I have also used this method of fixation in fractures of the tibia, where the reduction of the fracture is oftentimes particularly troublesome. Perfect alignment here is essential to avoid permanent lameness, and such injuries are resistant to manipulation owing to the lack of control over the lower fragment. The lower end of the tibia is certainly not an ideal site for a bone clip, since the bone is largely subcutaneous and the clip cannot be buried as it can in other situations. Nevertheless, if the clip occupies a position above the level of the boot it is not likely to cause trouble, and only occasionally have I had to remove one on account of discomfort. If for any reason it is necessary to remove the clip after its duty has been performed the wound must be re-opened and the clip prised from the bone. No surgical instrument that I know of will cut through this spring steel, and to find the loops of the clip and insert the introducer is extremely difficult. This instrument, however, has been made so that it can be taken apart and introduced, one blade at a time, like midwifery forceps.

With regard to compound fractures, it is generally recognized that the use of internal splints should be avoided until sepsis is banished. There are, however, cases where the position obtained by external splintage is so unsatisfactory that some fixation of the bone becomes justifiable, and an apparatus which secures the position of the fragments without penetrating their substance is surely better than wire or plates.

As a matter of fact, I have used this clip in several compound fractures, mildly infected, as is common in civil practice, with the happiest results. In these cases the clip should be removed as soon as there is evidence that a reasonable amount of consolidation has taken place and there is no risk of recurrent deformity.

Description of Skiagrams on Special Plate.

FIG. 1.—C. H., male, aged 5 years. Fracture of femur on September 4th, 1917. Skiagram taken on September 10th, antero-posterior view. Traction failed to reduce the deformity; 3/4 in. shortening.

FIG. 2.—C. H. Clip applied September 14th, 1917. Skiagram taken on September 18th, antero-posterior view. October 8th: Firm union, no shortening; massage started. October 29th: Patient could raise leg in bed and flex knee through 30 degrees; began to walk on crutches and swing leg. November 5th: Walking on leg and bending knee freely.

FIG. 3.—C. H. Skiagram taken December, 1920, antero-posterior view. Both legs 24½ inches in length. Perfect movement of all joints. Small area of thickening to be felt in upper third of thigh.

FIG. 4.—L. N., female, aged 4 years. Oblique fracture of right femur on September 22nd, 1917. Skiagram taken on September 25th, lateral view. Traction failed to reduce the deformity; 1 in. shortening.

FIG. 5.—L. N. Clip applied October 1st, 1917. Skiagram taken on October 4th, lateral view.

FIG. 6.—L. N. Skiagram taken three years later, lateral view. Perfect functional result.

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THE RELATION OF THE CURVATURE OF VESSELS AND OF HOLLOW VISCERA TO THEIR INTERNAL PRESSURE.

BY

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In considering the pressure within hollow viscera and vessels it is constantly overlooked that the curvature of the containing wall influences the pressure within it.

Historic.

The general problem of the equilibrium of a stretched flexible membrane with the pressures on each side of it was solved by Lagrange¹ in his *Mécanique Analytique*, 1788. The subject was developed chiefly by Poisson² in memoirs dated 1812 and 1829. For surfaces of simple curvature the equations connecting pressure, curvature, and tension can be reduced to very simple forms, such as those given below, and their truth can be demonstrated without the use of higher mathematics. Such demonstrations are easily available in mathematical textbooks—for instance, Besant and Ramsey³ and Lamb.⁵ The formulae are also discussed in many textbooks of physics, such as Poynting and J. J. Thomson,⁶ or the more elementary books of Edser⁷ and of Wagstaff.⁸

They are usually quoted in connexion with the surface tension of liquids, since the surface films constitute, almost without complications, material illustrations of mathematical statements. Some further applications have been made in the domain of physical chemistry.

The physical principles are well known to engineers; in estimating the bursting pressures of boilers and steam-pipes the curvature of their surfaces has to be taken into account,⁹ and the formulae are embodied in the British Board of Trade regulations. The principles are still recognizable, though horribly disguised by the British units of measurements, in formulae used by artisans and to be found in "Mechanics" Pocket-books."¹⁰ When sheets, strings, or wires in tension are wound upon cores, or when a turn of rope is tightened round a bollard, the same principles are involved. Allied formulae are used by architects in estimating the stability of arches and domes.

Although they are thus familiar in many branches of science, the formulae and the physical facts which they express appear to be unknown to the majority of medical writers. An exception was the Rev. Samuel Houghton, M.D.,¹¹ who, in his *Animal Mechanics*, 1847, referred to Lagrange's work and applied it to the parturient uterus.

Formulae expressing the Relation between the Curvature of a Stretched Membrane and the Pressure which it Exerts.

