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Signal-to-Noise Measurements in Magnitude Images from NMR Phased Arrays

Chris D. Constantinides, Ergin Atalar, and Elliot R. McVeigh

From the Departments of Biomedical Engineering (C.D.C., E.R.M.) and Radiology (E.A., E.R.M.), Johns Hopkins University School of Medicine, Baltimore, Maryland.

Abstract

A method is proposed to estimate signal-to-noise ratio (SNR) values in phased array magnitude images, based on a region-of-interest (ROI) analysis. It is shown that the SNR can be found by correcting the measured signal intensity for the noise bias effects and by evaluating the noise variance as the mean square value of all the pixel intensities in a chosen background ROI, divided by twice the number of receivers used. Estimated SNR values are shown to vary spatially within a bound of 20% with respect to the true SNR values as a result of noise correlations between receivers.

Keywords

phased array coils; noise correlations; signal-to-noise

INTRODUCTION

Magnetic resonance images traditionally have been presented as the magnitude value of a complex data array. These images have been used in practical imaging systems for clinical interpretation because they are not susceptible to artifacts generated from phase shifts such as those from field inhomogeneities, chemical shifts, or RF penetration artifacts (1,2).

An important index of image quality in magnitude MR images is the signal-to-noise ratio (SNR). A common method for measuring SNR values compares the mean signal to the standard deviation of noise. Although it is possible to measure the signal as the ensemble average of the pixel intensities over a region of interest (ROI), the noise cannot be measured directly from the same region because possible signal variation in that ROI may bias the noise estimate.

Henkelman (1) introduced a mathematical analysis describing the effects of noise in a magnitude-reconstructed MR image obtained from a single-receiver unit system. He showed that the average signal measurement becomes biased by the partially rectified noise, thus leading to an overestimation of the signal strength (1). This analysis yielded signal and noise correction factors as a function of signal intensity, which facilitated the extraction of the true signal and noise estimates from measured values from background regions in the image.

Numerous other correction schemes have been proposed to reduce this noise bias in magnitude-reconstructed images from a single receiver unit. Bernstein *et al.* (3) suggested the use of the phased real reconstruction for improved detectability in low SNR images. Miller *et al.* (4) and

McGibney *et al.* (5) both proposed bias-correction schemes for power images. More recently, Gudbjartsson *et al.* (6) have revisited this topic and have proposed a simple postprocessing scheme to correct for the bias due to the Rician distribution of the noisy magnitude data.

With the introduction of multiple receiver coils (7) for simultaneous acquisition of NMR signals in phased array systems, a number of image reconstruction algorithms have been developed in an effort to enhance the SNR of the composite image, while maintaining a large field of view. The need, however, for detailed maps of the sensitivity and phase shift factors of the RF field for most of these algorithms led to the employment of the sum-of-squares algorithm as the most practical for use in MR imaging systems (7). In this algorithm, the signal in each pixel in the composite image is the square root of the sum of the squares of the pixel values from the images from individual coils in the array.

It is the purpose of this paper to extend the estimation method for SNR on magnitude-reconstructed images, originally proposed for single-receiver units, to composite sum-of-squares images reconstructed from multiple-receiver phased array systems. The theoretical probability distributions of the measured signal intensity in such images—both in the presence and absence of signal—are presented. Based on these distributions, the effects of noise bias are analyzed in magnitude-reconstructed images obtained from arrays consisting of n receivers. A simple method is also proposed for estimating the inherent standard deviation of noise from an appropriate ROI analysis in the air background of such images. This method is verified with experimental measurements. In addition, correction plots are provided for the measured SNR estimates obtained in such images, similar to Henkelman's analysis (1) for the single receiver case.

THEORETICAL DERIVATION

Statistical Analysis in Magnitude Sum-of-Squares Images

Signal intensities in an MR image are corrupted by noise. In this section, the mathematical treatment describing the extraction of signal amplitudes measured in the presence of noise is presented for a system that uses multiple receiver units.

