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Hardware assessment using the multi-module, multi-resolution system (M^3R): A signal-detection study

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Abstract

The multi-module, multi-resolution system (M^3R) is used for hardware assessment in objective, task-based signal detection studies in projection data. A phantom capable of generating multiple realizations of a random textured background is introduced. Measured backgrounds from this phantom are used along with simulated lumpy and uniform backgrounds to investigate signal-to-noise ratio as a function of exposure time. Results are shown to agree with theoretical predictions, exhibiting a power-law like dependence previously seen for studies performed either in simulation or without an imaging system, and help validate the use of simulated lumpy backgrounds in observer studies. A second study looks at signal-detection performance, measured by AUC (area under the receiver operating characteristic curve), in lumpy backgrounds for 20 M^3R aperture combinations as a function of lump size and signal size. Observer performance reveals an improvement in AUC for certain ranges of signal and lump combinations through the use of multiplexed, multiple-pinhole apertures, indicating a need for task-specific aperture optimization. The channelized Hotelling observer is used with Laguerre-Gauss channels for both observer studies. Methods for selection of number of channels and channel width are discussed.

Keywords

image quality; hardware assessment; channelized Hotelling observer; signal detection

I. INTRODUCTION

Researchers in medical imaging are continually striving to improve imaging systems. In many ways, imaging hardware represents the cornerstone of these improvements.¹ No matter how refined reconstruction and processing algorithms become, their effectiveness will always depend on the quality of the data gathered by the imaging system itself. Therefore, the optimization of imaging hardware must constitute one of the principal goals in improving an imaging system.

We have designed and built a small-animal imaging system, known as the multi-module, multi-resolution system (M^3R)² that is well-suited for hardware assessment and evaluation. M^3R allows for easy modification of parameters such as magnification, pinhole diameter, pinhole number, and degree of multiplexing. This flexibility enables the relatively rapid design, construction, and evaluation of different apertures. The knowledge gained through these studies is intended to move towards the eventual goal of true, task-specific hardware optimization.

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The aims of this paper are manifold. First, it introduces a phantom capable of generating multiple realizations of a random, textured background for use in observer studies. Second, it presents a study on signal detection as a function of exposure time using measured backgrounds from this phantom as well as simulated lumpy and uniform backgrounds for multiple M^3R apertures. Third, signal detection in lumpy backgrounds is evaluated as a function of signal size and lump size for multiple configurations of M^3R . Finally, the paper is intended to be illustrative of the kind of studies for which M^3R has been designed and is now being implemented.

The remainder of the paper is structured as follows. Section II describes methods used for these studies, including imaging system and configuration, a brief discussion of image quality including a description of the channelized Hotelling observer (CHO) and figures of merit, data collection and calibration procedures, objects, and procedures for the two observer studies. Section III presents results of the two observer studies and a brief discussion of the methods used to determine channel width and number of channels. The final section provides a brief discussion of the results and outlines ongoing and future image quality studies involving M^3R .

II. METHODS

A. Imaging system and configuration

M^3R 's system configuration may be modified through the interchange of pinhole plates between slots machined into the system's shielding which is pictured in Fig. 1.² The system configuration used in this study consisted of three different magnifications, two different pinhole diameters, and four different pinhole numbers. Details of the configuration are shown in Table I. Angle, shown in the last column of the table, is defined as the full opening angle of the pinhole, resulting from a combination of the countersink and keel.

B. Image quality

The evaluation of an imaging system requires rigorous, objective, quantitative analysis. The techniques that comprise this analysis are collectively known as the objective assessment of image quality and have been well described by Barrett *et al.*^{3–5} No imaging system exists that will perform ideally for all tasks. Therefore, the optimization of hardware should pertain to a specific task or set of tasks. The evaluation of an imaging system to perform a given task requires identification of the task, knowledge of the objects to be imaged, a description of the observer that will perform the task, and a figure of merit by which to quantify the observer's ability to perform the task.

We consider two signal detection tasks in which an observer attempts to classify two-dimensional (2D) projection images, generated by taking different background and signal objects through different aperture configurations of M^3R , as either signal present or signal absent. In one study, the effect of varying exposure time on detection performance was explored for three different background types as a means to practically implement a phantom capable of generating random, textured backgrounds and compare its effect on observer performance with that of other background types. In the other study, detection performance of different M^3R aperture configurations was explored in simulated lumpy backgrounds as a function of lump size and signal size.

