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The Missing Censoring Indicator Model and the Smoothed Bootstrap

Sundarraman Subramanian^{a,2,1} and Derek Bean^b

^a Center for Applied Mathematics and Statistics, Department of Mathematical Sciences, New Jersey Institute of Technology, Newark

^b Department of Statistics, University of California, Berkeley

Abstract

For right censored data with missing censoring indicators, sub-density function kernel estimators play a significant role for estimating a survival function. Data-driven bandwidths for computing these kernel estimators are proposed. The bandwidths are obtained as minimizers of certain estimates of the mean integrated squared error (MISE). It is shown that the smoothed bootstrap offers a motivation for choosing the proposed MISE estimates for minimization. The efficacy of the proposed procedures is investigated through simulation studies and some illustrations are provided.

Keywords

Epanechnikov kernel; Integrated squared bias; Inverse probability weighted; Least squares cross validation; Missing at random; Pilot bandwidth

1 INTRODUCTION

In this article we focus on bootstrap-based bandwidth selection, required for the computation of certain conditional probabilities, leading to the provision of efficient survival function estimates when there are missing censoring indicators (MCIs). Let T denote a failure time, C an independent censoring time, and let X denote the minimum of T and C . Suppose that ξ is a “missingness” indicator, assuming the value 1 or 0 according as whether the censoring indicator $\delta = I(T \leq C)$ is observed or not. The data for the MCI model are n iid replicates of $Y = (X, \xi, \sigma)$, where $\sigma = \xi\delta$. Note that when $\sigma = 1$, the observation is uncensored and, when $\sigma = 0$, we observe a censored observation ($\xi = 1, \delta = 0$) or a missing indicator ($\xi = 0$).

For the MCI model, Subramanian (2004, 2006) introduced two asymptotically efficient kernel-based estimators of the survival function under the missing at random (MAR) frame-work. Denote the empirical estimators of $H(x) = P(X \leq x)$, $\bar{H}(x) = 1 - H(x)$ and $H_{11}(x) = P(X \leq x, \xi = 1, \sigma = 1)$ by $\hat{H}(x)$, $\hat{\bar{H}}(x)$ and $\hat{H}_{11}(x)$ respectively. Let $p(x) = P(\delta = 1|X = x)$ which equals $p_1(x) = \pi_1(x)$, where $p_1(x)$ and $\pi_1(x)$ are the derivatives with respect to x of $P(X \leq x, \delta = 1)$ and $P(X \leq x, \xi = 1)$ respectively. Let $\pi(x) = P(\xi = 1|X = x)$ which, under MAR, takes the form $\pi_1(x)/h(x)$, where $h(x)$ denotes the density of X . Let K denote a kernel function, a_n , b_n , and c_n denote three bandwidth (BW) sequences, and let $K_{a_n}(x) = a_n^{-1}K(x/a_n)$. The standard kernel-based estimators of $p(x)$ and $\pi(x)$ are given by

²Corresponding author: Tel.: +1 973 642 4496; fax: +1 973 596 5591; Email: sundars@njit.edu.

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$$\begin{aligned}\widehat{p}(x) &= \frac{n^{-1} \sum_{i=1}^n K_{a_n}(x - X_i) \sigma_i}{n^{-1} \sum_{i=1}^n K_{b_n}(x - X_i) \xi_i} \equiv \frac{\widehat{p}_1(x)}{\widehat{\pi}_1(x)}, \quad \widehat{\pi}(x) \\ &= \frac{n^{-1} \sum_{i=1}^n K_{b_n}(x - X_i) \xi_i}{n^{-1} \sum_{i=1}^n K_{c_n}(x - X_i)} \equiv \frac{\widehat{\pi}_1(x)}{\widehat{h}(x)}.\end{aligned}$$

