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Experimental Measurement and Quantification of Frictional Contact between Biological Surfaces Experiencing Large Deformation and Slip

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Abstract

Interactions between contacting biological surfaces may play significant roles in physiological and pathological processes. Theoretical models have described some special cases of contact, using one or more simplifying assumptions. Experimental quantification of contact could help to validate theoretical analyses. The objective of this study was to develop a general mathematical approach describing the dynamics of deformation and relative surface motion between contacting bodies and to implement this approach to describe the contact between two experimentally-tracked tissue surfaces. A theoretical formulation (in 2-D and 3-D) of contact using the movement of discrete tissue markers is described. The method was validated using theoretically-generated 3-D datasets, with < 1% error for a wide range of parameters. The method was applied to the contact loading of opposing articular cartilage tissues, where displacements of cell nuclei were tracked optically and used to quantify the movements and deformations of the surfaces. Compared to tissues with matched material properties, tissues with mis-matched material properties exhibited increased disparities in lateral expansion and relative motion (sliding) between the contacting surfaces.

Keywords

articular cartilage; biomechanics; contact

1. INTRODUCTION

Biomechanics plays a crucial role in the normal function of the human body. Stresses and strains within a tissue are known to influence cellular responses (Fung, 1981b, Guilak, et al., 1997) and contribute to processes such as growth, maturation, adaptation (Fung, 1981b, Kiviranta, et al., 1988), injury (Quinn, et al., 2001), and repair (Goldin, et al., 1980, Salter, et al., 1980). With mechanical loading, the details of such deformation are often governed by the biomechanical interactions between contacting tissue surfaces, and such interactions may also have direct implications for both normal and pathological processes. Shearing of cartilage

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during joint articulation has been shown to significantly affect the cellular response (Nugent, et al., 2006, Nugent-Derfus, et al., 2007). Such interactions, which take place throughout the body (e.g. heart and pericardial sac; and lungs and peritoneal cavity (Loring, et al., 2005)) may also affect physiological processes. To fully understand the role that mechanics play in physiological function, it is important to assess both the internal mechanical states of tissues and the interactions occurring between them.

Theoretical analyses of contact mechanics have been developed to predict the deformations and sliding in a variety of specific conditions. The topic of contact mechanics was introduced by Heinrich Hertz to describe the normal, frictionless contact between non-conforming, perfectly elastic bodies. Much research since then has focused on relaxing one or more of the assumptions of the classical Hertzian solution and extending the applicability to more practical problems, including inhomogeneous or anisotropic materials, large deformations, and frictional contact (Johnson, 1985). In recent years, advancements in the field of contact mechanics have been driven by the formulation of robust finite element approaches and increased computing power. Models have been developed for a variety of physiological (Herzog and Federico, 2006), and even patient-specific (Fernandez and Hunter, 2005), geometries, and more detailed macro-scale boundary conditions have been prescribed by real-time tracking of movement in human subjects (Fernandez and Hunter, 2005). In biological tissues, the complex nature of the materials (e.g. cartilage, which is viscoelastic, anisotropic, and inhomogeneous) and their surfaces (which may include multiple modes of lubrication) (Fung, 1981a) makes it challenging to predict contact behaviors. With continued improvements in imaging and image processing, it is increasingly feasible to complement and validate theoretical predictions with experimental studies.

Kinematics and deformation of tissues and organs have been experimentally quantified at a variety of length-scales. Video dimensional analysis (Intaglietta and Tompkins, 1972) and biplane cineradiography (Meier, et al., 1980) have each been used to visualize displacement fields in tissues. In the heart, biplane cineradiography has been used to assess muscle strains (Hashima, et al., 1993, McCulloch and Omens, 1991) using embedded surface markers. More recently, MRI has been used to measure changes in wall thickness and chamber volume (Costandi, et al., 2007) during heart contractions. Similar studies have monitored changes in cartilage thickness (Eckstein, et al., 2005), and joint kinematics (Freeman and Pinskerova, 2005) during joint articulation. In vitro studies with video microscopy have revealed marked spatial variation in the deformation and strain in articular cartilage (Schinagl, et al., 1997, Schinagl, et al., 1996, Wang, et al., 2002), and the deformation of single cells (Gilchrist, et al., 2007). However, experimental investigations of the local deformations and surface interactions arising between dynamically contacting biological bodies have not been reported previously.