1. Signal-to-noise ratio (SNR) and the channelized Hotelling observer (CHO)—

For a binary classification task, there exists a linear discriminant that maximizes the signal-to-noise ratio, an objective figure of merit defining the separation between two classes of objects. This optimal linear discriminant, also known as the Hotelling discriminant, is given by

$$w_{\text{Hot}} = K_g^{-1} \Delta \bar{g}, \quad (1)$$

where K_g is the covariance of the data (assumed equal for both hypotheses) and $\Delta \bar{g}$ is the average difference of the data under the two hypotheses.

Computational difficulties arise when attempting to apply the Hotelling discriminant. The primary difficulty lies in the need to estimate the covariance matrix on the data (written as $\hat{K}_g$). For a detector array that is only 79×79 pixels (i.e., $M = 6241$, the dimension typically used in our modular gamma cameras), a covariance matrix of dimension 6241×6241 must be estimated and inverted. For a covariance matrix of dimension $M \times M$, at least $M + 1$ images are required to obtain an invertible estimate. In practice, it has been shown that many times this minimum sample size may be required to ensure an acceptable amount of error in estimating the similarity between two distributions.⁶ Also, once this covariance matrix has been generated, it must be inverted. While methods exist for computing this inverse, they are computationally burdensome, time consuming, and sometimes inaccurate.

An alternative to the full Hotelling observer exists in the form of a channelized Hotelling observer.^{7–10} Application of a well-chosen channel template can greatly reduce dimensionality while preserving the majority of the information in an image. A channel template T is typically an $M \times P$ matrix whose P columns are the individual channel profiles. The data dimension M will often be in the thousands while the channel dimension P typically lies in a range from 4 to 15.

A channelized Hotelling observer was used to generate two figures of merit for this study. Signal-to-noise (SNR) (as opposed to AUC which is bounded between 0.5 and 1) was used as the figure of merit for the signal detection as a function of exposure time study because of the wide range of anticipated SNR values. An estimate of the Hotelling SNR on the channel outputs is given by

$$\text{SNR}_{\text{CHO}}^2 = s^\dagger T^\dagger \hat{K}_T^{-1} T s, \quad (2)$$

where s is the signal vector and $\hat{K}_T$ is an estimate of the $P \times P$ covariance on the channel outputs.

AUC was used as the figure of merit for the signal detection study as a function of signal and lump size. In the case of AUC analysis, test statistics under each hypothesis were formed according to

$$t_i = s^\dagger T^\dagger \hat{K}_T^{-1} T g_i, \quad (3)$$

where t_i is a test statistic for the i th class and g_i is a projection data vector belonging to the i th class. In both cases (SNR and AUC), the covariance matrix on the channel outputs must still be estimated; however, its dimension has now been reduced to $P \times P$. This reduction in dimensionality pares the number of images required to achieve adequate sampling and eliminates difficulties in matrix inversion. Standard deviation was calculated for the AUC values using a method developed for the multiple-reader, multiple-case problem.^{11,12}

The difficulty in implementing the channelized Hotelling observer lies in the selection of appropriate channels. Different channels have been shown to be appropriate for different tasks. For example, anthropomorphic channels like the Gabor channels have been shown to be excellent models on which to base the human vision system.¹³ Abbey and Barrett showed that difference-of-Gaussian channels with proper inclusion of internal observer noise succeeded in matching human observer performance in a lesion detection study.¹⁴ In addition to mimicking human performance, properly chosen channels may be used to approximate the ideal linear

observer. In this study, known, radially symmetric, signals were placed in lumpy backgrounds having no preferred orientation. These characteristics imply that the ideal linear template will be centered at the known position of the signal and be rotationally symmetric. Laguerre-Gauss channel profiles are a good choice for use in the channelized Hotelling observer (to approximate the Hotelling observer) for such a detection task because they form a basis on the space of rotationally symmetric square-integrable functions in $\mathcal{R}^{2,1}$. These channels have been employed in similar detection-task studies and shown to produce excellent agreement between the channelized and ideal linear observer performance with a small (5–30) number of channels.¹⁵

C. Data collection

Two methods were employed to generate projection data from M^3R . One method of data collection was through direct measurement. The bead phantom described in Sec. II D 2 was placed in the object space, and data were collected using the group's standard modular gamma cameras.¹⁶ The second method by which data were generated was through the use of a measured system matrix. Projection data are produced according to the imaging equation given by

$$g = Hf + n, \quad (4)$$

where g is an $M \times 1$ vector of projection data, f represents the $N \times 1$ discretized object space, n is noise, and H is the $M \times N$ system matrix, responsible for mapping an object to an image.