For the derivation of these formulae works on mathematics may be consulted: here they are accepted as data.

In a stretched membrane let t (tension) represent the pull on either side of a line of unit length drawn on the surface, in a homogeneous membrane t will be everywhere the same; let p represent the excess of pressure on the concave side of the membrane over that on the convex; and let r and r' represent radii of curvature such that they lie in two planes at right angles to each other whose intersection is normal to the surface. If the C.G.S. system of units is used t will be dynes per centimetre, p will be dynes per square centimetre; and r and r' will be centimetres. Then—

$$p = t \left(\frac{1}{r} + \frac{1}{r'} \right). \quad 1.$$

If the content of the membrane is fluid, p will be everywhere the same.

As the sum $\frac{1}{r} + \frac{1}{r'}$ is independent of the aspect of the planes in which the radii lie (Euler), it is convenient to choose r and r' as radii of the principal curvatures—that is, the curvatures which are maximal or minimal. For the sphere $r = r'$, therefore, the pressure inside a spherical stretched membrane is given by the equation—

$$p = 2 \frac{t}{r}. \quad 2.$$

For the cylinder with straight axis, r' of the curvature parallel to the axis is infinite, and $\frac{1}{r'}$ is zero; therefore the pressure inside a cylindrical stretched membrane is given by the equation—

$$p = \frac{t}{r}. \quad 3.$$

The length of the cylinder does not affect the pressure.

The reciprocal of the radius is a direct measure of the amount of curvature, and the two terms are used synonymously in mathematical works; the equations can therefore be expressed in words simply—the pressure inside a stretched membrane is proportional to its tension and to its curvature, and the influence of curvature as such is emphasized.

Here a protest must be made against the practice, still common among medical people, of using the word tension to designate what is properly termed pressure. For exact definitions of these words reference must be made to the philosophical literature of mechanics; broadly a tension is an internal force which increases with expansion of the material bearing it, and a pressure is one which decreases. To speak of the "tension" of gases, of dissolved bodies, or of pulses is confusion of language and leads to confusion of thought. In the case of the pulse the arterial wall is in tension, the contained blood is under pressure; the pressure is a force directed outwards, the tension is a force directed along the surface of the wall; since the wall is curved there is a resultant force directed inwards; this force balances the pressure of the blood outwards. The wall may be said to "exert" pressure as a result of its tension and curvature. Another common source of confusion must be indicated. The word pressure as used by mathematicians and physicists designates the ratio of a force to an area, real or imaginary. The total force acting on a surface is not a measure of pressure, but the magnitude of the force divided by the

area of the surface is. In such matters we should adhere strictly to the usage of the more basal sciences.

Using these terms in the physical sense, a simple demonstration of the truth of the equations can be given. Consider a stretched spherical membrane hemisected by an imaginary plane, the total force on either side of this circular plane tending to separate the hemispheres is the pressure on the plane multiplied by its area; the total force pulling the hemispheres together is the tension of the membrane on the circumference cut by the plane, and its total amount is the tension per unit length multiplied by the length of the circumference. These opposed forces are in equilibrium.

$$\therefore p \cdot \pi r^2 = t \cdot 2\pi r$$

$$\text{and } p = \frac{2t}{r}.$$

Similar reasoning can be applied to the cylinder.

Static.

Static deductions from the principle are simple.

(a) If in different membranes the tension is the same, the smaller the radii of curvature the greater is the pressure inside the membrane. The pressure inside a spherical membrane is twice that inside a cylindrical of the same radius. For the purpose of medicine it is important to emphasize that the pressure will be different inside viscera whose walls are of the same thickness and at the same tension if they are different in size or shape.

(b) Supposing the pressure to be the same within a number of membranes, the less the curvature of each membrane the greater must be its tension. A cylindrical membrane will have twice the tension of a spherical of the same radius, and a spherical membrane with twice the radius of curvature of another must also have twice the tension.

For instance, the tension of the cornea must be less than that of the sclerotic; where two blood vessels are in immediate communication, so that the blood in them is at the same pressure, the wall of the larger vessel must be in greater tension than that of the smaller.

Where the thickness of the wall is related to its tension the same statements apply to the thickness. For instance, the intercostal arteries as they leave the aorta have the merest fraction of its thickness of wall, although they withstand the same pressure. The same is true of all small branches of large arteries, the thinness of their walls is related to the sharpness of their curvature, by virtue of which small tension can exert large pressure. This feature of the build of vessels is justified, though not accounted for, by physical principles. With this in mind the tenuity of the minute lenticulo-striate arteries becomes less alarming, and the reproach commonly made that they are small branches of a big artery loses significance;¹² the smaller the stouter, to put it paradoxically. Their tragic failure occasionally must be otherwise explained. The comparatively frail glass gauge tube of a boiler is enabled by the sharpness of its curvature to withstand full steam pressure.

