Dynamical properties of the two-process model for sleep-wake cycles in infantile autism

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Abstract The two-process model is a scheme for the timing of sleep that consists of homeostatic (Process S) and circadian (Process C) variables. The two-process model exhibits abnormal sleep patterns such as internal desynchronization or sleep fragmentation. Early infants with autism often experience sleep difficulties. Large day-by-day changes are found in the sleep onset and waking times in autistic children. Frequent night waking is a prominent property of their sleep. Further, the sleep duration of autistic children is often fragmented. These sleep patterns in infants with autism are not fully understood yet. In the present study, the sleep patterns in autistic children were reproduced by a modified two-process model using non-linear analysis. A nap term was introduced into the original two-process model to reproduce the sleep patterns in early infants. The nap term and the time course of Process S are mentioned in the present study. Those parameters led to bifurcation of the sleep-wake cycle in the modified two-process model. In a certain range of these parameter sets, a small external noise was amplified, and an irregular sleep-wake cycle appeared. The short duration of sleep led to another irregular sleep onset or waking. Consequently, an irregular sleep-wake cycle appeared in early infantile autism.

Keywords Autism · Two-process model · Sleep cycle

Introduction

Sleep-wake alternation has homeostatic and circadian aspects. Since the EEG power density in the 0.7 to 25 Hz domain during sleep depends on the duration of the period of prior waking, it is considered a homeostatic aspect of sleep regulation (Daan et al. 1984b). Such EEG power density during sleep, which is primarily due to a decrease in slow-wave sleep (0.7–2.5 Hz), exponentially decreases with time after sleep onset (Daan and Beersma 1984a). On the other hand, the length of sleep varied with the time of day; rate of sleepiness is significantly high during the evening, while minima occur during the morning (Åkerstedt and Gillberg 1981). This is the circadian aspect of sleep regulation. A model for the timing of human sleep has been proposed that includes homeostatic and circadian aspects of sleep regulation (Borbély 1982; Daan et al. 1984b). It is a two-process model that consists of a sleep-dependent process (Process S) and a sleep-independent process (Process C). Process S shows exponential decay during sleep and an increase while awake. Process C shows periodic oscillation of circadian rhythms, and determines the upper and lower thresholds for Process S. The two-process model shows not only typical circadian sleep-wake alternations but also sleep fragmentation or internal desynchronization in the absence of time cues (Dann et al. 1984b).

Early infants with autism tend to show early waking in the morning or later sleeping in the evening (Takase et al. 1998). Such sleep patterns show a large variation in total sleep time. Inanuma (1984) has reported large day-by-day changes in sleep onset or waking times in autistic children, especially in those less than 4 years old. Frequent waking at night has also been reported. Autistic children also often take naps. The sleep duration seems fragmented

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in autistic children and their sleep patterns are irregular. However, those sleep patterns are likely to be neither the delayed sleep phase syndrome nor the advanced sleep phase syndrome (Takase et al. 1998). The features of the sleep patterns of autistic children are not fully understood yet.

Delay in the development of homeostatic regulation in sleep is one possible cause for irregular sleep patterns. Spindle frequency activity (10.5–15.0 Hz) during sleep in the first two rapid eye movement (REM) sleep periods is found in the sleep EEGs of babies until after their first year of life. Spindle frequency activity during REM sleep periods rapidly decreases with age in normal children. Ornitz (1972) has shown that decrement of the spindle frequency activity begins later in life in autistic children. Spindle frequency activity typically occurs during non rapid eye movement (NREM) sleep periods. This indicates that the development of NREM sleep is immature in autistic children when compared to age-matched normal children.

Naps must be considered in children's sleep propensities. A certain duration of daytime sleep occurs until four years of age (Iglowstein et al. 2003). Spontaneous napping and its duration depend on the body temperature of a circadian periodicity (Zulley and Campbell 1985). Naps beginning with high body temperature are terminated within several hours. The rate of sleepiness has two peaks in typical sleep-wake cycles (Broughton and Mullington 1992). Further, it has been reported that obligatory naps led to delayed bedtimes in nursery school children (Fukuda and Asaoka 2004). Although the subjects were adults, sleep latency in the postnap night was prolonged, and the EEG power density in NREM was decreased in the delta and theta bands (Werth et al. 1996). This indicates that naps modulate the bedtimes of night sleep. Such modulations by naps must be considered in the two-process model for sleep patterns in autistic children.