Our group chooses to directly measure the system response, following techniques developed by Chen *et al.*¹⁷ This procedure involves moving a point source through the object space on a Cartesian grid inscribed in a cylinder and recording the system response at each location. One advantage to this method is that issues such as position-dependent detector variations, system misalignment, inaccuracies in pinhole construction, and complicated system geometry are built into the procedure. Other modeling errors are also avoided through this use of real data. The system matrix used in this study consists of 41,888 voxels. Each voxel is a cube 0.9 mm on a side. The object space cylinder has a diameter of 30.6 mm (corresponding to 34 voxels) and a height of 39.6 mm (corresponding to 44 voxels). This particular system matrix was collected with an initial source of activity of 12 mCi and initial collection time of 0.1 s. Collection time was adjusted to compensate for the decay of the source, doubling every 6 h. The scan duration was approximately 22 h. Once in possession of the system matrix H , it becomes possible to generate projection data through the mapping of simulated objects. Simulated objects are generated on the grid of points described by the system matrix (or on a more finely sampled grid). Discretization error occurs as a result of simulating objects in this way but in this study, we have chosen to accept this penalty as a trade-off for the advantages gained through use of a measured system matrix. The reader is referred to Barrett and Myers¹ for further discussion of discretization error.

Because of the Poisson nature of the physical processes governing data collection, noise can be minimized in the measured system matrix by using long exposure times. This low-noise system matrix means that simulated objects taken through the measured system matrix better represent noise-free projection data given by

$$\bar{g} = Hf. \quad (5)$$

Poisson noise can be added to the simulated projection data by sampling a Poisson random vector with mean $\bar{g}$. Scatter and attenuation effects were ignored in this study because of the small object sizes.

D. Objects

1. Simulated signals—A set of 11 spherical objects was generated. The diameters of these spheres ranged from 0.45 to 7.2 mm. All spheres were located in the center of the object space. The objects were generated on a finely sampled grid and interpolated onto the grid of points used to measure the system matrix.

The $\mathcal{L}_2$ norms of the signals were held constant because doing so forces equal detectability across signals for the signal known exactly/background known exactly case in Gaussian noise.¹ While some parameters of this study differ from that ideal case, the idea was that inclusion of this trait would insure that differences in performance across signal and lump size may be attributed to variations in background and aperture configuration. Example projections of signal objects (diameter of 1.8 mm) for each aperture are shown in Fig. 2.

The signal objects were taken through the measured system matrix. Signal widths were measured for each signal size for each aperture. The signal center was measured once for each pinhole (because it does not vary as signal size increases). This information was used to generate the channelized data used by the channelized Hotelling observer. For the case of Aperture 2, in which signal projections overlapped for large signals, channels were allowed to overlap.

2. Measured bead backgrounds—Both clinical and small-animal single photon emission computed tomography images vary from subject to subject. This object variability has been shown to have a significant effect on observer performance for signal detection tasks.^{14,18} A phantom was desired to produce object variability experimentally for use in observer studies. This phantom could be used in addition to simulated lumpy backgrounds to compare observer performance for the two background types. The hope—because our measured system matrix incorporates real data, without modeling error—was to see similar performance between the two types of backgrounds, indicating that simulated objects are sufficient for our studies. Also, the phantom may be of interest for use in observer studies by other groups that use modeled system matrices.

To address these needs, we developed a hollow cylindrical phantom with a diameter of 30 mm, roughly that of a mouse, capable of generating multiple realizations of a random background in a relatively short span of time. The cylinder was filled with nonuniform, ellipsoidal polystyrene beads ~4 mm in diameter as shown in Fig. 3. A dilute radioactive source filled the space around the beads. Images of the phantom taken with this configuration resulted in the desired random background. Stirring of the bead mixture altered the arrangement of the beads and allowed for acquisition of many random background images. Inspiration for the phantom was found in previous work conducted by Sain.¹⁹ Example bead projections are shown in Figs. 4(a) and 4(b). The projection image in Fig. 4(a) is dominated by counting noise while the image in Fig. 4(b) is dominated by structured bead noise. Projection data were collected for 150 realizations (i.e., stirs) of the bead background.

Note that a signal was not present in the bead phantom at the time of data acquisition. Because the system mapping from object space to projection space is linear, background and projection data may be combined in projection space. This feature enables greater flexibility in choosing signal sizes and strengths.

3. Simulated lumpy backgrounds—A second set of backgrounds was produced in simulation. These are the lumpy backgrounds developed by Rolland and Barrett.^{3,18} Lump centers were randomly selected from within the object space. For the exposure time study, it was desired that the mean number of counts remain relatively constant across background types. Because obtaining measured data from the bead phantom is more difficult than

generating simulated data, the amplitude of the simulated objects—lumpy and uniform—was modified to match the average number of counts in the measured bead data. This modification is illustrated by

$$\alpha \bar{g} = \alpha H f, \quad (6)$$

where α is a multiplicative factor. The final realization, with the desired number of counts, was formed by sampling a Poisson random vector with mean $\alpha \bar{g}$. Example images for a lumpy background are shown in Figs. 4(c) and 4(d). The projection image in Fig. 4(c) is dominated by counting noise while the image in Fig. 4(d) is dominated by structured lumpy noise.

