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Experimental Investigation and Computational Modeling of Hydrodynamics in Bifurcating Microchannels

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Abstract

Methods involving microfluidics have been used in several chemical, biological and medical applications. In particular, a network of bifurcating microchannels can be used to distribute flow in a large space. In this work, we carried out experiments to determine hydrodynamic characteristics of bifurcating microfluidic networks. We measured pressure drop across bifurcating networks of various complexities for various flow rates. We also measured planar velocity fields in these networks by using particle image velocimetry. We further analyzed hydrodynamics in these networks using mathematical and computational modeling. Our results show that the experimental frictional resistances of complex bifurcating microchannels are about 30% greater than that predicted by Navier-Stokes' equations. Experimentally measured velocity profiles indicate that flow distributes equally at a bifurcation regardless of the complexity of the network. Flow division other than bifurcation such as trifurcation or quadruplication can lead to heterogeneities. These findings were verified by the results from the numerical simulations.

Keywords

Microfluidics; Flow Visualization; Transport; Fluid Dynamics; Optimal Transport; Frictional Losses; Velocity Distribution; Biotransport; Particle Image Velocimetry; Capillary Flow

Introduction

Recent advances in microfabrication and micromachining technologies have enabled the fabrication of miniaturized devices and systems. The principal components of a vast majority of such systems are micro-scale flow channels that are involved in fluid, heat and/or mass transfer in the system. Typical applications are found in liquid-phase microchannel reactors (Brody *et al.*, 1996; Burns *et al.*, 1999), micro-total-analysis-systems (Van Den Berg *et al.*, 2000), inkjet print heads (Allen *et al.*, 1985; Krause *et al.*, 1996) and as heat exchangers for cooling computer chips (Tuckerman *et al.*, 1981; Wu *et al.*, 1984). Micro-scale flow channels form the backbone of microfluidic and MEMS devices for life science applications, including

DNA analysis (Woolley *et al.*, 1996; Burns *et al.*, 1998), enzyme and immunoassays (Hadd *et al.*, 1997; Cheng *et al.*, 2001) and cell-based biosensors (Debusschere *et al.*, 2001). In the above applications, the physics of the microscale is utilized to decrease assay times, reduce reagent volumes and increase sensitivities (Beebe *et al.*, 2002). More complex network systems with multiple flow divisions are found in gradient generating chambers (Baskaran *et al.*, 2002), cell culture devices (Walker *et al.*, 2002) and for microvascular tissue engineering (Kaihara *et al.*, 2000).

The mode of flow division prevalent in most natural network systems involves bifurcating channels. For example more than 90% of the flow branching in the arterial networks in the pig coronary region occurs through bifurcations (Kassab *et al.*, 1993). It is also the predominant form of branching in the air passages of the human lung (Weibel, 1963). Such configurations can be optimized to achieve maximum transport efficiency combined with minimum driving cost in a system with limited space. It has been shown that when space is constrained, dichotomous branching poses less resistance than branching with more tributaries (Bejan, 2000). One of the main applications of bifurcating flow devices is in the area of microvascular tissue engineering. It is now clear that providing a tissue-engineered product with a built-in engineered vasculature, to convectively deliver nutrients to the implanted cells, is essential for tissue engineering of complex thicker tissues (Griffith *et al.*, 2002; Shin *et al.*, 2004; Fidkowski *et al.*, 2005; Janakiraman *et al.*, 2007).

Two important criteria for the design of bifurcating microfluidic systems are pressure drop and flow distribution. These are primarily dependent on the fluid properties and the geometry of the bifurcating network. The cost involved in driving the flow through the microchannels is proportional to the pressure drop across the channels. The final configuration must minimize the pressure drop for the given flow requirements. Accordingly, a thorough understanding of the hydrodynamics of microscale fluid flow is required for the design of more efficient systems. Despite extensive theoretical and experimental studies on the hydrodynamics of single-phase flows in straight microchannels in various geometries (Papautsky *et al.*, 1999; Qu *et al.*, 2000; Kohl *et al.*, 2005; Nath *et al.*, 2005), there is currently little information available on fluid flow characteristics in bifurcating microchannel networks.