In phased array systems, multiple receiver units enable simultaneous, parallel NMR signal acquisitions with one receiver for each surface coil in the array. In commercially available MR systems (e.g., Signa 1.5T®, General Electric), the intensity of any pixel in the reconstructed image is the square root of the summation of the squares of all the signal intensities at that position, as detected from each of the receiving elements. Thus, for an n receiver array, the

measured pixel intensity in the composite image is given by $M_n = \sqrt{\sum_{k=1}^n (M_{Rk}^2 + M_{Ik}^2)}$, where M_{Rk} and M_{Ik} represent the measured pixel intensities (sum of the signal and noise intensities) in the real and imaginary parts of the complex image reconstructed from the k th receiver, respectively. In the presence of noise, the probability density function for M_{Rk} and M_{Ik} is a Gaussian with mean A_{Rk} and A_{Ik} , respectively, and with a standard deviation, σ . A_{Rk} and A_{Ik} denote the image pixel intensities in the absence of noise in each of the real and imaginary parts of the complex image reconstructed from the k th receiver.

Although there is evidence for the existence of noise correlations in phased array systems (7-12), previously reported noise correlation values of ≤ 0.3 in humans and phantoms (7,13) further support the argument that such effects are minimal and are therefore neglected in the present analysis. A complete discussion on noise correlations is presented in a subsequent section.

Assuming absence of noise correlations and that the n different receivers are statistically independent, the probability density function for the composite random variable, M_n , in a sum-of-squares magnitude image is given by the noncentral chi distribution (14-16) defined by:

$$p(M_n) = \frac{A_n}{\sigma^2} \left(\frac{M_n}{A_n} \right)^n e^{-[(A_n^2 + M_n^2)/2\sigma^2]} I_{n-1} \left(\frac{M_n \cdot A_n}{\sigma^2} \right) \quad [1]$$

where I_{n-1} is the modified $(n-1)$ th order Bessel function of first kind, and A_n is the total image pixel intensity in the absence of noise contributed by all of the elements in the array, defined

by $A_n = \sqrt{\sum_{k=1}^n (A_{Rk}^2 + A_{Ik}^2)}$. In the case of a single-receiver system ($n = 1$), Eq. [1] reduces to the Rician distribution (17) proposed by Rice for applications to communications but also by Bernstein *et al.* (3) and Gudbjartsson *et al.* (6) for MR imaging. Equation [1] is plotted for the cases of one, two, and four receivers for different values of A_n/σ as shown in Figure 1. As can be seen, the probability density function is far from symmetric for small SNR values, although use of more receiver units reduces the skewness.

The first moment of the probability distribution $p(M_n)$ was evaluated (18) to be:

$$\bar{M}_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n-1} (n-1)!} \sigma \sqrt{\frac{\pi}{2}} {}_1F_1 \left(-\frac{1}{2}, n, -\frac{A_n^2}{2\sigma^2} \right) \quad [2]$$

where ${}_1F_1(a, b, c)$ is the confluent hypergeometric function (19). Figure 2a shows plots of the first moment for all three distributions corresponding to cases of one, two, and four receivers ($\bar{M}_1, \bar{M}_2, \bar{M}_4$) generated from Eq. [2] for different SNR values of A_n/σ . The second moment of each of the three distributions, $E(M_n^2)$, was also calculated (18) to be $E(M_n^2) = 2n\sigma^2 + A_n^2$ and was used in association with Eq. [2] to yield the standard deviation (σ_{Mn}) of the total noise as

$$\sigma_{Mn} = \sqrt{E(M_n^2) - \bar{M}_n^2} \quad [3]$$

depicted in Fig. 2b.