4. Simulated uniform background—A uniform object filling the object space was taken through the system matrix, producing a noise-free projection image reflecting aperture response to the entire object space. The total number of counts in the projection data was controlled using a multiplicative factor as described by Eq. (6). Again, noise was added (creating different noise realizations of the uniform background) by sampling a Poisson random vector with mean $\alpha H f$.

5. Calculation of noise—A useful decomposition was used for the covariance term in all components of this study. Without loss of generality, the estimate on the covariance on the data may be written

$$\widehat{K}_g = \widehat{K}_n + \widehat{K}_{\bar{g}}, \quad (7)$$

where $\widehat{K}_n$ is an estimate of the mean noise covariance and $\widehat{K}_{\bar{g}}$ is an estimate of the covariance on the noise-free projection data.¹ Also note that this decomposition does not assume statistical independence between the mean noise covariance and the covariance on the noise-free projection data.

In the case of channelized Hotelling observer, the estimated covariance on the channel outputs [Eq. (2)] may be written as

$$\widehat{K}_T = T \widehat{K}_n T^\dagger + (TW)(TW)^\dagger, \quad (8)$$

where W is a $N_S \times M$ matrix composed of the N_S sample images, scaled by $\sqrt{N_s}$ and having the mean sample image $\bar{g}$ subtracted.¹ The matrix $\widehat{K}_T$ will be $P \times P$ where P —the number of channels—is typically less than 20 and often as few as five or six, making the covariance matrix easily invertible. Also, the small dimension of $\widehat{K}_T$ eases requirements on sample number and noise characteristics necessary to estimate the matrix.

Practical considerations affected implementation of this method for each of the three different background types. The original objects in the case of the bead backgrounds were not known so the noise covariance $\widehat{K}_n$ was estimated using the mean of all collected projection images. The same projection images were also used to estimate $\widehat{K}_{\bar{g}}$ according to Eq. (8). Because the lumpy backgrounds were generated in simulation, the true lumpy objects were known and the mean object could be formed directly. The mean object was taken through the system matrix to form the noise covariance term $\widehat{K}_n$. As with the bead backgrounds, the covariance on the mean projection data was found according to the final term of Eq. (8). In the case of the constant uniform object, $\widehat{K}_{\bar{g}}$ contributes nothing to the overall covariance. Only the Poisson noise of the data collection process is present, meaning that the covariance on the noise is simply given by $\widehat{K} = \widehat{K}_n$.

E. Exposure time study

One hundred fifty realizations of each of the three background types were used to explore signal detection as a function of exposure time for different M^3R apertures. The hardware configurations used for this study included the four apertures used singly and the combination of all four apertures. Channel profiles were summed for an individual projection (i.e., the five pinholes present in Aperture 0) but concatenated across aperture to maintain reasonable dimensions on $\mathbf{K}_T$. The lumpy backgrounds were created with a Poisson distributed number of lumps with a mean of 20, a 2σ lump width of ~ 6.3 mm, and an activity of ~ 0.25 mCi per lump, making the total activity somewhat analogous to a mouse with 5 mCi of activity. Exposure times used in this study ranged from 1 ms to 10s.

Multiple motivations existed for performing the exposure time study. One motivation was to evaluate the practical implementation of the bead phantom in an observer study as it compares to other backgrounds (as described in Sec. II D 2). Another motivation lay in past work evaluating performance as a function of exposure time in backgrounds generated without an imaging system. This work adds another layer of complexity to those studies.

F. Signal detection—Varying signal size and lump size

The detection in lumpy backgrounds study began by generating 2000 lumpy background realizations for seven different lumpy object sizes. The lumpy object sizes ranged from 2σ widths of $L = 1.8$ – 6.3 mm. The average number of lumps per projection (Poisson distributed) was chosen to keep the average lumpy object activity constant. Average lumpy object activity is given by

$$\bar{L} = \frac{\bar{N}_L \int_{\infty} L d^3r}{V}, \quad (9)$$

where $\bar{L}$ is the average lumpy object activity, $\bar{N}_L$ is the average number of lumps per projection, V is the volume of the object space, and the integral represents the activity in a single lump. $\bar{N}_L$ ranged from 8 to 212 depending on lump size. Example projection images through Aperture 1 for the largest, middle, and smallest lump sizes are shown in Fig. 5.

