New stability criteria for uncertain neural networks with interval time-varying delays

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Abstract This paper is concerned with the stability analysis for neural networks with interval time-varying delays and parameter uncertainties. An approach combining the Lyapunov-Krasovskii functional with the differential inequality and linear matrix inequality techniques is taken to investigate this problem. By constructing a new Lyapunov-Krasovskii functional and introducing some free weighting matrices, some less conservative delay-derivative-dependent and delay-derivative-independent stability criteria are established in term of linear matrix inequality. And the new criteria are applicable to both fast and slow time-varying delays. Three numerical examples show that the proposed criterion are effective and is an improvement over some existing results in the literature.

Keywords Stability · Neural networks · Interval time-varying delays · Parameter uncertainties · Linear matrix inequality

Introduction

In the past decades, neural networks have received a great deal of interest due to their extensive applications in image

processing, quadratic optimization, fixed-point computation, pattern recognition, associative memory, another areas (Borkar and Soumyanatha 1997; Chua and Yang 1988; Cichocki and Unbehauen 1993; Michel and Liu 2002; Chen and Fang 2000). These applications strongly depend on the dynamic behavior of the network. However, in neural processing and signal transmission, significant time delays as a source of instability and bad performance may occur. Therefore, the stability of delayed neural networks has received considerable attention and a large amount of literature has been available (Arik 2000, 2002; Cao et al. 2005, 2006; Li et al. 2006, 2007; Liao et al. 2002; Liao et al. 2005; Wang 2007).

Liao et al. (2002) derives some sufficient conditions for asymptotic stability of neural networks with constant or time-varying delays. The Lyapunov-Krasovskii stability theory for functional differential equations and the linear matrix inequality (LMI) approach are employed to investigate the problem. Cao et al. (2006) considered the issue of global asymptotical stability for recurrent neural networks with mixed time-varying delays. They have derived several new sufficient conditions for checking the global asymptotical stability of recurrent neural networks with mixed time-varying delays. Li et al. (2006) have investigated the asymptotical and exponential stability of cellular neural networks with single and multiple delays. The method combining Lyapunov-Krasovskii functional, parameterized first-order model transformation and the linearization of the model under consideration has been adopted to study these issues. As a result, several novel delay-dependent and delay-independent asymptotical/exponential stability criteria for delayed cellular neural networks have been exploited. Particularly, the general delayed cellular neural networks have been shifted into a class of non-autonomic linear

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systems under the appropriate assumption on the activation functions.

On the other hand, the connection weights of the neurons depend on certain resistance and capacitance values that include uncertainties (modeling errors). When modeling neural networks, the parameter uncertainties (also called variations or fluctuations) should be taken into account. And in recent years, the stability analysis issues for neural networks in the presence of parameter uncertainties perturbations have stirred some initial research attention (Park 2007; Singh 2004; Zhang et al. 2005).

Recently, a special type of time delay in practical engineering systems, i.e., interval time-varying delay, is identified and investigated (Qiu et al. 2007; He et al. 2007). Interval time-varying delay is a time delay that varies in an interval in which the lower bound is not restricted to be 0. Qiu et al. (2007) have investigated the problem of robust stability of uncertain neural networks with interval time-varying delays. The delay factor is assumed to be time-varying and belongs to a given interval, which means that the lower and upper bounds of the interval time-varying delays are available. Based on the Lyapunov-Krasovskii functional approach, a new delay-dependent stability criteria is presented in terms of linear matrix inequalities (LMIs). It is worth noting that these stability criteria in Qiu et al. (2007) leave much room for improvement. A significant source of conservativeness that could be further reduced lies in the calculation of the time-derivative of the Lyapunov-Krasovskii functional. To the best of our knowledge, very few papers investigate the stability problem of neural networks with interval time-varying delays, which remains open but challenging. Therefore, it is of great significance to consider the stability of neural networks with interval time-varying delays. He et al. (2007) have studied the stability problem for neural networks with time-varying interval delay. Some less conservative stability criteria have been obtained by considering the relationship between the time-varying delay and its lower and upper bounds when calculating the upper bound of the derivative of Lyapunov functional.

In this paper, we deal with the robust stability problem for neural networks with interval time-varying delays and parameter uncertainties by choosing an appropriate Lyapunov functional. Some delay-derivative-dependent and delay-derivative-independent stability criteria are derived based on the new Lyapunov functional and the consideration of range for the time-delay. The resulting criteria are applicable to both fast and slow time-varying delay. Finally, three numerical examples are given to demonstrate the effectiveness and the merit of the proposed method.