The hemispherical end-caps of cylindrical boilers are made only half the thickness of the sides. One end of the arterial system is capped, during rather more than half of each cardiac cycle, by the aortic valves; owing to the three-lobed arrangement this end-cap is more curved than if it were simply hemispherical. This curvature goes some way towards justifying the thinness of the aortic valves to which Dr. Harry Campbell¹³ has drawn attention.

Dynamic.

The dynamics of the subject are less simple. Two purely physical examples will be discussed briefly as introductions to the complex problems which biology presents.

(a) Where the tension of the membranes is not altered by stretching or contraction. This is the case in liquid surface films, whose tension remains constant whatever changes in size or shape the films may undergo. Soap bubbles afford very convenient and almost uncomplicated experimental examples. The pressure within such membranes varies only with their curvature. Of two spherical bubbles, the pressure is greater within the smaller in inverse proportion to its diameter. If the cavities of the bubbles are connected, the smaller will empty itself into and distend the larger. It also follows that the more a bubble is blown up the smaller becomes the pressure inside it, a fact which may be surprising; but anyone who has blown a bulb on a glass tube has experienced the diminution of effort needed as the bulb

increases in size. The pressures exerted by curved liquid surfaces are probably not negligible in the minute mechanics of the living body.

The amount of external work done by the contraction of the membrane when a portion of its contents is discharged is of great theoretical importance. It is measured by the volume discharged multiplied by the pressure; as the pressure varies during the process it is not easy to estimate the work done without recourse to the calculus.

Let V_1 and V_2 be the volumes respectively before and after the discharge of a definite volume of contents, and let the energy lost or work done be W , then

$$W = \int_{V_1}^{V_2} p \cdot dv.$$

$$\text{For the sphere } p = \frac{2t}{\sqrt{\frac{4}{3}\pi} V}$$

$$\therefore W = \int_{V_1}^{V_2} \frac{2t}{\sqrt{\frac{4}{3}\pi}} \cdot V^{-\frac{2}{3}} \cdot dv$$

$$= \frac{4t}{3\sqrt{\frac{4}{3}\pi}} (V_2^{\frac{1}{3}} - V_1^{\frac{1}{3}})$$

t being constant here.

(b) The case of extensible membranes which obey Hooke's law, that is, equal linear stretchings produce equal additions to the tension; or, tension is proportional to stretch.

Indiarubber may be taken as an example; although it does not follow Hooke's law exactly, its great extensibility is convenient.

Consider a cylindrical membrane which at radius R_0 has no tension and therefore exerts no pressure. Suppose that the membrane is then distended so that the radius R is increased by the addition of a variable amount n , then the circumference is increased by $2\pi n$, and this represents a linear stretch of the membrane at right angles to the axis. According to Hooke's law the tension is increased in proportion to the stretch; let $2\pi nK$ represent the tension, where K is a constant; (K multiplied by the original circumference of the cylinder and divided by the area of cross section of the membrane at right angles to the circumference is Young's Modulus of Elasticity). Then the pressure within the cylinder is

$$p = \frac{2\pi nK}{R_0 + n}$$

Similarly it can be shown that the pressure within a spherical membrane is

$$p = \frac{4\pi nK}{R_0 + n}$$

Rewriting these equations—

$$\frac{p}{2\pi K} = \frac{n}{R_0 + n} \quad \text{and} \quad \frac{p}{4\pi K} = \frac{n}{R_0 + n} \quad \text{respectively,}$$

it is obvious that while n is small compared with R_0 , the pressure will rise rapidly as n increases, but that when n becomes large the less does further increase of n augment the pressure,

$$\left(\frac{dp}{dn} = \frac{R_0}{(R_0 + n)^2}\right)$$

and that the pressure approaches a limit as the fraction $\frac{n}{R_0 + n}$ approaches unity in value. The two apparently contradictory statements—that the rise in pressure is very nearly proportional to the increase in radius, and that the pressure remains very nearly constant in spite of variation in the radius—are both true. The first statement is true of a membrane slightly stretched, the second of one greatly stretched.

The work done during a discharge of contents can be calculated in the same way as for a surface film, but the resulting expression is rather cumbersome. The relation of the pressure to the varying thickness of the wall also can be expressed, but no advantage is gained.

It is easily deducible from the equations that a cylindrical membrane with fluid contents tends to become the same diameter throughout its length, and that if the cavities of two similar spherical membranes are connected they will become the same size. This is in contrast with liquid films, where the smaller sphere dilates the larger, and the cylinder is unstable when its length exceeds its circumference. The force directed towards uniformity of size becomes smaller with greater dilatation, so that when a membrane is greatly distended small interfering factors can produce marked irregularities in size and shape.