In a previous study (Janakiraman *et al.*, 2007), we developed a strategy for designing bifurcating flow networks with optimal transport properties. The resulting designs consisted of planar bifurcating networks in rectangular duct geometry with the assumption of symmetric flow distribution in the network channels. In this work, we discuss the experimental and numerical characterization of hydrodynamics, including pressure drop and flow distribution, in bifurcating flow channels fabricated using these designs. Our results show that overall frictional resistances of complex bifurcating microchannels measured experimentally are 25–30% higher than those predicted by Navier-Stokes' equations. This corresponds well with similar studies carried out in straight microchannel geometries previously. Numerical simulations of velocity fields in these microchannels indicate that flow distributes equally at a bifurcation. This is further verified by experimental measurement of velocity profiles in the bifurcating microchannels using particle image velocimetry (PIV).

Methods

Design of Bifurcating Flow Networks

The design of vasculature consisting of planar bifurcating flow networks with optimal transport characteristics to be integrated within tissue-engineering scaffolds is described elsewhere (Janakiraman *et al.*, 2007). Briefly, an optimization model was developed and solved to maximize the mass transport efficiency of the network design for a fixed allowable pressure drop across the network. Rectangular duct geometry was used and the optimal designs were

obtained for different generation numbers and porosity. Here, the porosity represents the percentage of volume occupied by the vasculature and the number of generations indicates the extent of branching in the design: 2^i branches in the i^{th} generation. For example, a generation 3 device consisted of 2^3 or 8 channels at its maximum level of branching. Table 1 lists the various device designs that were tested in this investigation.

Fabrication of the Bifurcating Microfluidic Devices

The flow network designs were sent to Advance Reproductions (North Andover, MA), where they were printed onto mylar transparency photo masks using a high resolution (>3000 dpi) laser printer. Flow networks were then obtained on silicon wafer templates by standard techniques of photolithography. We used Su-8 2075 (MicroChem Corp., MA), a negative tone near-UV epoxy based photoresist with a high aspect ratio (Lorenz *et al.*, 1997). The silicon wafer templates were then used as negative molds to obtain bifurcating microchannels in poly (dimethyl siloxane) (PDMS) through soft-lithography (Whitesides *et al.*, 2001). The channel dimensions, including the depths at different sections, were verified under a light microscope. Closed channel microfluidic devices were fabricated by bonding the patterned PDMS slab to a flat one using oxygen plasma bonding (Chaudhury *et al.*, 1991; Owen *et al.*, 1994) in a plasma etcher (Structure Probe Inc., PA). Holes were punched for the inlet and outlet prior to the bonding, and silicone tubing was attached to the inlet and outlet to provide access for flow. Microfluidics was established through the devices and the flow rate was controlled using a PHD 2000 syringe pump (Harvard Apparatus, MA).

Experimental Measurement of Pressure Drop

The pressure drop characteristics of bifurcating networks were investigated by experimentally measuring the differential pressure drop across the microfluidic devices at different flow rates. The flow rate to the bifurcating microfluidic device was controlled using a programmable syringe pump (PHD 2000, Harvard Apparatus). De-ionized water at a temperature of 20 °C was used as the test fluid. The pressure drop was measured using a differential pressure sensor, (All Sensors, CA), with an operating range of ± 5 inch water column pressure. Calibration of the pressure sensor was verified using an analog manometer (Dwyer Instruments, Inc., IN). The microfluidic devices were fabricated with additional ports to connect to the pressure transducer (Figure 1). The sensor voltage readings at different inlet flow rates were acquired onto the computer through a data acquisition card (E series, National Instruments Corporation, Austin, TX). The schematic of the entire setup is shown in Figure 1. For every second, ten voltage readings were recorded (Labview National Instruments Corporation, Austin, TX) and the data were stored in the computer for subsequent analysis. Two sets of pressure drop measurements for increasing and decreasing flow rates were taken for every device. The flow rates ranged from 0 to either 1000 $\mu\text{L}/\text{min}$ or a value corresponding to the upper limit of the sensor, whichever was lesser. For each design, up to four devices were fabricated and tested. The data were analyzed using Microsoft ExcelTM.