Four angular views of each lumpy object were generated for each M^3R aperture. This angular sampling allowed the use of any combination of M^3R apertures simultaneously. For example, to simulate M^3R being loaded with apertures 0, 1, 3, and 0 again just requires loading the angle-appropriate projection data for each of those apertures. The 20 aperture combinations evaluated are listed in the first column of Table II. These combinations represent each aperture used individually, the combination of all four apertures, all possible combinations in which two apertures were each used twice, and all combinations using apertures 0, 1, and 3. One motivation for using different combinations of apertures was to see if there was a potential gain in observer performance through the simultaneous use of multiple magnifications and/or pinhole geometries as opposed to using identical apertures with each of the four cameras in the system.

1. Selection of channel width and number of channels—A fast method for choosing channel width as a function of aperture and signal size was desired. The proposed method of channel width selection involved finding a channel width that returned the minimum mean square error (MSE) between the 0th order Laguerre–Gauss polynomial and the signal projection. Four aperture combinations (0000, 1111, 2222, and 3333) and four signal sizes were selected for this test. AUC was calculated for a range of channel widths surrounding the width that returned the lowest MSE. AUC values were also calculated for a fixed width (that which returned the minimum MSE) over a range of channel numbers for the same aperture

configurations and signal-lump pairs. This test was performed to investigate the impact of number of channels on AUC.

III. RESULTS

A. Exposure time study

The results from the exposure time study corroborate previous studies into the area of signal detection with changing exposure time for lumpy backgrounds.^{14,18,20,21} Rolland and Barrett¹⁸ were the first to note the power-law relationship between exposure time τ and SNR² in lumpy backgrounds. Figure 6 exhibits behavior indicating that the relationship between SNR and exposure time for lumpy backgrounds may be characterized well by such a relationship. This relationship was later explained analytically for a Gaussian signal in a Gaussian lumpy background.²⁰ For that case, the slope of the power law behaves as $1 - (\sigma_s^2/\sigma_\ell^2)$ where σ_s^2 and σ_ℓ^2 are the width of the signal and lumps, respectively. The slopes of the SNR data for the lumpy backgrounds demonstrate a similar reduction in slope with increasing signal size. Several parameters in this study differ from previous studies—spherical signals, measured system matrices, 3D objects—making direct comparison difficult but it is worthwhile to note the similar trends.

SNR values for the bead phantom backgrounds also exhibit a linear region of slope, but the slope flattens at high exposure times. Slope values for the bead (linear region) and lumpy backgrounds are similar for the small signal [Figs. 6(a) and 7(a)], but where the lumpy background SNR exhibits a marked decrease in slope with increasing signal size, the bead background SNR remains relatively constant across signal size. This behavior may be explained if the bead phantom backgrounds were in some way analogous to lumpy backgrounds with a large σ_ℓ^2 in which case the slope $1 - \frac{\sigma_s^2}{\sigma_\ell^2}$ would vary little with changing signal size.

In both background types, shifts appear in the order of aperture performance for large signals and long exposure times. For all exposure times in the small signal case, the combination of all four apertures performs the best, followed by Apertures 3, 1, 0, and 2, respectively. This order holds for the larger signals at small exposure times but as exposure time increases the performance of aperture 0 begins to match ($D=4.0$ mm signal) or exceed ($D=7.2$ mm signal) the performance of Aperture 1 and near that of Aperture 3. It is interesting that, although the shapes and slopes of the curves differ between the background types, the performance behavior between apertures is consistent.

The SNR was also calculated for the uniform background with respect to exposure time. The lack of object variability results in a power-law relationship with a slope of 1 for all exposure times. Figure 8 displays performance in all three backgrounds for a few representative aperture combinations and signal sizes. It is difficult to compare the performance in lumpy backgrounds directly to that of bead backgrounds because they represent two different object models. However, it is reassuring to see that both models return performances below that returned by the uniform background.

B. Signal detection—Varying signal size and lump size

1. Selection of channel width and number of channels—Detection performance appears to be robust with respect to both number of channels and channel width as shown in Fig. 9. The AUC as a function of channel width does indicate a decrease in performance for extremely narrow channel widths. The arrows in Fig. 9(a) represent the channel width that returned the minimum MSE between the 0th order Laguerre-Gauss channel and the signal in question for that particular aperture. Given the results of these four test cases, this method was

used for channel selection for all signal, aperture, and lump combinations. Based on the results from the three test cases shown in Fig. 9(b), six Laguerre-Gauss channels (indicated by the arrow) were chosen for the computation of all AUC values.