The objective of this study was to develop a general mathematical approach to quantify dynamic deformation and local surface interactions between experimentally-tracked bodies. First, a mathematical approach that is applicable to both 2-D and 3-D datasets is presented and validated using theoretically-generated 3-D datasets. Then, opposing cartilage surfaces were compressed, the dynamics of tissue deformation were recorded by video microscopy, and the deformations and surface interactions of the cartilage layers were quantified using the presented approach.

2. METHODS

2.1 Discrete Analysis of Experimentally-Tracked Contact

The time-dependent positions of experimentally-tracked mesh-points are used for all calculations of displacements, displacement gradients, strains, and surface interactions. Components of the formulation described here are analogous to those from continuum

formulations developed for large deformation frictional contact (Laursen, 1994, Pietrzak and Curnier, 1999, Simo and Laursen, 1992), with simplifications motivated by experimental measurements. Unlike theoretical predictions of contact mechanics, where the deformation of contacting bodies and the interaction between their surfaces are related by constitutive equations and must be solved for simultaneously, analysis of experimentally-derived data allows decoupling of deformation and sliding calculations. Therefore, the current approach is presented in two parts: 1) analysis of the internal deformations of each body, individually and 2) calculation of the interactions between the body surfaces. Calculations for 3-D analyses are described here, while the analogous 2-D formulation is included in the electronic supplement.

Intra-tissue Strain Equations—The initial position of each point in the body is given by $\mathbf{a}$, and the corresponding positions at all subsequent times in the deformed state, by $\mathbf{x} = f(\mathbf{a}, t)$. Deformation of each body is calculated independently (without consideration of the opposing surface) by calculation of the displacements of each point, $\mathbf{u} = \mathbf{x} - \mathbf{a}$. Displacement gradients in the three orthogonal directions are calculated by finite difference approximations and used to calculate Lagrangian strains, as detailed in the electronic supplement.

Surface Interactions—Analysis of surface interactions requires consideration of those points lying closest to the surface. Since the two surfaces are allowed to move independently, the kinematics of points on each surface must be considered simultaneously to determine their interactions. Theoretical solutions have accomplished this by description of one surface's motion as a convective velocity (Laursen, 1994) of the other surface or by using mixed Lagrangian-Eulerian (Boman and Ponthot, 2004) approaches. For the current analysis, the two movements will be tracked independently and the contacting regions will be matched at each time-point.

Analysis of surface interactions (Figure 1) was performed by describing motion in reference to one surface, referred to as a slave surface. For each point on the slave surface, the length of the opposing, master surface that slides over it during each time-step was determined.

The position, orientation, and movement of the master surface was described with respect to each point on the slave surface. At each time-point, the projection, $\mathbf{x}^P$, of the slave point onto the master surface was determined. The four mesh-points on the master surface that surround the projection of the slave point ($\mathbf{x}^{M1}$, $\mathbf{x}^{M2}$, $\mathbf{x}^{M3}$, and $\mathbf{x}^{M4}$; ordered from closest to furthest from the slave point) was determined. Four vectors were defined describing the relative positions of the slave point and master points: $\mathbf{g}_{M1}$ (equal to $\mathbf{x}^S - \mathbf{x}^{M1}$), $\mathbf{d}^{12}$ ($= \mathbf{x}^{M2} - \mathbf{x}^{M1}$), $\mathbf{d}^{13}$ ($= \mathbf{x}^{M3} - \mathbf{x}^{M1}$), and $\mathbf{g}_{M4}$ ($= \mathbf{x}^S - \mathbf{x}^{M4}$). The unit vectors ξ_1 and ξ_2 are then defined in the directions of $\mathbf{d}^{12}$ and $\mathbf{d}^{13}$, respectively, representing the local coordinate axes of the contacting, master surface sub-region. The position of $\mathbf{x}^P$ within the sub-region (with respect to $\mathbf{x}^{M1}$) is obtained from the equations:

$$\delta_1 = (\mathbf{g}_{M1} \cdot \xi_1) / \|\mathbf{d}^{12}\|, \quad \delta_2 = \mathbf{g}_{M1} \cdot \xi_2 / \|\mathbf{d}^{13}\| \quad (7)$$

$$\mathbf{x}^P = \delta_1 \|\mathbf{d}^{12}\| \xi_1 + \delta_2 \|\mathbf{d}^{13}\| \xi_2 \quad (8)$$

The displacement of $\mathbf{x}^P$ over a time-step can be determined by bilinear interpolation of the incremental displacements of the four surrounding master points.

$$\Delta \mathbf{u}^P = (1 - \delta_1)(1 - \delta_2)\Delta \mathbf{u}^{M1} + \delta_1(1 - \delta_2)\Delta \mathbf{u}^{M2} + (1 - \delta_1)\delta_2\Delta \mathbf{u}^{M3} + \delta_1\delta_2\Delta \mathbf{u}^{M4} \quad (9)$$

Since during sliding, the location of the projected point on the master surface changes during a single time-step, Δt , it is possible to define the displacement of the contacting surface using either the point that begins in contact, $\mathbf{x}^P(t-1)$, or the point that ends in contact, $\mathbf{x}^P(t)$. Here, the displacement of the contacting master surface was estimated by averaging the

displacements of those two points. The relative displacement of the master surface with respect to the slave point, in turn, is the difference of that value and the displacement of the slave point.

$$\Delta \mathbf{U} = (\Delta \mathbf{u}^P(t - \Delta t) + \Delta \mathbf{u}^P(t)) / 2 - \Delta \mathbf{u}^S \quad (13)$$

Definition of Contact—The gap distance between the slave point and its projection can be used to describe whether two surfaces are in contact. This distance is obtained by taking the dot product of $\mathbf{g}_{M1}$ and the unit normal to the master surface, defined by the cross product of the two tangent vectors.

$$\mathbf{n} = |\xi_1 \times \xi_2| \cdot \text{sign}_n \quad (15)$$

$$\text{sign}_n = \begin{cases} +, \text{if } |\xi_1 \times \xi_2| \cdot \mathbf{g}_{M1} \geq 0 \\ -, \text{if } |\xi_1 \times \xi_2| \cdot \mathbf{g}_{M1} < 0 \end{cases} \quad (16)$$

$$\mathbf{g}_n = (\mathbf{n} \cdot \mathbf{g}_{M1}) \mathbf{n} \quad (17)$$

Contact is typically defined as a gap distance of zero (Fernandez and Hunter, 2005), (Pietrzak and Curnier, 1999), and negative magnitudes, representing overlapping surfaces, are prohibited. However, for experimental data where the outermost tracked points lie slightly below the surface, it is practical to define contact as a gap distance below a prescribed threshold, D_{contact} ; negative gap distances would still be impossible, and distances lower than the contact threshold would be possible and represent compression of the surface-most tissue layers. In the current approach, surfaces were defined as contacting when the following three relations were true:

$$\|\mathbf{g}_n\| < D_{\text{contact}} \quad (18)$$

$$\|\mathbf{g}_{M1}\| < \sqrt{\|\mathbf{g}_n\|^2 + 0.5h^2} \quad (19)$$

$$\|\mathbf{g}_{M4}\| < \sqrt{\|\mathbf{g}_n\|^2 + 2h^2} \quad (20)$$

The first criterion sets the threshold for normal gap distance, while the second and third criteria set a limit on the distance to the closest and furthest of the surrounding points on the master surface, so that the normal vector represents a projection onto the slave surface.