Experimental Measurement of Flow Distribution

The velocity flow field in the microfluidic devices was obtained experimentally using particle image velocimetry (PIV). The experimental setup consisted of a flow loop and an optical diagnostic system (Figure 2).

Flow Loop—The flow loop consisted of the test section and a syringe pump (PHD 2000, Harvard Apparatus, MA) used to deliver a controlled flow rate. The test sections were bifurcating microfluidic devices of different generations and porosity (Table 1). The working fluid was an optically clear homogeneous mixture of aqueous sodium iodide solution and propylene glycol. Refractive index (R.I.) matching technique was used by matching refractive

indices of the working fluid and the PDMS test section (R.I. of PDMS = 1.4105), thereby eliminating scattering and optical distortion of light at the interface of the test section and the fluid (Kadambi *et al.*, 1990). The kinematic viscosity of the fluid was calculated using a Cannon-Fenske viscometer (Cannon Instrument Company, PA). Two types of seed particles, poly(methyl methacrylate) (PMMA) beads (Presperse Inc., NJ) and silicon carbide (TSI, Inc., MN) were selected as appropriate for the wide-ranging complexities of the devices requiring varying resolutions. The physical properties of the particles and the working fluid are summarized in Table 2.

Since the particle densities were slightly different than that of the working fluid, particle settling velocity was calculated by Stokes Law (Raffel *et al.*, 1998) as per the equation:

$$V_s = \frac{2R^2 [\rho_p - \rho_f] g}{9\mu} \quad (1)$$

Here, V_s is the settling velocity of particles, R is the mean radius of the particle, μ is the dynamic viscosity of the fluid and ρ_p and ρ_f are the densities of the particle and fluid respectively. The maximum settling velocity was estimated to be 9.47 $\mu\text{m/s}$. This implies that the particles in the smallest channel with width 100 μm will settle out of the flow in 10.5 s. The maximum inter frame time between the capture of two subsequent images during any experimental run was 30 ms. Hence the seed particles were not deflected downwards noticeably in the area of interest, during acquisition of images.

Optical Diagnostic System—The experimental setup (Figure 2) incorporated a 120 mJ/pulse Nd-YAG laser (532 nm wavelength and 5 ns pulse width), laser light sheet generating optics, a CCD camera (Kodak ES 1.0, resolution: 1008 $\times$ 1018 pixels) equipped with a 200 mm Macro Nikkor lens (Nikon, Inc.) and image acquisition and processing software (PIVACQ and PIVBATCH, Wernet, M.P., National Aeronautics and Space Administration, GRC, OH). The laser beam was formed into a light sheet using a cylindrical lens (focal length -6.35 mm) and a spherical lens (focal length 300 mm). A mirror was used to reflect the light sheet on to the test section. The width of the light sheet was greater than the depth of the microchannels and illuminated the entire flow field of the test section. The CCD camera was mounted on a 2-D traverse and had its focal axis perpendicular to the plane of the laser light sheet to acquire flow images. The camera lens aperture setting was fixed at f# 4 and the field of view (FOV) was set such that it encompassed the entire flow network. In order to obtain flow field data in individual channels, FOV of either 3 mm $\times$ 3mm (4 generation number) or 1mm $\times$ 1mm (5 generation number) was required. In such cases, a microscopic lens assembly (Navitar, Rochester, NY) consisting of a Zoom 6000 standard adapter (2X), a Zoom 6000 zoom lens (6.5X Mod.I), and a Zoom 6000 lens attachment (2X) was used. Silicon carbide particles were used as seed particles since they scattered more light, due to their higher refractive index.