Notations

The notations used throughout the paper are fairly standard. The superscript “ T ” stands for matrix transposition; $\mathbb{R}^n$ denotes the n -dimensional Euclidean space; the notation $P > 0$ means that P is real symmetric and positive definite; I and 0 represent identity matrix and zero matrix. In symmetric block matrices or long matrix expressions, we use an asterisk ($\star$) to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

Problem formulation

Consider the following neural networks with interval time-varying delays and parameter uncertainties:

$$\dot{u}(t) = -(C + \Delta C)u(t) + (A + \Delta A)g(u(t)) + (B + \Delta B)g(u(t - d(t))) + I, \quad (1)$$

where $u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^T$ is the neural state vector, $g(u(t)) = [g_1(u_1(t)), g_2(u_2(t)), \dots, g_n(u_n(t))]^T$ denotes the bounded neuron activation function with $g(0) = 0$, $I = [I_1, I_2, \dots, I_n]^T$ is a constant input vector. $C = \text{diag}[c_1, c_2, \dots, c_n] > 0$, $A = (a_{ij})_{n \times n}$, $B = (b_{ij})_{n \times n}$ are the inter connection matrices representing the weight coefficients of the neurons.

Assumption 1 The time-varying delay $d(t)$ satisfies.

$$0 \leq h_1 \leq d(t) \leq h_2, \quad (2)$$

$$\dot{d}(t) \leq \tau \quad (3)$$

where h_1, h_2, τ are constants.

Remark 1 Obviously, when $\tau = 0$ and $h_1 = h_2$ then $d(t)$ denotes a constant delay; the case when $h_1 = 0$, it implies that $0 \leq d(t) \leq h_2$ which is investigated in almost all the reported literature.

Assumption 2 The parameter uncertainties $\Delta C, \Delta A, \Delta B$ are of the form:

$$[\Delta C \quad \Delta A \quad \Delta B] = HF(t)[E_c \quad E_a \quad E_b], \quad (4)$$

in which H, E_c, E_a, E_b are known constant matrices with appropriate dimensions. The uncertain matrix $F(t)$ satisfies $F^T(t)F(t) \leq I$ for $\forall t \in \mathbb{R}$. (5)

In addition, the activation function $g(\cdot)$ is bounded, and satisfies that

$$(H) \quad 0 \leq \frac{g_i(x) - g_i(y)}{x - y} \leq l, \quad \forall x, y \in \mathbb{R}^n, x \neq y, \\ i = 1, 2, \dots, n \quad (6)$$

In the following, we always shift the equilibrium point u^* to the origin by transformation $x(\cdot) = u(\cdot) - u^*$ puts system (1) into the following form:

$$\dot{x}(t) = -(C + \Delta C)x(t) + (A + \Delta A)f(x(t)) \\ + (B + \Delta B)f(x(t - d(t))), \quad (7)$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$ is the state vector of the transformed system, with

$$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t))]^T$$

$$\text{and } f_j(x_j(t)) = g_j(x_j(t) + u_j^*) - g_j(u_j^*), \quad j = 1, 2, \dots, n$$

Note that functions $f(\cdot)$ here satisfy condition (H). Which is equivalent to

$$f_i(x_i)[f_i(x_i) - lx_i] \leq 0 \quad (8)$$

Now, we give the following lemma that is useful in deriving our LMI-based stability criterion.

Lemma 1 [Schur complement] *Given constant symmetric matrices $\sum_1, \sum_2, \sum_3$ where $\sum_1 = \sum_1^T$ and $0 < \sum_2 = \sum_2^T$, then $\sum_1 + \sum_3 \sum_2^{-1} \sum_3 < 0$ if and only if*

$$\begin{bmatrix} \sum_1 & \sum_3^T \\ \sum_3 & -\sum_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\sum_2 & \sum_3 \\ \sum_3^T & \sum_1 \end{bmatrix} < 0. \quad (9)$$

Lemma 2 *For any $z, y \in \mathbb{R}^{n \times m}$ and a positive scalar ε , the following inequality:*

$$2z^T y \leq \varepsilon z^T z + \varepsilon^{-1} y^T y \quad (10)$$

holds.

Main results

In order to discuss robust stability of system (1), which has parametric uncertainties (4) and (5), first, we consider the case in which the matrices C , A and B are fixed, i.e., $\Delta C = 0$, $\Delta A = 0$ and $\Delta B = 0$. For this case, the following theorem holds.