The relative displacement of the two surfaces can be divided into components tangent and normal to the master surface. An average unit normal was determined for each time-step and used to define motion in- and out-of the contact surface plane.

$$\mathbf{n}_{\text{avg}} = (\mathbf{n}(t - \Delta t) + \mathbf{n}(t)) / \|\mathbf{n}(t - \Delta t) + \mathbf{n}(t)\| \quad (23)$$

The gap displacement is oriented in the direction of the average unit normal vector, and its magnitude is determined by taking the dot product of the relative displacement with the average unit normal vector.

$$\Delta \mathbf{g}_n = (\Delta \mathbf{U} \cdot \mathbf{n}_{\text{avg}}) \mathbf{n}_{\text{avg}} \quad (24)$$

The sliding displacement during an individual time-step was then calculated by subtracting the gap displacement from the relative displacement.

$$\Delta \mathbf{S} = \Delta \mathbf{U} - \Delta \mathbf{g}_n \quad (26)$$

The mathematical formulations were validated using theoretical data-sets (Figure 2), as is described in detail in the electronic supplement.

2.2 Experimental Model of Cartilage Contact

An experiment and 2-D analysis were performed to demonstrate the utility of the developed formulation. Opposing osteochondral blocks ($10 \times 10 \times 2.5 \text{ mm}^3$; $L \times W \times H$) were loaded in uniaxial compression (1%/s to a total strain of 20%) and tissue deformation and surface interactions were tracked using video microscopy (Figure 3). Near the viewing plane, variations out-of-plane were minimized because of the fluid-impermeant nature of the glass base. Also, with the 2.5mm thickness, no out-of-plane buckling was observed, as might be expected with very thin samples, and the characteristic time constant for fluid movement out of plane is predicted to be much longer than the test duration (Frank and Grodzinsky, 1987; Mow, et al., 1980). Two experimental groups were investigated: 1) osteochondral blocks taken from the patellofemoral groove (PFG), with ~matched mechanical properties, were tested against each other and 2) a block from the PFG was tested against a block with grossly softer cartilage, from the uncovered region of the tibial plateau (UTP). The expectation of the testing was that the mismatched mechanical properties would result in greater axial strain in the tibial block, producing greater lateral expansion and, in turn, sliding between the two surfaces. Detailed sample preparation and test methods are provided in the electronic supplement.

Image Analysis—Tissue deformation and sliding were calculated from digital images using a custom program written in MATLAB 6.5 (Figure 4; validation provided in the electronic supplement) using functions from the Image Processing Toolbox (Mathworks, Inc., Natick, Massachusetts). A subset of fluorescently stained cell nuclei, which were spaced at intervals of ~15 pixels (~55 μm), served as fiducial markers of material points in digital images of the tissue during loading and stress relaxation. The translation of each point was tracked through each frame by iteratively maximizing normalized cross-correlation with both the preceding and initial frames. The relative movements of these nuclei were used to describe tissue contact and deformation. Approximately 5% (in surfaces with low to moderate deformation) to 50% (high deformation) of the points did not correlate sufficiently (correlation coefficients < 0.6 with the previous frame and < 0.4 with the initial frame) or crossed the surface boundary and were excluded from subsequent analyses; points continued to be excluded in subsequent frames, unless reacquired in a later frame by correlating with the initial undeformed image.

Points along the contacting surfaces were chosen manually, at lateral intervals of ~50 μm , in the initial image and then automatically tracked through the image sequence. A 100 pixel wide (in the lateral, x-direction) $\times$ length of the image (in the axial, z-direction) rectangular region was calculated around each surface point. The width-averaged intensity profile vs. axial (z-) position was calculated for each rectangular region. This intensity profile was then compared to the profile at the same lateral (x-) position in the subsequent frame and the axial position was adjusted to minimize the squared error terms between the intensity and intensity gradient profiles. A smoothing spline was fit to the surface points to define the surface boundary. The gross accuracy of surface tracking was checked visually for each frame.