PIV Image Acquisition and Processing

At each test condition 250 image pairs were acquired and analyzed using the softwares PIVACQ and PIVBATCH respectively (Wernet, M.P., National Aeronautics and Space Administration, GRC, OH). The displacements of solid particles between the first exposure image and the second exposure image were determined by the cross-correlation technique. The image analysis was carried out using a subregion size of 32 $\times$ 32 pixels with 50% overlap, thus satisfying the Nyquist sampling criterion. Cross-correlation of each image pair using PIVBATCH resulted in a vector output file with 61 $\times$ 62 vectors equally spaced over the entire field of view. Spatial resolutions ranging from 256 μm to 16 μm were obtained depending on the particular field of view used for an experimental run. Data was obtained at volumetric inlet flow rates of 50 $\mu\text{l/min}$, 75 $\mu\text{l/min}$, 100 $\mu\text{l/min}$, 200 $\mu\text{l/min}$ and 500 $\mu\text{l/min}$. At these flow rates,

the Reynolds number was calculated to be of the order 1 indicating that the flow was in the laminar regime and was dominated by viscous forces rather than inertial forces.

The average velocity for each experimental run was obtained from 250 data sets. The velocity output files were graphically represented using TecPlot Focus v10.2 (TecPlot, Inc., WA). The systematic and random uncertainties (Raffel *et al.*, 1998) associated with the velocity vectors were $\pm 0.6\%$ and $\pm 1.67\%$ respectively, resulting in an overall uncertainty of $\pm 1.77\%$.

Analytical and Numerical Simulation of Pressure Drop and Velocity Field

Analytical—Assuming a symmetric flow distribution and fully developed velocity profile in each channel, an analytical expression for the pressure drop across the network can be obtained for an incompressible Newtonian fluid (Janakiraman *et al.*, 2007):

$$\Delta P = \left(\frac{3\mu Q}{2} \right) \sum_{i=0}^n \left(\frac{L_i}{2^i a_i^3 b_i M_i} \right) \quad (2)$$

$$\text{where, } M_i = \left(1 - \frac{192a_i}{\pi^5 b_i} \sum_{n=1,3,5}^{\infty} \frac{\tanh\left(\frac{n\pi b_i}{2a_i}\right)}{n^5} \right)$$

The above equation represents the equivalent of Hagen-Poiseuille law for symmetric bifurcating flow networks of rectangular ducts. Note that the frictional resistance, ratio of pressure drop and flow rate, is linearly dependent on the viscosity and is independent of the flow rate.

Numerical—We numerically simulated the hydrodynamics in bifurcating microchannels using the finite element software, COMSOL Multiphysics™ (COMSOL Inc., MA). The geometries of the bifurcating network designs (of different generation numbers and porosities) were imported into the Fluid Dynamics Module of the software. The system of incompressible Navier-Stokes PDE's was solved numerically in the rectangular channels of the network designs. Boundary conditions included no-slip at all the individual channel walls in the network, an inlet velocity profile corresponding to the analytical velocity distribution over a cross-section for a given flow rate and atmospheric pressure at the outlet. The fluid viscosity and density of de-ionized water used for the simulations were $\rho = 1000 \text{ kg/m}^3$ and $\mu = 0.001 \text{ Pa}\cdot\text{sec}$. An iterative solver based on generalized minimum residuals (GMRES) with an Incomplete LU pre-conditioner was used for the computations. A steady state analysis was performed to obtain the pressure and velocity field distributions in the networks. The pressure drop across the network designs was determined for comparison with the experimental measurements. Flow rates were obtained at representative cross-sections in the geometry and the flow distribution efficacy of the bifurcating networks was quantified numerically. All the simulations were performed on a Windows XP Pro (Microsoft Corp., Redmond, WA) machine with Intel® Pentium IV 3.2 GHz processor and 2 GB RAM. A convergence ratio of less than 1×10^{-6} was typically obtained in 5 to 6 Newton iterations, each comprised of several hundred GMRES iterations. The computational times varied from about 1 min for solving 54,251 degrees of freedom in a generation 1 design to about 10 min for 193574 degrees of freedom in higher generation designs.