Theorem 1 *For given scalars $0 \leq h_1 \leq d(t) \leq h_2$ and τ , the neural network (7) is asymptotically stable, if there exist definite matrices $P > 0$, $Q_r = Q_r^T \geq 0$, $r = 1, 2, 3, 4$, $Z_j = Z_j^T > 0$, $j = 1, 2$, $N = [N_1^T N_2^T N_3^T N_4^T N_5^T N_6^T]^T$, $S = [S_1^T S_2^T S_3^T S_4^T S_5^T S_6^T]^T$, $J = [J_1^T J_2^T J_3^T J_4^T J_5^T J_6^T]^T$, $K = \text{diag}(k_1, k_2, \dots, k_i)$, $i = 1, 2, \dots, n$ such that the following LMI holds:*

$$\Xi = \begin{bmatrix} \Psi & h_2 N & \delta S & \delta J & W^T U \\ \star & -h_2 Z_1 & 0 & 0 & 0 \\ \star & \star & -\delta(Z_1 + Z_2) & 0 & 0 \\ \star & \star & \star & -\delta Z_2 & 0 \\ \star & \star & \star & \star & -U \end{bmatrix} < 0, \quad (11)$$

where

$$\Psi = \Psi_1 + \Psi_2 + \Psi_2^T,$$

$$\Psi_1 = \begin{bmatrix} \Psi_{11} & 0 & 0 & 0 & \Psi_{15} & PB \\ \star & \Psi_{22} & 0 & 0 & 0 & T \\ \star & \star & -Q_1 & 0 & 0 & 0 \\ \star & \star & \star & -Q_2 & 0 & 0 \\ \star & \star & \star & \star & \Psi_{55} & KB \\ \star & \star & \star & \star & \star & \Psi_{66} \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} N_1 & -N_1 + S_1 - J_1 & J_1 & -S_1 & 0 & 0 \\ N_2 & -N_2 + S_2 - J_2 & J_2 & -S_2 & 0 & 0 \\ N_3 & -N_3 + S_3 - J_3 & J_3 & -S_3 & 0 & 0 \\ N_4 & -N_4 + S_4 - J_4 & J_4 & -S_4 & 0 & 0 \\ N_5 & -N_5 + S_5 - J_5 & J_5 & -S_5 & 0 & 0 \\ N_6 & -N_6 + S_6 - J_6 & J_6 & -S_6 & 0 & 0 \end{bmatrix}$$

and

$$\Psi_{11} = -PC - C^T P + Q_1 + Q_2 + Q_3,$$

$$\Psi_{15} = PA - C^T K + R,$$

$$\Psi_{22} = -(1 - \tau)Q_3,$$

$$\Psi_{55} = Q_4 + KA + A^T K - 2I^{-1}R$$

$$\Psi_{66} = -(1 - \tau)Q_4 - 2I^{-1}T,$$

$$U = h_2 Z_1 + \delta Z_2,$$

$$\delta = h_2 - h_1.$$

Proof The Lyapunov functional of system (7) is defined by:

$$V(x(t)) = \sum_{i=1}^4 V_i(x(t))$$

$$V_1(x(t)) = x^T(t)Px(t) + 2 \sum_{i=1}^n k_i \int_0^{x_i} f_i(s)ds$$

$$V_2(x(t)) = \int_{t-h_1}^t x^T(s)Q_1x(s)ds + \int_{t-h_2}^t x^T(s)Q_2x(s)ds$$

$$V_3(x(t)) = \int_{t-d(t)}^t [x^T(s)Q_3x(s) + f^T(x(s))Q_4f(x(s))]ds$$

$$V_4(x(t)) = \int_{-h_2}^0 \int_{t+\theta}^t \dot{x}^T(s) Z_1 \dot{x}(s) ds d\theta \\ + \int_{-h_2}^{-h_1} \int_{t+\theta}^t \dot{x}^T(s) Z_2 \dot{x}(s) ds d\theta.$$

where $P > 0$, $Q_r = Q_r^T \geq 0$, $r = 1, 2, 3, 4$ and $Z_j = Z_j^T > 0$, $j = 1, 2$, are to be determined. From the Leibniz–Newton formula, the following equations are true for any matrices N, S and J with appropriate dimensions,

$$2\xi^T(t)N \left[x(t) - x(t-d(t)) - \int_{t-d(t)}^t \dot{x}(s) ds \right] = 0, \quad (12)$$

$$2\xi^T(t)S \left[x(t-d(t)) - x(t-h_2) - \int_{t-h_2}^{t-d(t)} \dot{x}(s) ds \right] = 0, \quad (13)$$

$$2\xi^T(t)J \left[x(t-h_1) - x(t-d(t)) - \int_{t-d(t)}^{t-h_1} \dot{x}(s) ds \right] = 0, \quad (14)$$

