimated the translations more accurately than the principle-axes method, while the principle-axes method gave more accurate rotation than our method. The summed images with and without taking into account the rigid-body motion estimated from our method were plotted in Figure 6. Summing with motion compensation greatly reduced the motion artifact.

IV. Discussion

Our method of using the generalized center-of-mass points reduces the registration of sets of images to the registration of sets of points. However, this method could be sensitive to the noise and other imperfectness present in images, such as artifacts caused by lack of modeling the physics of imaging in reconstruction. By minimizing the RMSD between sets of images, our method optimizes the pre-smoothing Gaussian function and chooses the optimal three power functions that give three points used in estimation of the rigid-body motion. We applied

this method to the images reconstructed with no attenuation compensation, and found it worked quite well for all noise-levels (from 1 M to 0.1M) studied in this manuscript. An alternative approach is to use more than three points. We found that using more than three points resulted in very similar results for motion estimation compared to the approach of using the optimal three points which we concentrated on in this study.

We compared our method with the principle-axes method. We found that the principle-axes method in most cases gave more accurate estimate of the rotation than our method. In two of three cases, our method estimated the translations which agreed better with those measured by the Polaris system than the principle-axes method. One possible explanation could be that only the center-of-mass was used to calculate the translation in the principle-axes method, while our method adjusted the estimate using an optimization process.

As a short-coming of the generalized center-of-mass points method, it does not work for the binary images, since only one point can be obtained by Equation (1). The principle-axes method was shown to be able to successfully align binary images [23].

We tested two motion-compensation strategies once the rigid-body was known. One is summing the 3D reconstructions after correcting the motion in each set post-reconstruction. The second is reconstructing the motion-present data by modeling the motion in the iterative reconstruction process. Both approaches helped reduce the motion artifacts. The summing approach is faster, while the reconstruction approach is slower but might handle the statistics more appropriately. Since iterative reconstruction is a non-linear process, the two approaches generally result in different images. Further investigations are needed to evaluate these two approaches.

In our simulations with the gated MCAT phantom, the rigid-body motion was extracted approximately in the presence of the non-rigid-body wall motion. In realistic clinic studies, different organs might move in different fashions. Segmentation of the region-of-interest will be required before applying our method.

Like the degeneracy encountered in the registration of points, our method could face the problem of degeneracy if symmetry exists in some directions. In the case of existence of the symmetry in one direction, motion along that direction is undermined, but will not cause motion artifacts in that direction.

It is of most interest whether our method could apply to the clinical patient study. In PET, by dividing the acquisition time into short intervals, a series of 3D images could be available. Our method could be applied to sum these images up with motion estimated and compensated. As future work, we will investigate application of this method in PET imaging.

V. Conclusion

With simulations and experimental measurement we tested the method of using the generalized center-of-mass points to detect and correct the rigid-body motion in imaging, and we observed our method helped in reducing motion artifacts.

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Fig. 1.

Summed images were generated for the MCAT phantom simulations. (Upper row) No motion compensation in summing images at t and $t+dt$. (Lower row) Motion compensated summing. From left to right, results for 4 noise-levels are plotted. No post-smoothing was applied to the summed images.

Fig. 2.

Reconstructions of the motion-present simulations for several noise-levels (Bottom row, from left to right), using the motion estimated from the optimal three generalized center-of-mass points (listed in Table 1). As a comparison, the same data were reconstructed with the ideal motion compensation (Upper row) and without motion compensation (Middle row). No post-smoothing was applied to these images.

Fig.3.

(Left) Reconstruction of the motion-free baseline acquisition. (Middle) The same transaxial slice for the image summed for the 4 reconstructions of the data acquired in 4 sequential acquisitions, without motion compensation in summing. (Right) The same slice with compensation for motion. No post-smoothing was applied to the images. Note that the summed images had 4 times the counts of the no motion image.

Fig.4.

(Left) Reconstruction of the motion-present data with compensation for the motion which was detected by Polaris. (Middle) No motion Compensation. (Right) Motion compensation with the motion estimated from the generalized center-of-mass points. No post-smoothing was applied to the images.