Results

Channel Dimensions and Microfluidics

The bifurcating flow networks used in this work consisted of designs with generations ranging from 0 to 6. All the flow networks spanned an overall length of 2 cm, an overall width of 1 cm and a constant depth. The bifurcating microchannels are fabricated in PDMS using standard

replica molding techniques. For a network with 6 generations, the smallest channels are 106 μm wide. Figure 3A is a digital camera image of the entire device obtained when a food dye was flowing. The inlet and outlet ports can be seen along with the ports for the two ports (arrow heads) for the pressure transducer. Figure 3B show a micrograph of the the smallest channels (bar = 100 μm). The dimensions of the fabricated channels match well with the original design dimensions. Figure 3C shows a micrograph of the channel cross section. The channels have a rectangular geometry and the depth is uniform everywhere. Depending on the design, the bifurcation of channels leads to a range of aspect ratios (depth/width) of the channels, from 0.02 at the inlet to 0.90 at the smallest channels. Table 1 summarizes the depths and minimum and maximum aspect ratios of all the devices used.

Frictional Losses

To study frictional losses in bifurcating microfluidic networks, we measured pressure drop across these networks. Figure 4-1 shows typical experimental results of pressure drop measurements in the networks. Pressure drop (ΔP , mmHg) is plotted as a function of flow rate (Q , ml/min) for two device designs corresponding to generations 4, and 6. As expected, the pressure drop in these devices increased with flow rate. The increase was linear and this behavior was observed for all the devices. The linear relationship indicates that each bifurcating microchannel device can be characterized by a constant frictional resistance, the slope of the straight line fitted to the experimental data. Figure 4-1 also shows pressure drop versus flow rate for a sample design (generation 6 device) for increasing (triangles) and decreasing (squares) flow rates. The results suggest that there is no hysteresis effect involved in the pressure drop characterization. Figure 4-1 further shows results from the numerical simulations for a generation 3 device. The results show a linear relationship and flow rate similar to experiments. Figure 4-2 shows the frictional resistances (mmHg/(ml/min)) of all the networks tested. The experimental frictional resistance varied from a value of 3.5 mmHg/(ml/min) for a simple design consisting of one channel to a value as high as 33 mmHg/(ml/min) for a relatively more complex design consisting of four generations. It should be mentioned that each design is different; for the corresponding generation number and porosity each design is optimized for maximum mass transport area subject to a frictional loss constraint. Figure 4-2 also shows analytical (Eq. 2) and numerical simulation results. The numerical simulations results of pressure drop also varied linearly with the flow rates (Figure 4-1). While the results show reasonably good agreement between the three values; the experimentally measured pressure drop values were, however, consistently greater than analytical (25%) and numerical results (31%). Interestingly, the simple analytical expression predicted the frictional resistance as well as the more complex numerical simulations.

Flow Distribution

To investigate the flow distribution characteristics of bifurcating networks, we carried out experimental measurements of flow in the networks using PIV. Figure 5-1A shows PIV results of a generation 2 device for a flow rate of 200 $\mu\text{l/min}$. The X-Y velocity field distribution obtained from the PIV measurements in the entire device is shown. The spatial resolution for the PIV measurement of the entire device is 256 μm . The results show very good distribution of fluid flow in the devices. One of the primary requirements for good flow distribution is that the flow rates in individual channels at any generation should all be the same. We measured the ratio of flow rate in individual channels to that of one of the channels in the same generation (numbers on dashed lines). They indicate very good symmetry and distribution of flow in the device. There is symmetry between the converging and diverging halves of the device (along the line $x \sim 7 \text{ mm}$) as well as between the top and bottom halves of the device (along the line $y \sim 5 \text{ mm}$) Figure 5-1B shows results from the numerical simulations in the same device. As shown, the theoretical and experimental results are in very good agreement with each other.

Figure 5-2A shows PIV results of a device of generation 4 for a flow rate of 50 $\mu\text{l}/\text{min}$. For clarity, results from only one half of the device are shown. The flow field in the other half (data not shown) is the mirror image of the one shown. Flow heterogeneity is observed in this device as the individual channels at the second bifurcation have varying flow rates. The differences in flow rates were, however, small (30%) and diminished considerably in the downstream channels. Results from the numerical simulations (Figure 5-2B) predicted such heterogeneity suggesting that the geometry of the network was the primary cause of the heterogeneity.