Noting that, for any diagonal matrices $R > 0$ and $T > 0$, from Eq. 8 there exist that

$$0 = 2x^T(t)Rf(x(t)) + 2x^T(t-d(t))Tf(x(t-d(t))) \\ - 2x^T(t)Rf(x(t)) - 2x^T(t-d(t))Tf(x(t-d(t))) \\ \leq 2x^T(t)Rf(x(t)) + 2x^T(t-d(t))Tf(x(t-d(t))) \\ - 2l^{-1}f^T(x(t))Rf(x(t)) \\ - 2l^{-1}f^T(x(t-d(t)))Tf(x(t-d(t))) \quad (15)$$

Calculating the derivative of $V(x(t))$ along the solutions of system (7)

$$\dot{V}_1(x(t)) = 2x^T(t)P\dot{x}(t) + 2f^T(x(t))K\dot{x}(t) \\ = -2x^T(t)PCx(t) + 2x^T(t)PAf(x(t)) \\ + 2x^T(t)PBf(x(t-d(t))) \\ - 2f^T(x(t))KCx(t) + 2f^T(x(t))KAf(x(t)) \\ + 2f^T(x(t))KBf(x(t-d(t))) \quad (16)$$

$$\dot{V}_2(x(t)) = x^T(t)(Q_1 + Q_2)x(t) - x^T(t-h_1)Q_1x(t-h_1) \\ - x^T(t-h_2)Q_2x(t-h_2) \quad (17)$$

$$\dot{V}_3(x(t)) = x^T(t)Q_3x(t) + f^T(x(t))Q_4f(x(t)) \\ - (1 - \dot{d}(t))f^T(x(t-d(t)))Q_4f(x(t-d(t))) \\ - (1 - \dot{d}(t))x^T(t-d(t))Q_3x(t-d(t)) \\ \leq x^T(t)Q_3x(t) + f^T(x(t))Q_4f(x(t)) \\ - (1 - \tau)f^T(x(t-d(t)))Q_4f(x(t-d(t))) \\ - (1 - \tau)x^T(t-d(t))Q_3x(t-d(t)) \quad (18)$$

$$\dot{V}_4(x(t)) = \dot{x}^T(t)(h_2Z_1 + (h_2 - h_1)Z_2)\dot{x}(t) \\ - \int_{t-h_2}^{t-d(t)} \dot{x}^T(s)(Z_1 + Z_2)\dot{x}(s) ds \\ - \int_{t-d(t)}^t \dot{x}^T(s)Z_1\dot{x}(s) ds \\ - \int_{t-d(t)}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s) ds \quad (19)$$

Combining Eqs. (15–19) and adding the left side of Eqs. (12–14) into the derivative of $V(x(t))$

$$\dot{V}(x(t)) \leq \xi^T(t)[\Psi_1 + \Psi_2 + \Psi_2^T + W^T(h_2Z_1 + \delta Z_2)W \\ + h_2NZ_1^{-1}N^T + \delta S(Z_1 + Z_2)^{-1}S^T + \delta JZ_2^{-1}J^T]\xi(t) \\ - \int_{t-d(t)}^t [\xi^T(t)N + \dot{x}^T(s)Z_1]Z_1^{-1}[N^T\xi(t) + Z_1\dot{x}(s)]ds \\ - \int_{t-d(t)}^{t-h_1} [\xi^T(t)J + \dot{x}^T(s)Z_2]Z_2^{-1}[J^T\xi(t) + Z_2\dot{x}(s)]ds \\ - \int_{t-h_2}^{t-d(t)} [\xi^T(t)S + \dot{x}^T(s)(Z_1 + Z_2)](Z_1 + Z_2)^{-1} \\ \times [S^T\xi(t) + (Z_1 + Z_2)\dot{x}(s)]ds \quad (20)$$

where

$$\xi(t) = \begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-h_1) \\ x(t-h_2) \\ f(x(t)) \\ f(x(t-d(t))) \end{bmatrix}, \quad N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{bmatrix}, \quad S = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix}, \\ J = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix},$$

$$W = [-C \ 0 \ 0 \ 0 \ A \ B],$$

$$\delta = h_2 - h_1.$$

Since $Z_i > 0, i = 1, 2$, then the last three parts in Eq. 20 are all < 0 . So, if

$$\Psi_1 + \Psi_2 + \Psi_2^T + W^T(h_2 Z_1 + \delta Z_2)W + h_2 N Z_1^{-1} N^T + \delta J Z_2^{-1} J^T + \delta S(Z_1 + Z_2)^{-1} S^T < 0,$$

which is equivalent to Eq. 11 by Schur complements, then $\dot{V}(x(t)) < -\varepsilon \|x(t)\|^2$ for a sufficiently small $\varepsilon > 0$ and $x(t) \neq 0$, which ensures the asymptotic stability of system (7), see e.g. Hale and Verduyn Lunel (1993). The proof is completed. $\square$

Remark 2 It is worth noting that the results in Qiu et al. (2007) are only applicable to systems with fast time-varying delay. In fact, in many cases, the derivative of time-varying delays is known and may be small. Thus, the results in Qiu et al. (2007) may have limited use. In our Theorem 1, τ can be any value or unknown. Therefore, Theorem 1 is applicable to both fast and slow time-varying delays.