Fig. 5. Polar maps generated for the images of anthropomorphic phantom shown in Fig. 4. (Left) Motion compensation with Polaris. (Middle) No motion Compensation. (Right) Motion compensation with the generalized center-of-mass points.

Fig. 6.

In simulations with the gated MCAT phantom, the end-diastolic (ED) (Upper left) and the end-systolic (ES) (Upper right) images were used. The ES image was translated and rotated (Upper right), and summed into the ED image without compensation for the motion (Lower left), and with compensation for the motion estimated from the generalized center-of-mass points (Lower right). Note the full 3D heart was employed in this study. Translation and rotation were also 3D. Thus the summed ED + ES slice at lower right is not the sum of the ED and ES slices shown at the top of the figure, but the sum of the ED slice at top and its appropriate counterpart after 3D translation and rotation.

Table 1

In the MCAT phantom simulations, the motion was estimated from images for 4 noise-levels, using the smoothing function and three generalized center-of-mass points determined to minimize the RMSD (root-mean-square difference). Note ROT_Y is Y-axis rotation.

Optimal Parameters & Estimated Motion	Noise-Free	1 M Counts	0.5 M Counts	0.1 M Counts
Optimal Smoothing Parameter	$\sigma = 2.0$	$\sigma = 2.5$	$\sigma = 1.5$	$\sigma = 5.0$
Optimal Power Function Indexes	3, 4, 5	1, 2, 3	1, 2, 3	1, 2, 3
Minimum Relative RMSD (10^{-6})	0.0578	0.213	0.279	0.567
Translations – Δx , Δy , Δz (cm)	1.96, 3.01, 2.50	2.00, 3.16, 2.60	1.99, 3.10, 2.54	1.58, 3.27, 2.60
ROT_Y (deg)	14.6	14.4	15.8	12.0

True Motion: (Horizontal) $\Delta x = 2$ cm, (Vertical) $\Delta y = 3$ cm, (Axial) $\Delta z = 2.5$ cm. Rotation by (Vertical) Y-axis = 15 deg.

Table 2

In the MCAT phantom simulations, the motion was estimated from images for 4 noise-levels, using the principle-axes.

Note RMSD is root-mean-square difference and ROT_Y is Y-axis rotation.

Estimated Motion	Noise-Free	1 M Counts	0.5 M Counts	0.1 M Counts
Relative RMSD (10^{-6})	1.23	1.25	1.25	1.35
Translations – Δx , Δy , Δz (cm)	2.01, 3.00, 2.50	2.01, 3.00, 2.50	2.03, 2.97, 2.48	1.96, 2.99, 2.57
ROT_Y (deg)	15.0	15.1	15.3	14.9

True Motion: (Horizontal) $\Delta x = 2$ cm, (Vertical) $\Delta y = 3$ cm, (Axial) $\Delta z = 2.5$ cm. Rotation by (Vertical) Y-axis = 15 deg.

Table 3

The relative mean-square error (RMSE) as calculated for the images in the middle and lower rows in Fig. 2.

Methods	Relative RMSE = RMSE/Counts (10^{-6})			
	Noise-free	1 M	0.5 M	0.1 M
No Motion Compensation	0.592	0.603	0.602	0.677
Motion Compensation	0.034	0.105	0.114	0.282

Table 4

The rigid-body motions of the anthropomorphic phantom were estimated from images reconstructed from the four datasets, by using the generalized center-of-mass points and the principle-axes, and compared with the motions detected by Polaris.

Dataset #	Motion by Polaris	Motion by the Generalized Center-of-Mass Points	Motion by the Principle-Axes Method
2	−6.19 cm translation by axial (Z) direction	−6.18 cm translation by axial (Z) direction	−6.09cm translation by axial (Z) direction
3	6.91 degree rotation by vertical (Y) axis	8.04 degree rotation by vertical (Y) axis	7.74 degree rotation by vertical (Y) axis
4	−15.6 degree rotation by horizontal (X) axis	−16.4 degree rotation by horizontal (X) axis	−17.2 degree rotation by horizontal (X) axis