Analysis of the dynamical properties of the sleep-wake cycle helps illuminate the irregularity of sleep patterns. Nakao et al. (1997) have suggested that the internal desynchronization of sleep is understood by the bifurcation of the two-process model using a circle map. Nonlinear analysis for the two-process model can reveal how homeostasis and circadian rhythm cause irregular sleep patterns, such as in autistic children. In the present study, we introduced a nap term into the two-process model to represent sleep patterns in infancy and reproduced the features of irregular sleep patterns in autistic children. One-dimensional maps derived from the modified two-process model revealed sensitivity of the sleep-wake cycle against external noise.

Methods

Two-process model with a nap term

Two-process model

The two-process model consists of Process C and Process S. Process C consists of $L(t)$ and $H(t)$ (Daan et al. 1984b). Process S rises during an awake time and decays during sleep. When Process S exceeds $H(t)$, sleep begins. When Process S crosses $L(t)$, sleep is terminated and Process S rises again: an awaking.

The two-process model is described as follows:

$$L(t) = A \left[0.97 \sin\left(\frac{2\pi(t-t_0)}{T}\right) + 0.22 \sin\left(\frac{4\pi(t-t_0)}{T}\right) + 0.07 \sin\left(\frac{6\pi(t-t_0)}{T}\right) + 0.03 \sin\left(\frac{8\pi(t-t_0)}{T}\right) + 0.01 \sin\left(\frac{10\pi(t-t_0)}{T}\right) \right] + \bar{L} + \xi(t) \quad (1)$$

$$H(t) = L(t) + D \quad (2)$$

$$S(t) = \begin{cases} 1 - (1 - L(t_w)) \exp(-\alpha(t - t_w)) & \text{wake} \\ H(t_s) \exp(-\beta(t - t_s)) & \text{sleep,} \end{cases} \quad (3)$$

where t_w is the waking time and t_s is the sleep onset time. Equations 1–3 were based on the mathematical expressions of Nakao et al. (1997). The shape of $H(t)$ and $L(t)$ was derived from the relation between sleep onset time and length in sleep deprivation experiments (Daan et al. 1984b). Since the subjects in Inanuma's paper (1984) were exposed to social environments, circadian period T was fixed at 24 h. We introduced t_0 into the two-process model ($t_0 = 10:00$) and redefined sleep onset time t_s or waking time t_w as ϕ_s or ϕ_w , respectively. ϕ_s (or ϕ_w) was given by the remainder of $\frac{t_s}{T}$ (or $\frac{t_w}{T}$). Amplitude A of Process C determines the coupling strength for the phase synchronization between the circadian rhythm and the sleep-wake cycle. Serum melatonin level shows circadian aspect: high concentration in night and low in daytime. Such alternations of melatonin level are phase-locked with sleep-wake cycle. Nighttime serum melatonin concentration is prominently high in infants (1–3 years old) (Waldhauser et al. 1988). Although functions of melatonin are not known, this might be linked to amplitude of circadian rhythms. Therefore, we assumed that the A 's of the infantile subjects were roughly twice as large as the adult subjects ($A = 0.12$; Daan et al. 1984b). Here, A was 0.236. Subsequently, the baseline was also increased ($\bar{L} = 0.27$). D is the distance between $L(t)$ and $H(t)$ ($D = 0.5$). The time course of declining Process S corresponds to an accumulation or a decline of EEG power density during sleep.

Rising rate α , which was originally extrapolated from the terminal EEG power density in the preceding sleep period, was not a measured quantity (Daan et al. 1984b). The EEG power density at the beginning of night sleep in children is several times larger than in adults (Gaudreau et al. 2001). Process S seems to rapidly rise in the daytime in children. Therefore, we assumed that rising rate α in children is larger than in adults. Here, α was fixed at 0.275 h^{-1} . Decay rate β , which represents the decay rate of the EEG power density of slow-wave activity during sleep, was used as a parameter. Gaussian noise $\xi(t)$ was added to Process C as environmental perturbation to the biological oscillators in a body. The considerable environmental factors are the time cues of sunlight and the conscious decisions in a framework of personal and cultural habits (Daan et al. 1984b; Beersma et al. 1985).