The survival function estimators are given by $\hat{S}_l(t) = \pi_{s \leq t} \{1 - d\hat{\Lambda}_l(t)\}$, for $l = D, I$, where π is the product integral and $\hat{\Lambda}_l(t)$ are the cumulative hazard estimators given by

$$\begin{aligned}\widehat{\Lambda}_D(t) &= \int_0^t \widehat{p}(x) \frac{d\widehat{H}(x)}{\widehat{H}(x-)}, \quad \widehat{\Lambda}_I(t) \\ &= \int_0^t \frac{1}{\widehat{\pi}(x)} \frac{d\widehat{H}_{11}(x)}{\widehat{H}(x-)}.\end{aligned}$$

We will refer to $\hat{S}_D(t)$ as a Dikta-type estimator (Dikta 1998) and $\hat{S}_I(t)$ as an inverse probability weighted (IPW) estimator (Robins and Rotnitzky 1992).

We implement the empirical smoothed bootstrap for subdensity BW estimation (Faraway and Jhun 1990; Taylor 1989). Alternatively, instead of the empirical criterion, we will compute the theoretical bootstrap expectation and obtain its minimizer. The latter approach avoids actual resampling for computing the BW based on the data at hand. Both approaches require an initial estimate of a given subdensity $h_1(x)$, however. We plug in the pilot kernel subdensity estimate $\hat{h}_1(x|g)$, where the pilot BW g is selected through least squares CV. For density estimation, this approach works well (Grund and Polzehl 1997).

The article is organized as follows. In Section 2, we present the smoothed bootstrap choices of BW estimation and an illustration. In Section 3, we present a simulation study. Section 4 contains some concluding remarks. Relevant calculations are in the Appendix.

2 Smoothed bootstrap for a subdensity bandwidth

We focus on estimating the conditional expectation of a generic binary variable η . Suppose that (X_i, η_i) , $i = 1, \dots, n$ are n iid pairs of (X, η) and that interest centers on estimating $v(t) = P(\eta = 1|X = t) = h_1(t)/h(t)$. Here $h_1(t)$ is the subdensity corresponding to $P(X \leq t, \eta = 1)$ and h

(t) is the density of X . The kernel estimator of $h_1(t)$ is $\widehat{h}_1(t|a_n) = n^{-1} \sum_{i=1}^n \eta_i K_{a_n}(t - X_i)$, where a_n is a BW sequence (simply a henceforth). The standard estimator of $h(t)$ is

$$\widehat{h}(t|a) = n^{-1} \sum_{i=1}^n K_a(t - X_i).$$

2.1 Theoretical smoothed bootstrap bandwidth

Let $R(f) = \int f^2(x) dx$ and $\rho = P(\eta = 1)$. The standard MISE criterion, given by

$$M_{h_1}(a) = E \int (\widehat{h}_1(x|a) - h_1(x))^2 dx, \quad (2.1)$$

can be shown to take the form [Here $*$ denotes the convolution operator.]

$$\begin{aligned}
 M_{h_1}(a) &= (na)^{-1} \rho R(K) \\
 &\quad - n^{-1} R(K_a * h_1) \\
 &\quad + R\{(K_a * h_1)(x) - h_1(x)\}.
 \end{aligned} \tag{2.2}$$

The first bandwidth estimate that we propose minimizes an estimate of M_{h_1} , obtained by plugging a pilot kernel estimate for h_1 and the sample proportion $\hat{\rho}$ for ρ in Eq. (2.2). In the Appendix, we show that this estimate of M_{h_1} , denoted by $\hat{M}_{h_1}$, is precisely the theoretical expectation over bootstrap samples, E^* , of the bootstrap integrated squared error (ISE):

$$\begin{aligned}
 \hat{M}_{h_1}(a) &= E^* \int (\hat{h}_1^*(x|a) - \hat{h}_1(x|g)) dx.
 \end{aligned} \tag{2.3}$$