2. Signal detection study—The maximum AUC values for all signal and lump pairs (across aperture combination) are displayed in Fig. 10(a). Maximum AUC values ranged from 0.686 to 0.979. A decrease in detectability was observed where lump and signal size were closely matched. This characteristic is evident in Fig. 10(a) as a dark band stretching from a signal diameter of ~3 mm and lump width of ~2 mm to signal diameter of ~6 mm and lump width of ~5 mm. A decrease in AUC is also seen for signals less than ~2 mm in diameter. This decrease in AUC is probably a result of poor system resolution.

Example data sets for two lump and signal pairs (out of 77 total) are shown in the first six columns of Table II. The data have been ordered by aperture configuration from highest performing to lowest performing with AUC and standard deviation values included. There are a few interesting points to note in these two data sets. First, Aperture 2 performed poorly. This poor performance occurs throughout the entire data set and is likely due to low sensitivity and a large degree of multiplexing. Because of this low performance, Aperture 2 will be largely ignored throughout the rest of the discussion.

In the first data set, aperture combinations containing Aperture 3 tend to perform best while in the second data set, aperture combinations including Apertures 1 and 0 tend to perform best (while those containing Aperture 3 are near the bottom of the rankings). While the performance difference between any two aperture combinations often lacked statistical significance (there were exceptions as seen in the first data set of Table II), such trends in performance as a function of aperture appear throughout all data sets.

The last three columns of Table II provide aperture combination ranking, AUC, and standard deviation values for the mean performance across all lump and signal combinations. While performance differences are small, there is consistency in the fact that combinations of Aperture 1 and Aperture 3 tend to rank highest. To quantify this ranking further, the frequency of occurrence of each aperture among the top five-performing aperture combinations for each signal/lump pair was calculated. For the example data set shown in the first three columns of Table II, this methodology results in scores of 1, 6, 0, and 13 for Apertures 0, 1, 2, and 3, respectively. The summary of this evaluation is shown in Fig. 10(b). Aperture 1 appears most frequently (774 occurrences), followed by Apertures 3 (589), 0 (173), and 2 (4).

Ranking by mean performance across all signal and lump combinations give some indication of overall performance. However, trends appeared throughout the data indicating improved performance for certain apertures in different regions of the lump/signal space. Table III illustrates these trends by displaying the aperture combination that returned the maximum AUC value for each signal/lump pair [the values displayed by Fig. 10(a)]. Examination of this table reveals a band spanning signal diameters in the 1.35–2.70 mm range for which aperture combinations dominated by Aperture 3 tend to return maximum AUC values. A second region near signal diameters of 4.50–6.30 mm and lump widths of 2.3–3.0 mm shows aperture combinations containing Aperture 0 to return maximum AUC values. These trends are illustrated graphically in Fig. 11. Aperture 0 was assigned the color green, Aperture 1 was assigned the color blue, and Aperture 3 was assigned the color red. The shading of the plot reflects the frequency of occurrence of each aperture among the top-five aperture combinations for each signal/lump pair. This qualitative plot further supports the trends evident in Table III with the band of red indicating increased observer performance for those aperture combinations containing Aperture 3 and the small segment of green indicating increased observer performance for those aperture combinations containing Aperture 0. Blue and purple regions

indicate increased observer performance for those aperture combinations containing Aperture 1. These findings indicate the potential for increased observer performance by selecting the proper aperture configuration for a given task.

IV. DISCUSSION AND CONCLUSIONS

M^3R has been successfully implemented as a tool for evaluating hardware configurations. The results of an exposure time study, using the channelized Hotelling observer with Laguerre-Gauss channels, support previous findings performed in simulation with lumpy backgrounds where Hotelling SNR and exposure time were shown to exhibit a power-law relationship. Lower than expected slope values for these lines may arise from differences in study parameters between this study and previous studies. Theory is currently being explored to help account for these differences.

The bead phantom was successfully employed to generate measured, random, textured backgrounds. These backgrounds were also implemented in the exposure time study and shown to exhibit behavior which shared similarities and differences with lumpy backgrounds, including a flattening at long exposure times that is not fully understood. We are actively pursuing research to answer this question. We believe the bead phantom would serve well as a source of real, random backgrounds for use in observer studies employing modeled system matrices. One difficulty in using the bead phantom is the necessary trade-off between rapidity of data acquisition and noise in the projection data. A model for computing the noise characteristics of the bead phantom is currently being developed to help overcome this difficulty.