In fact, Theorem 1 gives a criterion for system (1) with $d(t)$ satisfying (2) and (3). In many cases, the information of the derivative of delay is unknown. Regarding this circumstance, a rate-independent criterion for a delay only satisfying (2) is derived as follows by choosing $Q_3 = Q_4 = 0$ in Theorem 1.

Corollary 1 For given scalars $0 \leq h_1 \leq d(t) \leq h_2$, the neural network (7) is asymptotically stable, if there exist definite matrices $P > 0$, $Q_r = Q_r^T \geq 0$, $r = 1, 2$, $Z_j = Z_j^T > 0$, $j = 1, 2$, $N = [N_1^T \ N_2^T \ N_3^T \ N_4^T \ N_5^T \ N_6^T]^T$, $J = [J_1^T \ J_2^T \ J_3^T \ J_4^T \ J_5^T \ J_6^T]^T$, $S = [S_1^T \ S_2^T \ S_3^T \ S_4^T \ S_5^T \ S_6^T]^T$, and $K = \text{diag}(k_1, k_2, \dots, k_i)$, $i = 1, 2, \dots, n$ such that the following LMI holds:

$$\tilde{\Xi} = \begin{bmatrix} \tilde{\Psi} & h_2 N & \delta S & \delta J & W^T U \\ \star & -h_2 Z_1 & 0 & 0 & 0 \\ \star & \star & \Xi_{33} & 0 & 0 \\ \star & \star & \star & -\delta Z_2 & 0 \\ \star & \star & \star & \star & -U \end{bmatrix} < 0, \quad (21)$$

where

$$\tilde{\Psi} = \tilde{\Psi}_1 + \Psi_2 + \Psi_2^T,$$

$$\tilde{\Psi}_1 = \begin{bmatrix} \Psi_{11} & 0 & 0 & 0 & \Psi_{15} & PB \\ \star & 0 & 0 & 0 & 0 & T \\ \star & \star & -Q_1 & 0 & 0 & 0 \\ \star & \star & \star & -Q_2 & 0 & 0 \\ \star & \star & \star & \star & \tilde{\Psi}_{55} & KB \\ \star & \star & \star & \star & \star & -2I^{-1}T \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} N_1 & -N_1 + S_1 - J_1 & J_1 & -S_1 & 0 & 0 \\ N_2 & -N_2 + S_2 - J_2 & J_2 & -S_2 & 0 & 0 \\ N_3 & -N_3 + S_3 - J_3 & J_3 & -S_3 & 0 & 0 \\ N_4 & -N_4 + S_4 - J_4 & J_4 & -S_4 & 0 & 0 \\ N_5 & -N_5 + S_5 - J_5 & J_5 & -S_5 & 0 & 0 \\ N_6 & -N_6 + S_6 - J_6 & J_6 & -S_6 & 0 & 0 \end{bmatrix}$$

and

$$\Xi_{33} = -\delta(Z_1 + Z_2),$$

$$\Psi_{11} = -PC - C^T P + Q_1 + Q_2 + Q_3,$$

$$\Psi_{15} = PA - C^T K + R,$$

$$\tilde{\Psi}_{55} = Q_4 + KA + A^T K - 2I^{-1}R$$

$$U = h_2 Z_1 + \delta Z_2,$$

$$\delta = h_2 - h_1.$$

The following result provides the feasible robust stability criterion for systems with the admissible uncertainty.

Theorem 2 For given scalars $0 \leq h_1 \leq d(t) \leq h_2$ and τ , the neural network (7) is robustly, asymptotically stable, if there exist definite matrices $P > 0$, $Q_r = Q_r^T \geq 0$, $r = 1, 2, 3, 4$, $Z_j = Z_j^T > 0$, $j = 1, 2$, $N = [N_1^T \ N_2^T \ N_3^T \ N_4^T \ N_5^T \ N_6^T]^T$, $S = [S_1^T \ S_2^T \ S_3^T \ S_4^T \ S_5^T \ S_6^T]^T$, $J = [J_1^T \ J_2^T \ J_3^T \ J_4^T \ J_5^T \ J_6^T]^T$, $K = \text{diag}(k_1, k_2, \dots, k_i)$, $i = 1, 2, \dots, n$ and three positive scalars ε_i , $i = 1, 2, 3$, such that the following LMI holds:

$$\Pi = \begin{bmatrix} \tilde{\Psi} & h_2 N & \delta S & \delta J & W^T U & 0 \\ \star & -h_2 Z_1 & 0 & 0 & 0 & 0 \\ \star & \star & \Xi_{33} & 0 & 0 & 0 \\ \star & \star & \star & -\delta Z_2 & 0 & 0 \\ \star & \star & \star & \star & -U & HU^T \\ \star & \star & \star & \star & \star & -\varepsilon_3 I \end{bmatrix} < 0, \quad (22)$$

where

$$\tilde{\Psi} = \tilde{\Psi}_1 + \Psi_2 + \Psi_2^T,$$

$$\tilde{\Psi}_1 = \begin{bmatrix} \tilde{\Psi}_{11} & 0 & 0 & 0 & \tilde{\Psi}_{15} & \tilde{\Psi}_{16} & PH & 0 \\ \star & \Psi_{22} & 0 & 0 & 0 & T & 0 & 0 \\ \star & \star & -Q_1 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -Q_2 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \tilde{\Psi}_{55} & \tilde{\Psi}_{56} & 0 & KH \\ \star & \star & \star & \star & \star & \tilde{\Psi}_{66} & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\varepsilon_1 I & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\varepsilon_2 I \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} N_1 & -N_1 + S_1 - J_1 & J_1 & -S_1 & 0 & 0 \\ N_2 & -N_2 + S_2 - J_2 & J_2 & -S_2 & 0 & 0 \\ N_3 & -N_3 + S_3 - J_3 & J_3 & -S_3 & 0 & 0 \\ N_4 & -N_4 + S_4 - J_4 & J_4 & -S_4 & 0 & 0 \\ N_5 & -N_5 + S_5 - J_5 & J_5 & -S_5 & 0 & 0 \\ N_6 & -N_6 + S_6 - J_6 & J_6 & -S_6 & 0 & 0 \end{bmatrix},$$

and

$$\Xi_{33} = -\delta(Z_1 + Z_2),$$

$$\tilde{\Psi}_{11} = -PC - C^T P + Q_1 + Q_2 + Q_3 + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_c^T E_c,$$

$$\tilde{\Psi}_{15} = PA - C^T K + R - (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_c^T E_a,$$

$$\tilde{\Psi}_{16} = PB - (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_c^T E_b,$$

$$\Psi_{22} = -(1 - \tau)Q_3,$$

$$\tilde{\Psi}_{55} = Q_4 + KA + A^T K - 2I^{-1}R + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_a^T E_a,$$

$$\tilde{\Psi}_{56} = KB + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_a^T E_b,$$

$$\tilde{\Psi}_{66} = -(1 - \tau)Q_4 - 2I^{-1}T + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_b^T E_b,$$

$$U = h_2 Z_1 + \delta Z_2,$$

$$\delta = h_2 - h_1.$$

Proof By Lemma 1, the system is robustly, asymptotically stable if the following inequality holds:

$$\Pi = \Xi + \Omega_1 F(t) \Omega_2^T + \Omega_2 F(t) \Omega_1^T + \Omega_3 F(t) \Omega_4^T + \Omega_4 F(t) \Omega_3^T + \Omega_5 F(t) \Omega_6^T + \Omega_6 F(t) \Omega_5^T < 0 \quad (23)$$

where

$$\begin{aligned} \Omega_1 &= [H^T P \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T, \\ \Omega_2 &= [-E_c \quad 0 \quad 0 \quad 0 \quad E_a \quad E_b \quad 0 \quad 0 \quad 0 \quad 0]^T, \\ \Omega_3 &= [0 \quad 0 \quad 0 \quad 0 \quad H^T K \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T, \\ \Omega_4 &= [-E_c \quad 0 \quad 0 \quad 0 \quad E_a \quad E_b \quad 0 \quad 0 \quad 0 \quad 0]^T, \\ \Omega_5 &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad H^T U]^T, \\ \Omega_6 &= [-E_c \quad 0 \quad 0 \quad 0 \quad E_a \quad E_b \quad 0 \quad 0 \quad 0 \quad 0]^T. \end{aligned}$$