Here, $\hat{h}_1(x|g)$ is the pilot kernel subdensity estimate of $h_1(x)$ with BW g (see Section 2.4) and $\hat{h}_1^*(x|a)$ is the kernel subdensity estimate based on a smoothed bootstrap sample, that is, a sample drawn from the pilot subdensity estimate $\hat{h}_1(x|g)$ (see also section 2.2). Let $C(a) = (na)^{-1} \hat{\rho} R(K) + R(K_a * \hat{h}_1 - \hat{h}_1)$. From Eq. (A.3), the actual expression for the right hand side of Eq. (2.3) is given by

$$\hat{M}_{h_1}(a) = C(a) - n^{-1} R(K_a * \hat{h}_1). \tag{2.4}$$

The minimizer of $\hat{M}_{h_1}(a)$ given by Eq. (2.4) defines the *theoretical* smoothed bootstrap (TSB) choice of BW. On the other hand, the minimizer of $C(a)$ defines the *smoothed cross validation* (SCV) choice of BW, see, for example, Hall, Marron, and Park (1992).

2.2 Empirical iterated smoothed bootstrap bandwidth

Let ε denote an independent random deviate drawn from the kernel K . As before, let $\hat{h}_1(x|g)$ denote the pilot kernel subdensity estimate of $h_1(x)$ with BW g (see Section 2.4).

The *empirical iterated* smoothed bootstrap (EISB) works as follows:

Step 1: Resample from (X_i, η_i) , $i = 1, \dots, n$ to obtain $(\tilde{X}_i, \tilde{\eta}_i)$, $i = 1, \dots, n$.

Step 2: For each $i = 1, \dots, n$, obtain $X_i^* = \tilde{X}_i + g\varepsilon_i$. Also define $\eta_i^* = \tilde{\eta}_i$.

Step 3: Define the smoothed bootstrap kernel density estimate of h_1 as follows:

$$\hat{h}_1^*(x|a) = n^{-1} \sum_{i=1}^n K_a(x - X_i^*) \eta_i^* \tag{2.5}$$

Step 4: Repeat the above procedure L times, for L bootstrap samples and L estimators.

Step 5: Form the empirical approximation to Eq. (2.3), also denoted by $M_{h_1}(a)$ for convenience, by taking E^* as the empirical mean. That is,

$$\begin{aligned}\widehat{M}_{h_1}(a) \\ = L^{-1} \sum_{j=1}^L \int (\widehat{h}_{1,j}^*(x|a) - \widehat{h}_1(x|g))^2 dx.\end{aligned}\quad (2.6)$$

Step 6: Obtain the one-step bootstrap choice of the BW by minimizing $M_{h_1}(a)$ over a . If a two-step bootstrap BW is desired, the value obtained in step 5 can be assigned as the new initial choice g , and steps 2 through 5 can be repeated.

The CS1 resampling plan of Gonzalez-Manteiga, Cao, and Marron (1996) may be used to generate the smoothed resamples, but this would require two pilot BWs, one each for $\eta = 0$ and 1, increasing the computational burden. In fact, it suffices to ensure that the resampled X_i 's with $\eta_i = 1$ are properly smoothed, since the “improperly smoothed” X_i corresponding to $\eta_i = 0$ would be annihilated and do not figure in the analysis, see Eq. (2.5) above.

2.3 Asymptotic smoothed bootstrap bandwidth

For $l = 0, 1, 2$, let $\mu_l(K) = \int u^l K(u) du$. Note that $\mu_0(K) = 1$. After some calculations as in Cao (1993) or Wand and Jones (1995), assuming that $\mu_1(K) = 0$, it can be shown that $M_{h_1}(a) = \overline{M}_{h_1}(a) + o\{(na)^{-1} + a^4\}$, where the AMISE, $\overline{M}_{h_1}(a)$, is given by

$$\begin{aligned}\overline{M}_{h_1}(a) \\ = (na)^{-1} \rho R(K) \\ + \frac{1}{4} a^4 (\mu_2(K))^2 R(h_1'').\end{aligned}\quad (2.7)$$

Its minimizer, however, depends on the second derivative of h_1 , which is difficult to estimate:

$$a_{\text{AMISE}} = \left\{ \frac{\rho R(K)}{n R(h_1'') (\mu_2(K))^2} \right\}^{\frac{1}{5}}. \quad (2.8)$$