Two-process model with a nap term

An ontogenetic transition of polycyclic alternation between the states of sleep and awake are seen (Kleitman 1963). Two or more sleep-wake cycles are included in a day in childhood. Until four years of age, children regularly take naps that are accounted for by the decay of Process S (Werth et al. 1996). Transitory lowering of $H(t)$ accounts for the nap onset in the two-process model. On the other hand, under a condition without time cues, when the subject was allowed to take naps, spontaneous naps occurred halfway between successive body temperature minima (Zulley and Campbell 1985). Thus we transitorily lowered $H(t)$ near the peak of Process C. This represented a bicyclic alternation between the states of sleep and awake in a day.

A bell-shaped function $N(t)$ was added to $H(t)$ of the original two-process model in the present study:

$$N(t) = -A_{\text{nap}} \left[\frac{1}{1 + \exp(-\alpha_{\text{nap}}(\phi - \mu + \sigma/2))} - \frac{1}{1 + \exp(-\alpha_{\text{nap}}(\phi - \mu - \sigma/2))} \right], \quad (4)$$

where ϕ is the remainder of $\frac{t}{T}$. Bell-shaped function $N(t)$ represents a nap term. In Fig. 1, the shape of modified $H(t)$ and the time course of Process S are shown. $N(t)$ lowers the threshold of the sleep onset in the afternoon and causes a nap. μ , which represents the position of the nap term, was 12:30. σ and α_{nap} determined the time range of the nap term. $\sigma = 4 \text{ h}$. $\alpha_{\text{nap}} = 1.3 \text{ h}^{-1}$. A large A_{nap} derives a low threshold for rising Process S around $\phi = \mu$. A_{nap} is a control parameter.

$H(t)$, $L(t)$, and $S(t)$ were updated every second, and then sleep onset time ϕ_s^n was collected ($n = 0, 1, \dots$). Iterations were repeated 400 times, and the last 200 of ϕ_s^n were used to assess the period of the sleep-wake cycles. If the difference between ϕ_s^n and ϕ_s^{n-q} ($q = 1, 2, \dots$) was less than

Fig. 1 Two-process model with a nap term. The bar indicates the sleep duration. Process S ($S(t)$) rises during waking and decays during sleep. Sleep begins when Process S exceeds $H(t)$, and waking begins when Process S crosses $L(t)$. The decay rate of Process S is determined by β . $H(t)$ is distorted by the bell-shaped function of the nap term. When Process S meets the nap term, the short duration of sleep occurs in daytime

10 s, those values were regarded as the same. The minimum number of q s was taken as the sleep period during p days. The number of naps (l) and sleeps following night wakings (k) were also counted in q sleep episodes. Sleep episodes were classified into three types. When sleep is caused by $N(t)$, that sleep was treated as a nap. In fact, if $S(t)$ exceeded $H(t)$ during $N(t) \leq -0.01$, ϕ_s^n was counted as a nap. If sleep began during $N(t) > -0.01$ and it was the first sleep episode between successive minima of $N(t)$, that ϕ_s^n was treated as the nocturnal sleep onset. The other sleep episodes were considered sleep following night waking.

The sleep-wake patterns of the two-process model were plotted in the parameter plane with A_{nap} in increments of 0.001 and β in increments of 0.0005 h^{-1} . The initial ϕ_s^0 was 20:00. When the parameter plane was obtained, $\xi(t) = 0$.

Circle map derived from two-process model

When ϕ_s^n is given, ϕ_w^{n+1} is numerically derived in a two-process model. Consequently, ϕ_s^{n+1} is determined by ϕ_w^{n+1} . In the circle map, ϕ_w was plotted as a function of ϕ_s (F_{sw}) and vice versa (F_{ws}). F_{ws} was rotated and superposed onto F_{sw} (Fig. 2a). Figure 2a shows the circle maps derived from the two-process model without a nap term ($A_{\text{nap}} = 0$). In F_{sw} , ϕ_w is truncated around $\phi_s = 17:00$, and that edge of ϕ_w discontinuously connects to another branch.