In both bead backgrounds and lumpy backgrounds, the order of aperture performance was observed to change for larger signals and longer exposure times with Aperture 0 (low magnification, multiple-pinhole), exhibiting a relative increase in performance. The reasons for this change in performance are as yet unknown. The exposure time study has proven to be interesting in that it began as a straightforward validation study but has instead raised several questions for which future research is needed.

A second study employed the channelized Hotelling observer with Laguerre-Gauss channels to assess the performance of 20 M^3R aperture combinations. AUC was used to compare the performance of these different configurations for 77 lumpy background (seven lump sizes) and signal (11 signal sizes) combinations. The single-pinhole, high-magnification aperture was shown to appear most often among the top-performing aperture combinations. However, over a large region of lump/signal space, aperture combinations dominated by Aperture 3 were shown to be optimal. Over a smaller region of signal/lump space, the inclusion of Aperture 0 was shown to be preferable. Both Aperture 0 and Aperture 3 are multiplexed, multi-pinhole apertures. These results indicate the need for task-specific aperture design and the potential gain through the use of multiple-pinhole, multiplexed apertures. The difficult question then becomes how to determine the appropriate aperture design for a specific task. It is yet unclear why a specific aperture or aperture combination performs best for a specific lump/signal pair. With the emergence of adaptive imaging, the need to rapidly and empirically determine an optimal aperture configuration for a given task will grow in importance. Studies such as this one will help us deal with this challenge. For instance, a scout image of an object may provide the user with some information about the general structure of an object or the likely size of a tumor that one is trying to detect. This information may be used with the knowledge gained from our study to help choose the parameters of the final imaging aperture or set of apertures. The development of more specific observer studies and a broader range of aperture evaluation methods, for which M^3R is a well-suited tool, should also help answer this difficult question.

Future research involving M^3R , will include signal-detection and estimation studies including reconstructed images and, potentially, human observers. Another study will explore the use of rapid qualitative aperture assessment methods to predict the performance of the ideal observer (for which M^3R has been fully modeled) for signal-detection tasks. M^3R 's flexibility, ease of use, and unique system specifications have also attracted interest for a variety of other studies relating to sensitivity, dynamic imaging, and the development of rules to guide aperture selection in adaptive imaging.

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Fig. 1.
M³R system shielding. Slots machined into the system shielding allow the easy interchange of pinhole apertures (three plates are pictured, surrounding the object space). Four modular gamma cameras are used with the system with one abutting each of the ports seen in the shielding. A lid is affixed and the entire system is elevated so that a rotation stage may be used for tomographic data acquisition.

Fig. 2.

Example signal images (generated from a 1.8 mm signal) used in the study, (a) Aperture 0 had a magnification of 1.6 and five 1 mm pinholes. (b) Aperture 1 had a magnification of 3.33 and one 1 mm pinhole. (c) Aperture 2 had a magnification of 2.47 and nine 0.25 mm pinholes. (d) Aperture 3 had a magnification of 3.33 and four 1 mm pinholes.

Fig. 3. Bead phantom used to generate random background images. The phantom is filled with nonuniform polystyrene beads which are then stirred after each image to simulate object variability. Radioactive liquid occupies the space surrounding the beads.

Fig. 4. Projection images of an example bead background dominated by (a) counting noise, and (b) structured bead noise. Projection images of an example lumpy background dominated by (c) counting noise, and (d) structured lump noise.

Fig. 5.
Example 2D projections of 3D lumpy backgrounds for lump 2σ widths of (a) $L=6.0$ mm, (b) $L=3.0$ mm, and (c) $L=2.0$ mm.

Fig. 6. SNR for detection in a lumpy background with increasing exposure time. (a) Signal size is $D=0.9$ mm. (b) Signal size is $D=4.0$ mm (c) Signal size is $D=7.2$ mm. Please refer to Table I for aperture information. Note: Scales have been fixed across the three plots in this figure.

Fig. 7.

SNR for detection in a bead background with increasing exposure time, (a) Signal size is $D=0.9$ mm. (b) Signal size is $D=4.0$ mm. (c) Signal size is $D=7.2$ mm. Please refer to Table I for aperture information. Note: Scales have been fixed across the three plots in this figure.

Fig. 8. SNR for different backgrounds as a function of exposure time. (a) Signal diameter $D = 0.9$ mm, Aperture 1. (b) Signal diameter $D = 4.0$ mm, Aperture 3. (c) Signal diameter $D = 7.2$ mm, Aperture 0. Please refer to Table I for aperture information. Note: Scales have been fixed across the three plots in this figure.

Fig. 9.