However, it can be shown that $\widehat{M}_{h_1} = \overline{M}_{h_1} + o\{(na)^{-1} + a^4\}$, where the estimate $\overline{M}_{h_1}$ is obtained by the usual substitution of estimates ρ and $R(\widehat{h}_1'')$ into Eq. (2.7). The BW that minimizes $\widehat{M}_{h_1}$ is given by Eq. (2.8), modified by the indicated substitutions. We call this estimator *asymptotic smoothed bootstrap* (ASB) choice, but don't pursue it any further.

2.4 Cross validation BW

We specify the pilot BW through least squares CV. Here we give a justification for the approach. Denote by $\widehat{h}_{1,-i}$ the subdensity estimate based on all of $(X_j, \eta_j), j = 1, \dots, n$, except the pair (X_i, η_i) . We define the CV criterion by

$$\begin{aligned}\lambda(a) \int \widehat{h}_1^2(x|a) dx \\ - 2n^{-1} \sum_{i=1}^n \eta_i \widehat{h}_{1,-i}(X_i|a).\end{aligned}\quad (2.9)$$

$$E\left(\eta_i \widehat{h}_{1,-i}(X_i|a)|X_i, \eta_i\right)$$

Indeed, for each i , we have that $= \eta_i \int K_a(X_i - x)h_1(x)dx$; see, for example Silverman (1986). Then, by a conditioning argument, it follows that

$$\begin{aligned} E(\lambda(a)) &= E \int \widehat{h}_1^2(x|a)dx - 2n^{-1} \sum_{i=1}^n E\left(\eta_i \int K_a(X_i - x)h_1(x)dx\right) \\ &= E \int \widehat{h}_1^2(x|a)dx - 2E \int \left(n^{-1} \sum_{i=1}^n \eta_i K_a(X_i - x)\right) h_1(x)dx \\ &= E \int \widehat{h}_1^2(x|a)dx - 2E \int \widehat{h}_1(x|a)h_1(x)dx \\ &= M_{h_1}(a) - R(h_1), \end{aligned}$$

implying that the same value minimizes both $M_{h_1}(a)$ and $E(\lambda(a))$. This suggests that it is reasonable to choose the minimizer of $\lambda(a)$, which gives the pilot BW g .

2.5 An Illustration

We employ the version of Primary Biliary Cirrhosis (PBC) data (Fleming and Harrington 1991) available at <http://lib.stat.cmu.edu/datasets/psc>. We considered the 374 female patients for our analysis and imputed missingness by taking $\pi(t) = e^t/(1 + e^t)$, giving a missingness percentage of 44. The observed times were divided by 4800, to restrict them between 0 and 1, and the minimization was performed over a grid of 500 BW values in (0.001, 0.5). Numerical integrations were performed over a grid of 200 points in (0, 1). The number of bootstrap samples was 500. The BWs in days are given in Table 1 below.

Since missingness was artificially introduced, the benchmark in this case, namely the Kaplan {Meier estimator, can be computed. The IPW and Dikta-type estimated survival curves were calculated using the two-step EISB BWs. All the curves are shown in Figure 1.

3 Numerical results

We performed three different simulation studies to investigate the operating characteristics of the CV, EISB, and TSB choice of BWs. According to Faraway and Jhun (1990), “a good benchmark with which to measure the performance” of the above three BW estimators is the ISE choice, which provides the best possible BW given complete knowledge of the underlying subdensity. We have not provided results for the SCV and ASB choices of BWs. For all the simulations (as well as the illustration), we employed the Epanechnikov kernel. The sample size was 100, the number of bootstrap samples was 200, and the number of Monte Carlo replications was 1000. We only report the results of one of the simulation studies, referring the interested reader to our technical report (Subramanian and Bean 2008).