By Lemma 2, Eq. (23) holds if the following inequality satisfies:

$$\begin{aligned} \Xi + \varepsilon_1^{-1} \Omega_1 \Omega_1^T + \varepsilon_1 \Omega_2 \Omega_2^T + \varepsilon_2^{-1} \Omega_3 \Omega_3^T + \varepsilon_2 \Omega_4 \Omega_4^T \\ + \varepsilon_3^{-1} \Omega_5 \Omega_5^T + \varepsilon_3 \Omega_6 \Omega_6^T \\ \equiv \Xi + \Omega < 0 \end{aligned} \quad (24)$$

where $\varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0$,

$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & 0 & 0 & \Omega_{15} & \Omega_{16} & 0 & 0 & 0 & 0 \\ \star & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \Omega_{55} & \Omega_{56} & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \Omega_{66} & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \Omega_{1010} \end{bmatrix},$$

$$\Omega_{11} = \varepsilon_1^{-1} P H H^T P + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) E_c^T E_c,$$

$$\Omega_{15} = -(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) E_c^T E_a,$$

$$\Omega_{16} = -(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) E_c^T E_b,$$

$$\Omega_{55} = \varepsilon_2^{-1} K H H^T K + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) E_a^T E_a,$$

$$\Omega_{56} = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) E_a^T E_b,$$

$$\Omega_{66} = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) E_b^T E_b,$$

$$\Omega_{1010} = \varepsilon_3^{-1} U^T H H^T U.$$

Then, by Lemma 1, the inequality given in Eq. 24 is equivalent to the LMI Eq. 22. Thus, if the LMIs given in Eq. 22 holds, the system (1) is robust asymptotically stable. This completes the proof.

By setting $Q_3 = Q_4 = 0$, the Corollary 2 is established from Theorem 2. $\square$

Corollary 2 For given scalars $0 \leq h_1 \leq d(t) \leq h_2$, the neural network (7) is robustly, asymptotically stable, if there exist definite matrices $P > 0, Q_r = Q_r^T \geq 0, r = 1, 2, Z_j = Z_j^T > 0, j = 1, 2, N = [N_1^T \ N_2^T \ N_3^T \ N_4^T \ N_5^T \ N_6^T]^T, S = [S_1^T \ S_2^T \ S_3^T \ S_4^T \ S_5^T \ S_6^T]^T, J = [J_1^T \ J_2^T \ J_3^T \ J_4^T \ J_5^T \ J_6^T]^T, K = \text{diag}(k_1, k_2, \dots, k_i), i = 1, 2, \dots, n$ and three positive scalars $\varepsilon_i, i = 1, 2, 3$, such that the following LMI holds:

$$\begin{bmatrix} \hat{\Psi} & -h_2 Z_1 & 0 & 0 & 0 & 0 \\ \star & \star & \Xi_{33} & 0 & 0 & 0 \\ \star & \star & \star & -\delta Z_2 & 0 & 0 \\ \star & \star & \star & \star & -U & H U^T \\ \star & \star & \star & \star & \star & -\varepsilon_3 I \end{bmatrix} < 0, \quad (25)$$

where

$$\hat{\Psi} = \tilde{\Psi}_1 + \Psi_2 + \Psi_2^T,$$

$$\tilde{\Psi}_1 = \begin{bmatrix} \tilde{\Psi}_{11} & 0 & 0 & 0 & \tilde{\Psi}_{15} & \tilde{\Psi}_{16} & PH & 0 \\ \star & 0 & 0 & 0 & 0 & T & 0 & 0 \\ \star & \star & -Q_1 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -Q_2 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \tilde{\Psi}_{55} & \tilde{\Psi}_{56} & 0 & KH \\ \star & \star & \star & \star & \star & \tilde{\Psi}_{66} & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\varepsilon_1 I & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\varepsilon_2 I \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} N_1 & -N_1 + S_1 - J_1 & J_1 & -S_1 & 0 & 0 \\ N_2 & -N_2 + S_2 - J_2 & J_2 & -S_2 & 0 & 0 \\ N_3 & -N_3 + S_3 - J_3 & J_3 & -S_3 & 0 & 0 \\ N_4 & -N_4 + S_4 - J_4 & J_4 & -S_4 & 0 & 0 \\ N_5 & -N_5 + S_5 - J_5 & J_5 & -S_5 & 0 & 0 \\ N_6 & -N_6 + S_6 - J_6 & J_6 & -S_6 & 0 & 0 \end{bmatrix}$$

where

$$\tilde{\Psi}_{11} = -PC - C^T P + Q_1 + Q_2 + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_c^T E_c,$$

$$\tilde{\Psi}_{55} = KA + A^T K - 2I^{-1}R + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_a^T E_a,$$

$$\tilde{\Psi}_{66} = -2I^{-1}T + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)E_b^T E_b,$$

and other parameters are defined in Theorem 2.