Mesh-points, at which to quantify displacements and calculate displacement gradients, Lagrangian strains, area changes, and surface sliding, were defined in the manually-selected region of interest (ROI) at 100 μm spacing. At each time-point, the positions of mesh-points not located along the articular surface were updated using a local affine mapping of the surrounding tracked nuclei (~16 nuclei; $200 \times 200 \mu\text{m}^2$ region); lateral positions of mesh-points lying along the articular surface were determined similarly, and input into the surface spline equation to calculate the corresponding axial positions.

Calculations of deformation and sliding were performed as described above. For sliding calculations, one block (PFG in mismatched pairs) was chosen as the reference side. For each mesh-point along the surface, the contact condition (using equations 10–12 in the electronic supplement; $D_{\text{contact}} = 1 \text{ pixel} \approx 4 \mu\text{m}$) was checked at each time-point. Sliding distances were

calculated as described earlier. For points remaining in contact, the calculated distances were used. For points entering or leaving contact during the time-step, sliding distances were approximated as half of the calculated distances. For points not in contact, sliding distances were set to zero.

Data Analysis and Statistics—A total of $n=3$ matched and $n=3$ mismatched samples were tested. With $x=0$ defined as the center of the block (5 mm from each edge) in the lateral direction, average sliding was assessed over three surface regions along mismatched samples (center: $x = -250$ to $250 \mu\text{m}$; left edge: $x = -3.75$ to -3.25 mm; right edge: $x = 3.25$ to 3.75 mm). Sliding was quantified at the center region ($x = -250$ to $250 \mu\text{m}$) of the matched samples, as a control. For mismatched pairs, axial strains were also quantified for the superficial $500 \mu\text{m}$ of each surface in the center regions. The effect of sample type on sliding was determined using a t-test comparing the central region of matched and mismatched pairs. The effect of surface location on sliding between mismatched pairs was determined using a one-way ANOVA with a Tukey post-hoc test. Differences in axial and lateral strain at the surfaces of UTP and PFG blocks in mismatched samples were determined by t-tests. Data are presented as mean $\pm$ SEM.

3. RESULTS

3.1 Experimental Compression of Opposing Osteochondral Blocks

Qualitatively, sliding occurred between the surfaces of both matched and mismatched samples during compression, but in distinctive manners. In matched samples, the direction of sliding tended to be the same across the entire surface and appeared to be related to the surface curvatures of the particular blocks. For mismatched samples, the softer UTP surface experienced grossly greater lateral expansion, concomitant with sliding toward the block edges with respect to the PFG surface, especially in regions away from the center axis of contact. For all samples, the opposing surfaces came into contact within the first ~ 2 frames, after which the regions of interest remained in contact for the duration of testing.

The differences in contact mechanics for matched and mis-matched samples were apparent in quantitative deformation and sliding measures. For the matched samples, lateral strains (Figure 5B.i) in matched samples were similar for both surfaces and varied relatively little throughout the tissue depth analyzed; the axial strains (Figure 5A.i) near the surface were similar between the two contacting surfaces and decreased markedly with tissue depth. In contrast, for the mismatched samples, the UTP surface experienced greater lateral expansion than the PFG surface ($p<0.05$, Figure 5B.ii) and also higher peak axial strains ($p<0.05$; Figure 5A.ii). Sliding between the surfaces (Figure 5D) was consistent with the extent of differences in the average lateral strain of the surfaces (since the lateral displacement of a point at the surface is determined by the integrated sum of lateral strain components over the surface from that point to the center of the block, Figure 5C). In the center regions of mismatched samples, surfaces averaged $\sim 110 \mu\text{m}$ of sliding, more than double that experienced along the same region of matched samples ($p<0.05$). Sliding magnitudes varied significantly with lateral position along the surfaces of mismatched samples ($p<0.05$). Sliding was increased in left and right regions (-3.5 mm and $+3.5$ mm from the center) when compared to the center region ($p<0.05$), with magnitudes exceeding $200 \mu\text{m}$.