In order to study the effect of flow rate on flow distribution, we measured the X-Y velocity field distribution in a device of generation 3 at two flow rates: 100 and 200 $\mu\text{l}/\text{min}$. Figure 5-3 shows the results, and as before, results from only one half of the device are shown for clarity. The results show that while small flow heterogeneities (9–14%) were observed, changing the flow rate did not seem to affect the distribution within the channels.

To study the flow distribution in microchannels, we measured the X-Y velocity field distribution in the smallest channels of generation 4 (Figure 5-4A) and 5 (Figure 5-4B) microfluidic devices by acquiring and processing high resolution PIV data at specific locations of the network. A flow rate of 100 $\mu\text{l}/\text{min}$ was employed. The spatial resolutions were about 50 μm and 16 μm respectively for generation 4 and 5 devices. Figure 5-4 shows the X-Y velocity fields in the smallest channels of these devices. The results show that at the smallest channels the heterogeneities were small and the distribution was excellent locally despite flow heterogeneities upstream of these devices (data not shown) similar to the ones shown in Figure 5-2. This indicates that apart from such upstream flow heterogeneities, the flow divides equally in all subsequent bifurcations downstream.

We carried out numerical simulations to further investigate the flow distribution in these devices. Figure 6 summarizes the results from these simulations. Results from five designs with generations 1, 2, 3, 4 and 5 are shown. As before, except for generation 1 design, results are shown for only one half of the design. Ratios of the flow rates obtained at the smallest channels in the design are shown. The results show that the ratios range from 1.0 (perfect distribution) to 1.3 in the channels. Generation 1 design with two channels had the best flow distribution. In devices with increasing number of generations, the distribution seems to be bimodal; in the outer half of the channels an equal flow distribution was observed and in the inner half of the channels, an equal flow distribution, but greater than the outer channels, was observed.

Discussion

Bifurcations are the predominant mode of flow division in most natural distribution systems large and small, such as river deltas, roots and branches of most trees, air passageways in the lungs, and the arterial-venous networks in the body. They are nature's way of optimizing cost and efficiency in systems constrained by space. Optimization principles of bifurcating systems may be coupled with advances in microfabrication methodologies and a better understanding of physics of the microscale, for applications in the miniaturization of several life science systems. One such application is in microvascular tissue engineering. Without any intrinsic vasculature initially, current tissue-engineered (TE) products have to depend on the host vasculature for the supply of nutrients. This puts a serious limitation on the achievable thickness of a TE product, as the mass transfer of nutrients occurs by passive diffusion. The bifurcating flow networks used in this study are designed for the creation of TE products with builtin microvasculature for convective delivery of nutrients to overcome this limitation (Janakiraman *et al.*, 2007). In another study (Janakiraman *et al.*, 2007), these networks were fabricated in natural biomaterials, and endothelialized in bioreactors.

The two important criteria used in the design of any substrate distribution system are frictional resistance and flow distribution. The former relates to the cost of the distribution and the latter to the efficacy of the distribution. The frictional resistance is a function of geometry of the path and viscosity of the fluid flowing through it. In a network with multiple flow paths, the overall frictional resistance is dependent on the frictional resistances of individual segments of the network. Our results showed that in all bifurcating flow networks with a single inlet and a single outlet, the experimental frictional resistances were greater than the numerically or analytically predicted values by 25–31%. Others have measured 10–30% greater friction factor values compared to theory in straight non-branching microchannels (Wilding *et al.*, 1994; Jiang *et al.*, 1995; Papautsky *et al.*, 1999). In those studies, it was concluded that flow in microchannels deviated from traditional Navier-Stokes predictions. This was attributed to several variations in the flow phenomenon in the microscale, *viz.* fluid temperature variations, slip flow at the boundary and viscosity variations near the channel walls (Papautsky *et al.*, 2001), that were not observed in macroscale flow. While our results suggest that a similar behavior is true for bifurcating microchannels, other factors may have contributed to the increase in frictional resistance. These include entrance and exit losses, resulting in additional pressure drop across the devices, and pressure losses at bifurcations. Our results also show that the simple analytical expression (Eq. 2) predicted frictional resistance as well as the numerical method, and both the analytical and numerical methods yielded similar results. Since the analytical expression did not take into account losses at the exit, entrance and bifurcations, this suggests that the increased resistance, though small, is probably the result of microscale flow phenomena.