Noise Statistics in the Absence of Signal Intensities

To calculate the SNR from a magnitude image, measurements of the signal and the noise are necessary. The signal is easily measured as the mean intensity within an ROI in the object. Unlike the signal, however, the noise is not easy to obtain since the root-mean-square deviation (RMSD) around the average signal from an ROI in an object is dominated by the variations in the object signal intensity. What can be measured directly is the background standard deviation (σ_{Mn}) on a region of the image that contains no signal ($A_n = 0$). For an array of n coil elements, the measured pixel intensity of the composite image in regions of no signal, ($A_n = 0$), is

$M_n = \sqrt{\sum_{k=1}^n (N_{Rk}^2 + N_{Ik}^2)}$. N_{Rk} and N_{Ik} are the noise values in the real and imaginary parts of the complex image reconstructed from the k th receiver, respectively, assumed to be identically and independently randomly distributed with zero mean and with a standard deviation of σ . Assuming absence of noise correlations, the composite noise random variable, M_n , follows a central chi statistic with $2n$ degrees of freedom and with a probability distribution (15,16,20)

$$p_c(M_n) = \frac{2}{(\sigma\sqrt{2})^{2n} (n-1)!} M_n^{2n-1} e^{-(M_n^2/2\sigma^2)} \quad [4]$$

For a single-receiver system with noise signal only, the probability density function for the signal magnitude, M_1 , follows a Rayleigh distribution with a mean value of 1.25σ and a standard

deviation of 0.655σ (1,21). Similarly, the mean and the standard deviation values of M_n (in regions of the image where $A_n = 0$) in the case of two and four receivers can be computed from Eq. [4] (or from Eqs. [1]-[3]) to be $\bar{M}_2 = 1.88\sigma$, $\bar{M}_4 = 2.74\sigma$ and $\sigma_{M2} = 0.682\sigma$, $\sigma_{M4} = 0.695\sigma$, respectively. Although these values of background σ_{Mn} can be used to compute estimates of the true noise variance, σ , a better estimate for σ can be found from direct measurements in such images. The second moment of the measured pixel intensity in a background ROI is given by:

$$E(M_n^2) = 2n\sigma^2 \quad [5]$$

and is unaffected by noise correlations. A good estimator for $E(M_n^2)$ is the mean square value of the total number of pixels, L , in the selected ROI,

$$E(M_n^2) \approx \frac{1}{L} \sum_{i=1}^L (\text{pixel value})_i^2 \quad [6]$$

Combining Eq. [5] and Eq. [6] yields

$$\sigma \approx \sqrt{\frac{\sum_{i=1}^L (\text{pixel value})_i^2}{2Ln}} \quad [7]$$

Using Fig. 3, the measured average signal, $\bar{M}_n$ can be corrected to yield an estimate for A_n and the SNR can be calculated as A_n/σ . For systems using up to four receivers and for values of $\bar{M}_n/\sigma > 10$, the error in the measurement is $< 8\%$.

Practical Significance—Measuring SNR in Magnitude Images—The analyses presented above suggest that the necessary steps for correct calculation of SNR values in magnitude sum-of-squares images are:

- Compute the noise standard deviation, σ , as the root mean square value of all the pixel intensities in a selected background ROI divided by twice the number of receivers used (Eq. [7]). Avoid regions in the image corrupted by artifacts.
- Select a desired ROI within the object. Measure the mean intensity value, $(\bar{M}_n)$, of all the pixels within the selected ROI.
- Calculate A_n using Fig. 3 and compute the image SNR as A_n/σ .

NOISE CORRELATIONS

The existence of noise correlations in phased array systems causes a spatial variation of the total noise in the composite image (11,22,23). As indicated by Eqs. [5]-[7], noise correlations have no effect on estimates of σ obtained from a background ROI of the composite image. However, in cases where estimates of σ (and hence SNR) are sought for each pixel of the image, care must be taken in regions where $A_n \neq 0$. In this section, we describe quantitatively the effects of noise correlations on the composite pixel variance and provide upper and lower bounds for the expected variation of SNR.

In the original paper on NMR phased arrays (7), Roemer quantified noise correlations in terms of an electric coupling coefficient (or equivalently a correlation coefficient) between the array coils, with a maximum theoretical value of 41 %, although experimental measurements yielded values $\leq 30\%$ in humans and phantoms (7,13).