The failure time distribution was Weibull of the type $F(x) = 1 - \exp(-(ax)^2)$. The censoring distribution was exponential with mean 1. Then $p(x) = 2ax/(1 + 2ax)$. The conditional probability of nonmissingness was taken as the two parameter logit model: $\pi(x) = \exp(\theta_1 + \theta_2 x)/(1 + \exp(\theta_1 + \theta_2 x))$. The parameters α and θ_1, θ_2 were chosen to get the following combinations of CR and MR: (10%, 44%), (20%, 35%), (25%, 19%), and (42%, 15%). The numerical integrations were performed over a grid of 100 points. Minimizations were over BWs taken in a fine grid of values of the unit interval. From Tables 2 and 3, the two-step EISB choice of BW performs best in the sense of nearness to the ISE choice as well as in terms of relative efficiency.

In Table 4, we provide the MISE and standard deviation for the IPW and Dikta-type estimators, computed using the two-step EISB choice of BW. The Dikta-type estimator performs marginally better than the IPW estimator for Simulation 2.

4 Conclusion

Bootstrap-based BW selection procedures for computing kernel subdensity estimators were investigated in this paper. The methods mirror their counterpart for density estimation. The two-step empirical iterated smoothed bootstrap produces superior BW estimates as evidenced by several simulation studies. This “optimal” BW was plugged into the Dikta-type and IPW survival function estimators and a MISE analysis was performed. The Dikta-type estimator produces marginally better estimates compared to the IPW estimator. The methods were applied to a real data set with simulated MCIs.

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Appendix

Here we give a sketch of the derivation. Details can be found in a technical report (Subramanian and Bean 2008). Following convention, we denote P^* and E^* to be the probability and expectation, respectively, conditional on the observed sample points $\{(X_i, \eta_i), 1 \leq i \leq n\}$. Conditional on the sample, the bootstrap random variable (X, η) is distributed uniformly over each of the data points. Let E_K denote the expectation with respect to the distribution of ε . Assuming independence between ε and (X, η) , we can show that

$$P^*(X^* \leq x, \eta^* = 1) = E_K \left\{ \frac{1}{n} \sum_{i=1}^n \eta_i I(X_i + g\varepsilon \leq x) \right\}.$$

It follows that

$$P^*(X^* \leq x, \eta^* = 1) = \frac{1}{n} \sum_{i=1}^n P\left(\varepsilon \leq \frac{x - X_i}{g}\right),$$

from which we can deduce that the derivative with respect to x is $\hat{h}_1(x)$. That is, $(X^*, \eta^* = 1)$, conditional on the sample, are distributed with subdensity $\hat{h}_1(x)$. The representation for the bootstrap MISE with $\hat{h}_1$ plugged in for the true subdensity now follows readily. Specifically, denoting $\widehat{h}_1(x|a)$, $\widehat{h}_1^*(x|a)$ etc as $\widehat{h}_{1,a}(x)$, $\widehat{h}_{1,a}^*(x)$, we have that $E^*(\widehat{h}_{1,a}^*(x)) = (K_a * \widehat{h}_{1,g})(x)$. Therefore the bias and variance are given by

$$B^*(x) = (K_a * \widehat{h}_{1,g})(x) - \widehat{h}_{1,g}(x) \quad (\text{A.1})$$

$$\begin{aligned} \text{Var}^*(\widehat{h}_{1,a}^*(x)) \\ = \frac{1}{n} \left((K_a^2 * \widehat{h}_{1,g})(x) - (K_a * \widehat{h}_{1,g})^2(x) \right). \end{aligned} \quad (\text{A.2})$$

Using Eq. (A.1) and Eq. (A.2), we can show that

$$\begin{aligned} \widehat{M}_{h_1}(a) = & \frac{1}{n} \int (K_a^2 * \widehat{h}_{1,g})(x) dx + \left(1 - \frac{1}{n}\right) \int (K_a * \widehat{h}_{1,g})^2(x) dx \\ & - 2 \int (K_a * \widehat{h}_{1,g})(x) \widehat{h}_{1,g}(x) dx + \int \widehat{h}_{1,g}(x)^2 dx. \end{aligned}$$