For a design with n generations, there are 2^n possible flow paths from entrance to exit. While this is an attractive feature to enable transport to various locations, the efficacy of the transport depends on equal distribution of the flow. For example, if flow distributes unevenly at each bifurcation, this can lead to progressive maldistribution of flow in daughter channels thereby resulting in inefficient transport, or worse, waste of space. We used the technique of particle image velocimetry (PIV) for experimental measurement of velocity profile and quantification of flow distribution efficacy in the bifurcating network designs. This imaging technique is widely used for visualizing flow and obtaining information such as stagnation points and backflow eddies in the test section of interest (Raffel *et al.*, 1998). Methods used for estimating flow rates in microchannels include tracking meniscus displacement, use of a calibrated flow meter or syringe pump, and the weighing tank method. While these methods yield information about flow rates, they are difficult to adapt for a network of bifurcating channels. Optical methods for determination of fluid velocity in microsystems include particle-based flow visualization techniques, such as laser Doppler anemometry and particle image velocimetry (PIV), scalar-based flow visualization techniques that involve fluorescence, phosphorescence or photobleaching and point-detection scanning techniques (Sinton, 2004). PIV and *micro-PIV* (Meinhart *et al.*, 1999) techniques are useful in obtaining planar velocity fields with a high degree of spatial resolution (few microns) and are the most well characterized microscale flow visualization methods. We adapted the traditional PIV method for use in the microscale range as the range of average velocities is quite wide (0.08 cm/sec to 1.1 cm/sec).

In our technique, the laser light sheet illuminated the entire depth of the microchannels, thereby preventing us from obtaining velocity information as a function of depth. To obtain flow distribution information, therefore, we compared the flow rates through each of the channels in any i^{th} generation of a design to that of one of the channels in the same generation. Our results suggest that there was uniform flow distribution in the lower generation devices. There was also symmetry between the converging and diverging halves. For devices with higher generations (4 and 5), however, small flow heterogeneities were observed upstream and but not downstream (Figure 5-2). Specifically, the heterogeneities were noted at the 1st and 2nd generations. This discrepancy was probably due to the branching in the 1st generation which

appeared indistinct. As a result, these designs seemed to quadruplicate from a single inlet channel into 4 channels of the 2nd generation. It is possible to overcome this problem by making slight modifications in the design of the networks, such that there is distinct branching in the first generation. This along with uniform distribution elsewhere in the networks suggests that modes of flow division other than bifurcations, *viz.*, trifurcations and quadruplications lead to unequal flow distribution. Nevertheless, the observed flow heterogeneities were small and unaffected by changing flow rates (Figure 5-3).

The overall flow distribution obtained experimentally matched well with the numerical results. There was equal flow division in all the channels (*at every generation*) for lower generation designs (Figures 5-1 and 5-2). The heterogeneities seen in higher generation designs were also noted in the numerical results suggesting that the heterogeneities are probably due to device design and not due to microscale phenomena. The higher generation devices showed heterogeneities in the flow distribution in such a way that a *set* of outer channels exhibited lower flows than another *set* of inner channels (Figure 6). As explained earlier, this discrepancy was due to the branching in the 1st generation which appeared indistinct. The fact that bifurcations form the most efficient mode of equal flow division is reinforced by the observation that starting from the 2nd generation, the flow distribution is very even in the daughter channels.