Recently, noise correlations have been classified into two types: *extrinsic* and *intrinsic* (24). Extrinsic noise correlations are due to noise voltages that originate from the mutual coupling of the coil elements in the array, a phenomenon commonly referred to as “cross-talk.” Proper combination of signals from the different coil elements in the array and use of low input impedance preamplifiers eliminate this type of correlation. Most important are the intrinsic noise correlations. These correlations refer to noise voltages that originate from eddy currents induced in the sample that share common paths. Intrinsic noise correlations are completely separable in nature from the extrinsic noise correlations and are impossible to reduce in a lossless way.

Hayes *et al.* (11) have shown that the noise correlation values between coils p and q in a phased array, ρ_{pq} , can be expressed as:

$$\rho_{pq} = \frac{E \left[N_{R_p} N_{R_q} \cos(\phi_p - \phi_q) \right]}{\sqrt{E(N_{R_p}^2) E(N_{R_q}^2)}} \quad [8]$$

where ϕ_p, ϕ_q are the phases associated with the complex noise. It is also assumed that only the noise component that is colinear with the signal tends to alter the magnitude of the image and so the orthogonal component of the noise is completely neglected. The variance of the composite image pixel is given by (11):

$$\sigma_{M_{n(\text{cor})}}^2 = \sum_{p=1}^n \sum_{q=1}^n \left\{ E \left[N_{R_p} N_{R_q} \cos(\phi_p - \phi_q) \right] \cos(\theta_p - \theta_q) \right\} \left(\frac{A_p A_q}{A_p^2 + A_q^2} \right) \quad [9]$$

where θ_p, θ_q represent the angles between the xy component of the magnetic field B_I and the initial direction of the voxel magnetic moment for coils p and q , respectively. So, for a two-coil array,

$$\sigma_{M_{2(\text{cor})}}^2 = \left\{ 2\sigma^2 + 2E \left[N_{R_1} N_{R_2} \cos(\phi_1 - \phi_2) \right] \cos(\theta_1 - \theta_2) \right\} \cdot \left(\frac{A_1 A_2}{A_1^2 + A_2^2} \right) \quad [10]$$

Maximum deviation from the uncorrelated case thus occurs when the magnetic field lines from the two coils intersect either parallel or antiparallel ($\cos(\theta_1 - \theta_2) = \pm 1$) and $A_1 = A_2$. Using these conditions and Eq. [8] in Eq. [10] yields

$$\sigma_{M_{2(\text{cor})}}^2 \leq \sigma^2 (1 \pm \rho_{12}) \quad [11]$$

For a maximum correlation coefficient of $\rho_{12} = 0.3$, the measured SNR is expected to vary within a bound of 20% compared with the uncorrelated case. It is important to note that this variation is maximal when the signal contributions to the selected ROI from the two coils are equal. When the signal is dominated by a single coil, the noise correlations have no effect on the composite pixel variance.

In planar arrays with more than two coils, the error bounds will increase due to the smaller (yet finite) correlations from the nonadjacent coils, although such changes are not expected to deviate significantly from the two-coil array case. Further complexities are expected in the case of volume phased arrays or wrap-around strips where the correlation coefficient might be equally large for both adjacent coils and for coils that are further apart in the array.

EXPERIMENTS

All experiments were performed on a 1.5T Signa® (GE Medical Systems, Milwaukee, WI) imaging system. No filtering or correction for geometric distortion was performed during image reconstruction.

In an effort to test the validity of Eq. [7], two series of 90 images were acquired from the midaxial and midcoronal slices of a cylindrical water phantom (diameter = 27 cm, length = 36 cm) using two separate coil loops (5" General Purpose circular coils, GE Medical Systems, Milwaukee, WI) placed on either side of the phantom (connected to separate receivers as a two-loop phased array) and a four-loop phased array (Pelvic Phased Array®, GE Medical Systems, Milwaukee, WI). A typical image was used in each case, in association with the computation of σ , as proposed by Eq. [7], to generate an SNR image. The true SNR values were computed by averaging the entire series of 90 images, divided by the standard deviation of all 90 measured values about the mean intensity in each pixel of the image. Figures 4a and 4b show the images obtained by dividing the calculated and true SNR values in the case of two and four receivers. The background intensities in such images have been suppressed by the application of a thresholding algorithm. Profiles taken along the midline of the phantom clearly depict that calculated SNR values of a typical image match the true SNR values within the phantom, thus confirming the validity of Eq. [7] as an easy and practical method for computing σ from magnitude sum-of-squares images. Figure 4 also demonstrates the effects of noise correlations on SNR. In the case of the two single coil loops, which process the data for reconstruction in the same manner as a two-loop phased array, there is no effect from noise correlations (Fig. 4a). In the case of overlapping coils, as in the case of the pelvic coil, noise correlations cause a variation in the SNR values computed on a pixel-by-pixel basis. Such variation is bounded within the limits proposed above.