Furthermore, we can also show that the first term on the right side of the above equation is

$$\begin{aligned} \frac{1}{n} \int (K_a^2 * \widehat{h}_{1,g})(x) dx \\ = \frac{1}{na} \left(\frac{1}{n} \sum_{i=1}^n \eta_i \right) \int_{-M}^M K^2(u) du \end{aligned}$$

Therefore,

$$\begin{aligned} \widehat{M}_{h_1}(a) = & \frac{1}{na} \left(\frac{1}{n} \sum_{i=1}^n \eta_i \right) \int_{-M}^M K^2(u) du + \left(1 - \frac{1}{n}\right) \int (K_a * \widehat{h}_{1,g})^2(x) dx \\ & - 2 \int (K_a * \widehat{h}_{1,g})(x) \widehat{h}_{1,g}(x) dx + \int \widehat{h}_{1,g}(x)^2 dx. \end{aligned} \quad (\text{A.3})$$

An alternative derivation of the expression for $\hat{M}_{h1}(a)$ given by Eq. (A.3) is provided in our technical report (Subramanian and Bean 2008).

Figure 1.
Comparison of estimators for the Primary Biliary Cirrhosis data set. Missingness imputed using $\pi(t) = e^t / (1 + e^t)$.

Table 1BW_s (in days) for the PBC data.

Parameter	CV	TSB	EISB
h	451.2	537.6	734.4
π_1	312.0	537.6	873.6
p_1	316.8	662.4	916.8

Table 2Simulation 2. Mean and standard deviation (SD) of BW for π_1 and p_1 .

Parameter	CR	MR	ISE BW		CV BW		TSB BW		EISB BW	
			Mean	SD	Mean	SD	Mean	SD	Mean	SD
π_1	10%	44%	0.063	0.016	0.035	0.012	0.052	0.012	0.064	0.016
	20%	35%	0.124	0.028	0.068	0.026	0.092	0.028	0.120	0.029
	25%	19%	0.152	0.033	0.100	0.039	0.118	0.036	0.150	0.033
p_1	42%	15%	0.263	0.066	0.171	0.070	0.197	0.063	0.261	0.064
	10%	44%	0.056	0.014	0.034	0.011	0.052	0.012	0.065	0.017
	20%	35%	0.114	0.026	0.065	0.026	0.094	0.029	0.123	0.032
	25%	19%	0.198	0.043	0.090	0.038	0.121	0.038	0.154	0.039
	42%	15%	0.303	0.074	0.162	0.075	0.226	0.070	0.299	0.083

Table 3Simulation 2. Relative efficiencies of CV, TSB and EISB BWs for π_1 and p_1 .

Parameter	CR	MR	CV		TSB		EISB	
			Mean	SD	Mean	SD	Mean	SD
π_1	10%	44%	3.42	4.25	2.13	3.38	1.99	3.23
	20%	35%	5.16	10.35	3.12	6.35	2.35	5.74
	25%	19%	4.17	8.55	2.80	4.82	2.03	3.73
	42%	15%	3.13	5.05	2.32	2.97	1.76	3.02
p_1	10%	44%	1.55	1.05	1.28	0.86	1.32	0.84
	20%	35%	2.73	3.42	1.72	1.73	1.54	1.50
	25%	19%	3.76	5.00	2.42	3.21	1.88	2.76
	42%	15%	4.24	7.18	2.33	3.37	1.87	3.45

Table 4

Simulation 2. Comparison of IPW and Dikta-type survival function estimators using the two-step EISB choice of BW for various CRs.

CR	MR	IPW		Dikta-type	
		MISE	SD	MISE	SD
10%	44%	0.0005	0.0004	0.0004	0.0003
20%	35%	0.0010	0.0009	0.0008	0.0007
25%	19%	0.0012	0.0011	0.0011	0.0011
42%	15%	0.0029	0.0025	0.0027	0.0027