(a) AUC as a function of channel width for three different aperture and signal combinations. Channel widths producing the minimum MSE between the signal and the 0th order Laguerre-Gauss polynomial are indicated by arrows. (b) AUC as a function of number of channels for the same three aperture and signal combinations as were used in (a). The arrow indicates the number of channels for the signal-detection study in lumpy backgrounds. Channel widths used for this study are the same as indicated by the arrows in (a).

Fig. 10. (a) Maximum AUC values (out of 20 possible aperture combinations) as a function of signal diameter and lump size (a total of 77 combinations). (b) The total number of occurrences of each aperture in the top-five performing aperture combinations for all signal and lump combinations.

Fig. 11.

A map of the frequency of occurrence of Apertures 0, 1, and 3 among the top-five performing aperture combinations as a function of signal size and lump size. Green corresponds to Aperture 0, blue corresponds to Aperture 1, and red corresponds to Aperture 3. The plot is dominated by Aperture 1 (blue and purple sections) but there is a large region in which Aperture 3 appears most critical to generating high AUC and a smaller region where Aperture 0 is most critical.

Table 1
Example system configuration. Aperture number, magnification, number of pinholes, pinhole diameter, sensitivity (normalized to the most sensitive aperture), and full opening angle are provided.

Aperture	Magnification	Number of pinholes	Diameter	Sensitivity	Angle
0	1.60	5	1 mm	0.59	85.4°
1	3.33	1	1 mm	0.50	85.4°
2	2.47	9	0.25 mm	0.10	60.0°
3	3.33	4	1 mm	1.00	85.4°

Table II

The first six columns of the table provide aperture combinations, AUC, and standard deviation values for two example signal and lump combinations. Note the differences in rank order for the aperture combinations. The final three columns provide ranked aperture combinations, AUC and standard deviation values for all aperture combinations averaged over the 77 signal and lump pair combinations. D represents signal diameter while L refers to 2σ lump width.

$D=1.8$ mm, $L=3.6$ mm			$D=4.5$ mm, $L=2.0$ mm			Mean-all combinations		
Apertures	AUC	σ	Apertures	AUC	σ	Apertures	AUC	σ
3333	0.921	0.009	1111	0.813	0.013	1113	0.802	0.014
1333	0.890	0.010	1110	0.811	0.013	1313	0.801	0.014
1313	0.886	0.011	1113	0.805	0.014	1111	0.799	0.014
1113	0.884	0.011	1313	0.800	0.014	1333	0.795	0.014
0333	0.883	0.011	0113	0.796	0.014	0113	0.792	0.014
1111	0.865	0.012	0013	0.792	0.014	1110	0.790	0.014
0303	0.863	0.012	0133	0.788	0.014	3333	0.790	0.014
0133	0.862	0.012	1212	0.785	0.014	0133	0.788	0.014
0113	0.862	0.012	0101	0.784	0.014	0333	0.785	0.014
1110	0.856	0.012	0003	0.782	0.014	0013	0.777	0.014
0013	0.849	0.012	0001	0.781	0.014	0101	0.773	0.015
0123	0.846	0.012	0000	0.779	0.014	0123	0.769	0.015
2323	0.843	0.012	0123	0.776	0.014	0303	0.768	0.015
0003	0.841	0.012	0303	0.772	0.015	0003	0.755	0.015
0101	0.839	0.013	0333	0.769	0.015	0001	0.752	0.015
0001	0.825	0.013	1333	0.767	0.015	1212	0.749	0.015
1212	0.805	0.014	3333	0.748	0.015	2323	0.744	0.015
0000	0.789	0.014	0202	0.750	0.016	0000	0.725	0.016
0202	0.750	0.015	2323	0.727	0.016	0202	0.693	0.016
2222	0.656	0.017	2222	0.670	0.017	2222	0.632	0.017

Table III
Aperture combinations returning maximum AUC values for all signal and lump combinations.

Signal diameter (mm)	Aperture combinations generating maximum AUC for each signal/lump combination						
	2.0	2.3	2.6	3.0	3.6	4.5	6.0
0.45	0113	1110	1113	1313	1113	1333	1113
0.90	1111	1111	1313	1111	3333	1111	1111
1.35	1333	1313	1313	3333	3333	1113	1313
1.80	3333	0333	3333	1313	3333	3333	3333
2.25	1313	1333	3333	3333	3333	3333	1333
2.70	1113	0113	1113	1333	1333	1333	0133
3.15	0113	0333	1113	1111	1111	1113	3333
3.60	1113	1111	1111	1111	1111	1111	1113
4.50	1111	1111	0001	1113	1313	1111	1313
5.40	1113	1110	0001	1113	1110	1111	1111
6.30	1111	1110	1212	0101	1113	1113	1333