4. DISCUSSION

This paper has described a mathematical formulation to be used with experimental measurements to characterize the deformation and surface interactions between two deformable bodies. The developed approach has the ability to handle dynamic interactions of complex geometries. The contact formulation was validated using a series of theoretically-

generated 2-D and 3-D datasets (discussed in detail in the electronic supplement). It was then used to analyze an experimental dataset of 2-D cartilage contact. The approach is compatible with a variety of experimental techniques, with calculations based on the tracked movement of tissue. The reduction of experimental measurements using the methods described could be applicable broadly to surfaces and contribute to an improved understanding of contacting tissue surfaces.

The accuracy of the contact description will depend on the specific experimental setup and choices of parameters in both data collection and analysis. The spatial and temporal sampling of any imaging technique must be fine enough so that assumptions of the calculations hold true, including that 1) linear displacements of points over a single time-step are a good estimate of their actual path and 2) variations in strain between mesh-points are ~linear. Choices of mesh spacing and sampling rate should be determined by weighing the resolution necessary to sufficiently describe the material behavior against the computational cost, and may also be limited by the choice of experimental methods. Ideally, experimental parameters would be adjusted to avoid scenarios with high expected errors; for instance, if contact is expected to occur along a path with high curvature, it may be necessary to increase the temporal sampling rate. Choices in data analysis could also be used to maximize the accuracy of the results. Since one of the larger sources of error comes from slave points that enter or leave contact with the master surface, the slave surface should be chosen that minimizes that occurrence (the band in the current 3-D example), unless there is a specific experimental interest in the other surface (e.g. a certain side exhibits greater wear or physiological response not seen by the opposing side).

Under an arbitrary load, macroscopic sliding or a mixture of sliding and rolling may occur between contacting surfaces. However, even in the absence of macroscopic displacements between the opposing sides, local sliding may be induced near the contact point by differences in material properties or surface geometries. This second, more subtle, type of sliding depends on both the geometry and material properties of the bodies and the frictional characteristics of the surfaces and may be difficult to accurately predict using theoretical models. The complexity of such interactions highlights the need for experimental determination. Measuring the displacement between the initial and final frames only, would result in an underestimate of the sliding length.

Detailed descriptions of sliding may provide information on the frictional properties of the surfaces and have implications to tissue wear. The ability to simultaneously quantify the strains in tissues during sliding may also be important, by giving some indication of the applied normal force. The ability for tissue deformation and interactions to be experimentally quantified during physiological loading may be of use in determining normal or pathological biomechanical processes.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Figure 1.
Diagram of contact in 2 and 3 dimensions and definition of variables.

Figure 2.

Diagram of the theoretical dataset used for validation of 3-D contact definitions. A semi-infinite cylinder is allowed to slide axially, rotate, and expand inside a band that remains fixed at one end and deforms uniformly in the circumferential direction of the cylinder.

Figure 3. Sample preparation (A) and mechanical testing setup (B) of opposing cartilage blocks.

Figure 4.

Flowchart of image processing routine. A region of interest and bounding surface are manually selected in the initial frame image (**A.i**). At each subsequent frame (**A.ii**), fiducial markers (**B**), and the surface (**C**) are automatically tracked and areas of low correlation are excluded from further analysis (**D**). Movement and deformation of the tissue is calculated from the tracked points.

Figure 5.

Deformation and sliding between cartilage surfaces with matched (PFG-PFG) or mismatched (PFG-UTP) mechanical properties. Composite contour maps show average axial (**A**) and lateral (**B**) strains and lateral displacements (**C**) in $400 \times 900 \mu\text{m}^2$ (lateral distance $\times$ tissue depth; box $=100 \mu\text{m}$) tissue regions at the lateral center of matched samples (**A.i, B.i, C.i**) and at the center and near the edges of mismatched (**A.ii, B.ii, C.ii**) samples. Total sliding distances between the surfaces at those same locations along matched and mismatched samples are also shown (**D**). $n=3$. ([#] $p<0.05$ compared to center match; [†] $p<0.05$ compared to center mismatch).