Numerical examples

In this section, three examples are given to show the effectiveness and less conservativeness of our theoretical results.

Example 1 Consider the following neural networks with interval time-varying delays:

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, A = \begin{bmatrix} 0.7 & 0.8 \\ -0.5 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & -0.2 \end{bmatrix},$$

$$l = 0.5, \tau = 2.$$

The calculation results obtained by Theorem 1 in this paper for different cases are listed in Table 1.

Remark 3 From Example 1, we can see that our results are correct for fast time-varying delay. And, it is easy to verify that our theoretical result is also applicable to slow time-varying delay.

In the following, the example in Yang et al. (2006) and Qiu et al. (2007) is used to demonstrate that our result is better than some existing results in the literature.

Table 1 Allowable upper bound of h_2 with given h_1

	$h_1 = 0$	$h_1 = 0.1$	$h_1 = 0.5$	$h_1 = 0.8$	$h_1 = 1$
h_2	11.0183	11.0727	11.4128	11.7128	11.9128

Example 2 Consider the following neural networks with interval time-varying delays:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t - d(t))), \quad (26)$$

$$C = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, A = \begin{bmatrix} -0.3 & 0.3 \\ 0.1 & -0.1 \end{bmatrix}, B = \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & 0.3 \end{bmatrix},$$

$l = 1, h_1 = 0.1, h_2 = 2.3, \tau$ is unknown.

Using Matlab LMI Control Toolbox, by our Corollary 1, we can find that the system (17) asymptotically stable and a part of the solutions of LMI (21) are given as follows:

$$P = 10^3 \times \begin{bmatrix} 1.1745 & -0.1290 \\ -0.1290 & 0.8632 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 11.1176 & 6.2402 \\ 7.9867 & 14.6987 \end{bmatrix},$$

$$R = \begin{bmatrix} 251.8854 & 0 \\ 0 & 251.8854 \end{bmatrix},$$

$$T = \begin{bmatrix} 282.2503 & 0 \\ 0 & 282.2503 \end{bmatrix}.$$

Remark 4 It should be pointed out that, if we let $h_1 = 0.1, h_2 = 2.6$, the conditions in Qiu et al. (2007) are not feasible, but using Corollary 1 in this paper, we can find that system (26) is still asymptotically stable. Therefore, our method is less conservative in some degree than that in Qiu et al. (2007).

Remark 5 If we let $h_1 = 0, h_2 = 2.7104$, we can also find the neural network (26) is asymptotically stable. Here, the solution of the LMI (21) is omitted. The same neural networks have been considered in Yang et al. (2006) and Qiu et al. (2007), where the maximal admissible time delay for stability was obtained to be $h_2 = 0.2916$ and $h_2 = 2.2397$, respectively. By Corollary 1, we can have the maximal admissible time delay for stability is $h_2 = 2.7104$. It is clear that $2.7104 > 2.2397 > 0.2916$.

Further, another example is given to illustrate the effectiveness of Corollary 2.

Example 3 Consider the following neural networks with interval time-varying delays and parameter uncertainties:

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, A = \begin{bmatrix} 0.7 & 0.8 \\ -0.5 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 0.2 & 0.2 \\ -0.6 & 0.1 \end{bmatrix},$$

$$E_c = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.2 \end{bmatrix}, E_a = \begin{bmatrix} 0.1 & 0.6 \\ 0.6 & -0.3 \end{bmatrix},$$

$$E_b = \begin{bmatrix} -0.1 & 0.4 \\ 0.4 & -0.1 \end{bmatrix}, l = 0.5, H = I, \tau \text{ is unknown.}$$

Under different levels of the upper bounds of the time-delay, Table 2 lists the results of the maximum allowable delay bounds. It is seen from Table 2 that the results

Table 2 Allowable upper bound of h_2 with given h_1

	$h_1 = 0$	$h_1 = 0.1$	$h_1 = 0.5$	$h_1 = 0.8$
Qiu et al. (2007)	2.082	2.182	2.582	2.882
Corollary 2	3.883	3.905	4.139	4.333

obtained from our method are less conservative than those obtained from Qiu et al. (2007).

Therefore, we can say that the results in this paper are much effective and less conservative than the one in Yang et al. (2006) and Qiu et al. (2007).

Conclusion

The stability problem for neural networks with interval time-varying delays and parameter uncertainties is considered. Based on the Lyapunov-Krasovskii functional approach, some delay-dependent stability criteria are derived by introducing free weighting matrices, which are used to reduce the conservatism of the obtained criterion. As a result, the new stability criteria in terms of LMIs are applicable to both fast and slow time-varying delays. Numerical examples are given to show the effectiveness of the method.

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