In Fig. 2a, there is an intersection between F_{ws} and F_{sw} around 17:30. Due to the iterations, the intersection (ϕ_s^* , ϕ_w^*) goes to (ϕ_w^* , ϕ_s^*) and returns to (ϕ_s^* , ϕ_w^*). The intersection is a fixed point in the circle maps. A one-dimensional map of ϕ_s^n reconstructed from those circle maps had a fixed point ($\bullet$ in Fig. 2b). The one-dimensional

Fig. 2 Maps derived by the two-process model. $A_{\text{nap}} = 0$, $\beta = 0.2237 \text{ h}^{-1}$. **(a)** Circle maps. Iterations between ϕ_s and ϕ_w is performed by alternative application of $\phi_w = F_{\text{sw}}(\phi_s)$ and $\phi_s = F_{\text{ws}}(\phi_w)$. There is an intersection (ϕ_s^*, ϕ_w^*) between F_{sw} and F_{ws} . The intersection indicates the fixed point of ϕ_s (or ϕ_w). **(b)** One-dimensional map of ϕ_s . The black dot indicates a fixed point

map indicates that the intersection in the circle maps is a stable fixed point.

Sleep parameters in autistic children

The irregularity of sleep-wake patterns was assessed by the following five sleep parameters: (1) standard deviation (SD) of nocturnal sleep onset time, (2) SD of morning waking time, (3) mean number of sleep episodes, (4) SD of number of sleep episodes, and (5) total number of night waking. To obtain the sleep parameters, we performed a 30-day simulation with the two-process model under noisy conditions. The initial 10 days were eliminated, and ϕ_s^n and ϕ_w^n during the last 30 days were collected. $H(t)$, $L(t)$, and $S(t)$ were updated every 10 min. The initial sleep onset time ϕ_s^0 was 20:00. The mean value of white noise $\zeta(t)$ was

0 and the standard deviation was 0.006. The 30-day simulation was repeated 20 times.

The mean values of the above sleep parameters obtained by the model were compared with those reported by Inanuma (1984), who showed the month-long records of sleep patterns in 12 autistic children 1–3 years old and 16 normal children 1–3 years old. The criteria of autism were provided by Kanner (1943, 1944); Rutter (1968, 1978), and the World Health Organization (1978). The sleep diaries contained the sleep onset and waking times. He reported that although there was no difference in the mean number of sleep episodes between the two groups, autistic children showed irregular sleep-wake cycle such as large fluctuations of nocturnal sleep onset time or morning waking time or the number of sleep episodes. Night waking occurred in the autistic children, but not in the records of the normal children. Table 1 includes the sleep parameters reported by Inanuma.

The sleep parameters obtained by the model were compared with Inanuma's data in autistic or normal subjects. Tests of statistical significance were performed using t test. P -values < 0.01 were considered significant. The null hypothesis was that a series of ϕ_s^n (or ϕ_w^n) derived by the present two-process model was equivalent to that in early infantile autism. If significant differences were not found, the null hypothesis was not rejected. When there was no significant difference in the sleep parameters, we regard that the simulation well reproduced the sleep patterns in Inanuma's data. Statistical significance tests were also performed for the sleep parameters between models for normal and autistic children. For the statistical tests, we chose a (β, A_{nap}) -parameter set from each model group

Table 1 Sleep parameters obtained by the two-process model and Inanuma's clinical data

Sleep parameter	Autistic children		Normal children	
	Model $\beta=0.238 \text{ h}^{-1}$ $A_{\text{nap}}=0.21$	Inanuma (1984)	Model $\beta=0.2237 \text{ h}^{-1}$ $A_{\text{nap}}=0.24$	Inanuma (1984)
SD of nocturnal sleep onset time [h]	(N=20)	(N=11)	(N=20)	(N=16)
	0.96 ± 0.04	1.17 ± 0.41	0.80 ± 0.09	0.68 ± 0.24
	_____ n.s. _____		_____ n.s. _____	
SD of morning waking time [h]	(N=20)	(N=11)	(N=20)	(N=16)
	0.78 ± 0.07	0.96 ± 0.36	0.25 ± 0.03	0.49 ± 0.15
	_____ n.s. _____		_____ * _____	
Mean number of sleep episodes [day^{-1}]	(N=20)	(N=12)	(N=20)	(N=16)
	1.66 ± 0.02	1.64 ± 0.38	1.85 ± 0.04	1.70 ± 0.36
	_____ n.s. _____		_____ n.s. _____	
SD of number of sleep episodes [day^{-1}]	(N=20)	(N=12)	(N=20)	(N=16)
	0.48 ± 0.01	0.53 ± 0.11	0.36 ± 0.04	0.37 ± 0.13
	_____ n.s. _____		_____ n.s. _____	
Total number of night waking [per month]	(N=20)	(N=11)	(N=20)	(N=16)
	5.70 ± 1.17	3.55 ± 4.93	0.00 ± 0.00	0.00 ± 0.00
	_____ n.s. _____		_____ n.s. _____	

Difference in each sleep parameter was analyzed by t test (* $P < 0.01$).

which the null hypothesis was not rejected, and obtained the sleep parameters by those parameter sets. The sleep parameters of those two model groups were compared by t test.