In addition, the theoretical analysis outlined above suggests that the ratio of the mean to the RMSD of the background in a sum-of-squares image for a two-receiver system should be $1.88/0.682 = 2.76$, and $2.74/0.695 = 3.94$ for a four-receiver system. Noise images were obtained using the two single 5" coils (connected as a two-loop phased array), a two-loop spine coil (Cervical Thoracic Lumbar Phased Array coil with only two coils selected, GE Medical Systems, Milwaukee, WI), the four-loop pelvic phased array, and the body coil in the presence of the cylindrical water phantom. In experimental measurements, over a series of 10 images, the ratio was determined to be 2.72 ± 0.015^1 for the two single coils, 2.75 ± 0.022 for the two-receiver (two-loop spine array) system, and 3.86 ± 0.023 for a four-receiver (four-loop pelvic array) system. Using the same imaging protocol for the body coil, the ratio was found to be 1.92 ± 0.015 , which is in agreement with the results of Henkelman (1). Measurements were made in a 32×128 pixel square region carefully positioned in the middle of the image to avoid possible artifacts (1,25).

CONCLUSION

In this note, a method is proposed for correct calculation of SNR values in composite magnitude images obtained from phased array systems using the sum-of-squares reconstruction and employing n receivers. The theoretical probability distributions of the measured signal intensity in such images, both in the presence and absence of signal intensities, have been presented. A simple equation is also proposed for computing the true noise standard deviation, σ , from composite sum-of-squares images as the root mean square value of all the pixel intensities in a chosen background ROI, divided by twice the number of receivers used. The validity of this equation was verified through experimental measurements.

Correction plots have been provided to account for the noise bias effects on the measured signal in the case of one, two, and four receivers. Although in most imaging applications we are dealing with the correction of such bias at relatively high SNR values (>10), in which case such correction is within 8% of the measured value, correction, nevertheless, is important in low SNR regions of the image and also in low SNR images reconstructed from phased arrays in spectroscopic imaging of nuclei such as sodium, fluorine, and phosphorus.

Estimated SNR values (on a pixel-by-pixel basis) are shown to vary within a bound of 20% at different spatial regions of the image, with respect to the true SNR values, as a result of noise correlations.

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FIG. 1. The chi distribution of M_n/σ for several values of A_n/σ for a (a) single-receiver system, (b) two-receiver system, and (c) four-receiver system.

FIG. 2.

(a) The measured SNR $\left(\bar{M}_n/\sigma\right)$ and (b) the normalized standard deviation (σ_{M_n}/σ) plots for a magnitude sum-of-squares image obtained using single-, two-, and four-receiver systems, for several values of A_n/σ , in a uniform region of the image.

FIG. 3. Correction plots in sum-of-squares magnitude images for single-, two-, and four-receiver systems as a function of $\bar{M}_n/\sigma$. Corrected SNR values were obtained by subtracting the appropriate correction factor from $\bar{M}_n/\sigma$.

FIG. 4.

(a), (b) Midaxial and midcoronal images of a cylindrical water phantom generated by dividing the calculated SNR values by the true SNR values in the case of a (a) two-receiver and a (b) four-receiver system. The background intensities in both images were suppressed using a thresholding algorithm. (c), (d) Profiles taken along the midline of each of the images in (a) and (b) above.