In summary, we carried out experimental and theoretical investigation of fluid flow in bifurcating microchannel networks. Our results suggest the following:

1. Bifurcating networks are an efficient mode of fluid transport and delivery
2. Hydrodynamics in microchannels lead to slightly increased resistance that may be due to microscale phenomena
3. Analytical and numerical methods can be used to predict frictional resistance in bifurcating networks with reasonable accuracy
4. Flow divisions other than bifurcation such as trifurcation or quadruplication can lead to heterogeneities

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Figure 1. Schematic of the experimental setup to measure pressure drop across a network of bifurcating microchannels (see text for details)

Figure 2. Schematic of the experimental setup to measure flow and velocity distribution using PIV (see text for details)

Figure 3. Microfluidic test device and flow channels. **3A** is a digital image of an entire device. The blue color is a result of flow of a food dye. The arrow heads indicate ports for the pressure transducer. **3B** is a light microscope image of the smallest channel in the device (top view, bar = 100 μm). **3C** is a light microscope image of the cross section of the smallest channels (bar = 100 μm)

Figure 4.

Frictional losses in bifurcating microchannels. **4-1** shows plots of pressure drop across the networks as a function of flow rate for devices of generations 4 (circles), 6 (triangles and squares), and 3 (diamonds). Both experimental (circles, triangles, squares) and numerical (diamonds) results are shown. For generation 6 device, results are shown for increasing (triangles) and decreasing (squares) flow rates. Error bars represent standard deviations. **4-2** is

a summary of frictional resistances of all the devices measured using experimental, analytical and numerical methods. Error bars represent standard deviations.

Figure 5.

Velocity and flow distribution in bifurcating microchannels. **5-1** shows experimental (A) and numerical (B) results for a generation 2 device. **5-2** shows experimental (A) and numerical (B) results for a generation 4 device. **5-3** shows experimental velocity fields in a generation 3 device for flow rates 100 (A) and 200 (B) $\mu\text{l}/\text{min}$. **5-4** shows velocity fields in the smallest

channels of generation 5 (**A**) and 6 (**B**) devices. In all the figures, the velocity vectors are color coded for magnitude. The numbers on dashed lines are flow rate ratios (see text for details).

Figure 6. Numerical results of flow distribution in bifurcating networks. Flow rate ratio at the smallest channels is plotted for devices of generations 1–5. Due to symmetry, results from only one half of each device, except the generation 1 device, are shown.

Geometry characteristics of bifurcating network designs; n = Number of generations, θ = Network porosity, L = Initial length of network, W = Initial width of network, Y = Length scaling factor, X = Width scaling factor, $\alpha_{\min}$ = Minimum aspect ratio, and $\alpha_{\max}$ = Maximum aspect ratio

n	θ	Geometry Variables				Depth (Mean $\pm$ S.D.) (μ m)	$\alpha_{\min}$	$\alpha_{\max}$
		L (cm)	W (cm)	Y	X			
1	0.5	0.5313	0.3637	1.0000	1.0000	106.4 $\pm$ 1.5	0.03	0.03
2		0.3726	0.2453	1.0000	0.9701	100.6 $\pm$ 1.5	0.03	0.04
3		0.2936	0.2149	1.0000	0.8171	106.2 $\pm$ 2.2	0.03	0.09
4		0.2943	0.2497	0.9062	0.6906	103.2 $\pm$ 2.1	0.02	0.18
5		0.3555	0.3349	0.7693	0.5993	93.7 $\pm$ 2.4	0.02	0.36
6		0.3955	0.4011	0.7045	0.5448	95.8 $\pm$ 5.3	0.02	0.90
4	0.4	0.2550	0.0860	0.9929	0.9025	96.4 $\pm$ 5.6	0.02	0.17
5		0.3044	0.1719	0.8455	0.6857	90.1 $\pm$ 2.4	0.02	0.34
6		0.3389	0.2498	0.7728	0.5902	93.4 $\pm$ 1.3	0.02	0.88

Table 2
Properties of the seed particles and the working fluid used in the PIV experiments

	Seed Particles		Sodium Iodide	Working Fluid	
	PMMA	Silicon Carbide		Water	Propylene Glycol
Mean Diameter	6 μm	2 μm	23.8%	41.6%	34.6%
Specific Gravity	1.1–1.25	3.2		1.19	
Refractive Index	1.48	2.65		1.4105	
Settling velocity	3.82 $\mu\text{m/s}$	9.87 $\mu\text{m/s}$		3.5 cS	