Results

Dynamical properties of two-process model with a nap term

The phase diagram of the two-process model with a nap term is shown in Fig. 3. The sleep-wake cycle is represented by notation $p:q(l,k)$, which means that q sleep episodes are included during p cycles of Process C. These sleep episodes include l nap times and k sleep following night waking times. For example, 1:1(0,0) indicates that a nocturnal sleep is periodically taken everyday. Naps and night wakings are not seen. 1:2(1,0) indicates that a nap is taken every day. 1:2(0,1) indicates that a night waking is seen every day. In 2:3(0,1), a nap is taken every other day. In 2:3(0,1), night waking occurs every other day.

If A_{nap} increased, Process S tended to meet the nap term in $H(t)$. The nap frequency monotonically increased along A_{nap} ($\beta = 0.2$ in Fig. 3): no nap $\rightarrow$ once every other day $\rightarrow$ twice every three days $\rightarrow$ once a day. Since A_{nap} lowers $H(t)$, the time range of the morning waking followed by naps is extended in the two-process model with a nap term. Figure 4 shows circle maps derived from the two-process model with a nap term at $A_{\text{nap}} = 0.24$ and $\beta = 0.2237 \text{ h}^{-1}$. There is a trajectory of 1:2(1,0), which shows periodic

Fig. 3 The phase diagram of the two-process model with a nap term as a function of β and A_{nap} . The notation $p:q(l,k)$ means that q sleeps including l naps and k sleeps following night wakings are seen during p days. The area without the notation satisfies one or more following conditions: $p \geq 4, q \geq 6, l \geq 3, k \geq 3$. A represents the parameter set which the sleep parameters are not different from those of infantile autism. N represents the parameter set which the sleep parameters are not significantly different from those of normal children

Fig. 4 Circle maps derived by the two-process model with a nap term. $A_{\text{nap}} = 0.24$. $\beta = 0.2237 \text{ h}^{-1}$. An increment of A_{nap} induces nap branch and moves a tip of F_{ws} (pointed by $\blacktriangle$) upwardly. Periodic alternations between nocturnal sleep and nap (1:2(1,0)) occurs

alternations between nocturnal sleep and nap. A_{nap} introduced a branch representing nap in the circle map (nap branch in Fig. 4). An increase in A_{nap} also moves the terminal of F_{ws} upwardly (indicated by $\blacktriangle$ in Fig. 4). Therefore, the intersection between F_{sw} and F_{ws} was removed, and a channel between a branch of F_{sw} and the terminal of F_{ws} (indicated by $\blacktriangle$ in Fig. 4) was made. That channel allows iteration to the nap branch of F_{ws} .

When decay rate β was large, $S(t)$ reached $L(t)$ at night. This means night waking. The frequency of night waking increased with an increase in β . When $A_{\text{nap}} < \sim 0.2$, β led to the bifurcation of sleep-wake cycles from 1:1(0,0) to 1:2(0,1) by 2:3(0,1). 1:2(0,1) means that night waking occurs every night. When $A_{\text{nap}} > \sim 0.2$, β led to bifurcation from 1:1(0,0) and 1:2(1,0) through 2:3(1,0) and 3:5(2,0).

In the circle maps, β shifted F_{sw} horizontally (Fig. 5a). The F_{sw} shift removed the intersection between F_{sw} and F_{ws} ($\beta = 0.238 \text{ h}^{-1}$, $A_{\text{nap}} = 0.21$). Due to the disappearance of the intersection, the fixed point became unstable, and another periodic cycle appeared. In the one-dimensional map (Fig. 5b), there is a 2:3(1,0) periodic orbit. An increment of β also induced a short branch ($\star$ in Fig. 5b). An iteration to the short branch means sleep in the early evening followed by a nap in the next morning. The discontinuous point of F_{sw} , which corresponds to night waking, was also moved along the ϕ_s axis. Therefore, a small disturbance potentially causes night waking.

Sleep parameters of the two-process model with a nap term

White noise was applied to the two-process model with a nap term during iterations. One example of the sleep patterns over a 30-day period is shown in Fig. 6a ($\beta = 0.238 \text{ h}^{-1}$, $A_{\text{nap}} = 0.21$). These sleep patterns resemble to those in autistic children between 1 and 4 years

Fig. 5 One-dimensional maps derived from the two-process model with a nap term. $A_{\text{nap}} = 0.21$. $\beta = 0.238 \text{ h}^{-1}$. **(a)** Circle maps. An increment of A_{nap} and β shift a tip of F_{ws} (pointed by $\blacktriangle$) upwardly and shift F_{sw} to right along the ϕ_s axis, respectively. Those shifts remove

the intersection between F_{sw} and F_{ws} . **(b)** One-dimensional map of ϕ_s . The trajectory indicates the periodic orbit 2:3(1,0). Iteration to a short branch (indicated by $\star$) causes sleep in the early evening followed by a nap in the next morning

of age; frequent naps occurred (14.0 ± 1.2 times/month) and night wakings were found several times. The sleep parameters are summarized in Table 1. We found (β, A_{nap}) -parameter set in which the sleep parameters in the two-process model were not significantly different from those of autistic children between 1 and 4 years of age. Parameter set A in Fig. 3 shows the (β, A_{nap}) -parameter sets with which autistic sleep patterns were reproduced. Most of area A is encompassed in the 2:3(1,0) region. Although night waking does not occur in the 2:3(1,0) area, the noise caused night waking several times a month. There were 3:5(1,1) and 2:3(0,1) beside area A. In the circle maps (Fig. 5a), iterations from the branch of sleep in the early evening to the branch of night waking tended to be induced under noise. Fluctuations facilitate such iterations. In the one-dimensional map, the noise separates ϕ_s from the stable periodic cycle (Fig. 5b). During iterations, the short branch ($\star$ in Fig. 5b) was sometimes skipped in the one-dimensional map. Consequently, night waking occurred. The trajectory of iterations was sensitive to noise.

Parameter set N in Fig. 3 shows that the sleep parameters of the two-process model were not significantly different from those of normal children between 1 and 4 years of age except the SDs of morning waking times for normal children. In particular, we chose a parameter set with which no night waking was seen in 20 simulations. A nap occurred almost every day. A typical sleep-wake pattern is shown in Fig. 6b ($\beta = 0.2237 \text{ h}^{-1}$ and $A_{\text{nap}} = 0.24$). Naps were often seen. The noise caused a little fluctuation in the sleep-wake cycle. Although the sleep-wake cycle was not exactly 1:2(1,0), fluctuations of ϕ_s were small. Further, there was no night waking in the sleep-wake cycles. The sleep parameters are summarized in Table 1. In the circle maps (Fig. 7), a periodic sleep-wake cycle existed. The trajectory of the iterations was far from the branch of the night waking. Therefore, normal nocturnal sleep followed normal waking even if the noise exists.

The sleep parameters obtained by the simulation with one parameter set in area A ($\beta = 0.238 \text{ h}^{-1}$, $A_{\text{nap}} = 0.21$) were

Fig. 6 Day-by-day plot of the sleep patterns obtained by the two-process model with a nap term. Bar indicates the sleep duration. **(a)** Irregular sleep pattern. The sleep pattern represents that in autistic children. Sleep onset and waking times are irregular. Night waking is also seen. $A_{\text{nap}} = 0.21$. $\beta = 0.238 \text{ h}^{-1}$. **(b)** Periodic sleep pattern. This pattern represents that in normal children. $A_{\text{nap}} = 0.24$. $\beta = 0.2237 \text{ h}^{-1}$

Fig. 7 One-dimensional map derived by the two-process model with a nap term. $A_{\text{nap}} = 0.24$. $\beta = 0.2237 \text{ h}^{-1}$. The parameter set exhibits sleep patterns in normal children. The map has the periodic orbit 1:2(1,0). Iteration to a short branch (indicated by $\star$) causes sleep in the early evening. The sleep in the early evening is followed by a nap, but not night waking

compared with in area N ($\beta = 0.2237 \text{ h}^{-1}$, $A_{\text{nap}} = 0.24$). There are significant differences between simulation results for normal and autistic groups in all sleep parameters (Table 1). The statistic tests indicated that the sleep-wake cycle of a model for autistic case showed more irregular sleep-wake cycle than for normal case. However, there is significant difference in the mean number of sleep episodes in the simulation, while no differences is found in Inanuma's results.

Discussion

Irregular sleep-wake patterns were reproduced by the two-process model with a nap term when external noise was given. The noise caused large fluctuations in the sleep patterns in autistic cases, while fluctuations were limited in normal cases. The sensitivity of the sleep-wake cycle depended on A_{nap} and β in the two-process model. In the circle maps, A_{nap} and β determined the relative positions of F_{sw} and F_{ws} . A_{nap} introduced a nap branch in F_{ws} . A large A_{nap} extends the range of the morning waking times followed by a nap. The time range of waking followed by sleep in the early evening is narrowed. Further, β shifts F_{sw} along the ϕ_s axis. Such F_{sw} shift causes bifurcations in the sleep-wake cycle of ϕ_s . These bifurcations make the sleep-wake cycle sensitive to noise. A small noise disturbs branches in the circle map. That shift of the branch generates another trajectory in which night waking is included, and then sleep is fragmented.

According to parent answers to questionnaires (Hoshino et al. 1984), sleep disturbances were reduced in children who led a regular life. The bifurcation properties of the one-dimensional maps shown here indicate that a regular sleep-wake cycle is exhibited under less noisy environments in autistic children. The parent deviser seems to be returning dislocated sleep-wake rhythm to periodic orbit, and then sleep disturbances are diminished.

The bell-shaped function of the two-process model causes a nap. The physiological meaning of Process C is not clearly defined. Body temperature is one candidate for Process C because it has a circadian rhythm and is phase-locked with the 24-h period of sunlight. Sleep episodes tend to occur around the temperature minimum and appear once a day except naps. Internal desynchronization has revealed that naps are mainly seen around the temperature minimum and halfway between successive minima (Zulley and Campbell 1985). Naps that begin halfway between successive minima show a short duration of sleep, while the other type of naps shows relatively long sleep. Those features in naps may correspond to the nap term introduced here.

Decay rate β corresponds to the decline of EEG power density during NREM sleep (Daan et al. 1984b; Borbély

1982). For the irregular sleep patterns in autistic children, β needs to be larger than in normal children in the present two-process model. Ornitz (1972) has reported immaturity in the sleep spindle during NREM sleep in autistic children. Age-related reduction in slow-wave activity and theta during sleep in normal subjects has also been reported (Gaudreau et al. 2001). Children showed more slow-wave activity than adolescents and other older subjects. Gaudreau et al. (2001) have also reported that, although the subjects were 6 or older, the decay rate of the slow-wave activity was steeper than the older subjects, suggesting an attenuation of homeostatic sleep drive with age. Those results may indicate that decay rate β of the two-process model decreases with age, and a larger β is related to immaturity in the sleep architecture in autistic children.

The two-process model has been used to understand how coexisting sleep problems occur in depression (Borbély 1987). Prolonged sleep latency, night waking, and sleep interruptions are found in the sleep patterns in depression. Reduction of slow-wave sleep has been found in depressives. Borbély and Wirz-Justice (1982) have hypothesized that Process S in depressed patients does not rise to the level attained in healthy subjects. This deficiency in Process S accounts for the impairment of sleep onset and continuity in depressed patients. Simulations with a low rising rate of Process S also reproduced sleep continuity in depression (Beersma et al. 1985). Even if each developmental disorder had similar sleep propensities, sleep architecture causing such sleep propensities might be different. To apply the two-process model to other developmental disorders, parameters must be carefully adjusted.

Although autistic children show more irregular sleep patterns than normal children, the difference in the parameter plane was small in our results. Such a slight difference in the balance of homeostatic and circadian variables led to sensitivity of sleep-wake cycles to environmental stimuli in autistic children. Generally, sleep problems in autistic children diminish with age. Hoshino et al. (1984) reported that the duration period of sleep problems was 22 months on the average. Our results showed that there is robust periodic sleep-wake cycle beside noise sensitive orbits. If homeostasis and circadian rhythms are balanced, the sleep-wake cycle is periodic even under noise. Those results suggest that the sleep architecture matures in autistic children, and thus, sleep problems is diminished with age.

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