Experimenting with rubber it is not difficult to record rough agreement with theory, but precisely regular results are difficult to obtain. This is due to several disturbing agents, such as departure from Hooke's law, and the fact that in stretched rubber, as in many other materials, the tension diminishes with time, and at a rate which bears no simple relation to other conditions.

Application to Physiology.

The complex problems presented by the dynamics of curved muscular sheets will not be attempted here; it is intended to leave unobscured certain physical principles which must be included, explicitly or implicitly, in any exact consideration of the subject. A few notes only are added further to clear the ground.

It is frequently assumed that all solids follow Hooke's law sufficiently nearly for any practical purpose. I have made a long series of experiments on animal tissues and have found that they all deviate sensibly from Hooke's law, and that many deviate widely. Nor would a definition of "solids" which excluded animal tissues set the matter right. Hooke's law can only be assumed in physiology as a means to a first approximation.

The heart is the most interesting field for the application of these principles, but the bladder is more suitable. The left ventricle is usually said to contain about 100 c.cm. of blood at the beginning of systole, and to discharge about 60 c.cm. during its contraction; if it were spherical its internal radius would be roughly 2.9 cm. at the beginning of systole, and 2.1 cm. at the end. The radius of the bladder is subject to immensely greater variations. The bladder is also thin-walled, nearly spherical, and its pressure is more easily determined than that of the heart.

Everyday experience provides data of considerable interest. Other things being equal, the trajectory of a stream of liquid projected into free air varies with the velocity of outflow, and the velocity is proportional to the square root of the pressure at the opening of the reservoir. After the beginning of micturition, and independently of the abdominal musculature, the trajectory remains nearly constant for some time, and then shortens towards the end, showing that for the first half or so of micturition the pressure in the bladder is nearly constant. Tension and radius must decrease together, although the muscular wall is thickening as the bladder contracts, and it might therefore be expected to exert greater pressure.

Summary.

1. The pressure exerted by any structure comparable to a stretched membrane is dependent on its curvature.
2. The pressure within a cylindrical and a spherical membrane is $\frac{t}{r}$ and $\frac{2t}{r}$ respectively, where r is the radius of curvature and t is the tension.
3. With rubber-like materials, as a curved membrane is progressively distended, the pressure rises rapidly at first, then more and more slowly as it approaches a limiting value.
4. Such materials are only a rough guide to the behaviour of membranes made of muscle and other animal materials.
5. In any exact consideration of the mechanics of vessels and of hollow viscera the curvature of the wall must be taken into account.
6. When such allowance is made, purely physiological effects will be more clearly visible.

Note.—Since the above was written Dr. J. R. Gillespie of Belfast has pointed out the application of these principles in the study of the capillary circulation (BRITISH MEDICAL JOURNAL, 1921, vol. i, p. 873).

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FOCAL INFECTION IN RELATION TO THE ETIOLOGY OF SKIN DISEASES.*

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THE term "focal infection" may be understood in three senses: First, it may imply that living and virulent bacteria are carried by the blood stream to near or remote organs, there to set up active infection; secondly, that foci such as the tonsils or dental alveoli may act as carriers of bacteria which, harmless to the person who carries them, may by contact affect other persons; thirdly, we may understand by focal infection a collection of bacteria in a crypt, pocket, tube, or fold of tissue, which, while not actively growing or multiplying, may infect the body by their cell substance which is liberated by the action of ferments or enzymes, either by self-digestion (autolysis) or by the enzymes of the cells of the invaded tissue (heterolysis).

With the first two of these aspects of focal infection I am not at present concerned. The third aspect may be conveniently designated by the term introduced by Adams—namely, *subinfection*. Now, since the chief ingredient in the bacterial cell substance is some protein body, we might more accurately name the subject of this present discussion protein poisoning in relation to cutaneous diseases.

It will help to throw some light on this obscure subject if we start with the statement of a fundamental biological fact—namely, that terrestrial vertebrates and bacteria have entered into a symbiosis, in which for the return of food maintenance the enzymes of the bacteria are thrown into line with the enzymes of the body in the scheme of defence against foreign proteins. Our food consists of foreign proteins, carbohydrates, and fats, and it is essential for the welfare of the body that these should not be allowed to pass through the wall of the intestines into the interior without first being deprived of their toxicity. There exists in the body an organized system of defence against foreign proteins. The first line is taken by the cells of the mucosa of the stomach and the higher intestines, the second line by the bacteria of the colon, and the third line by the free moving cells of the lymphoid tissues. As regards the distribution of bacteria throughout the body we find the major portion of them in lymphoid tissue in parts to which food has access. But they are not wholly confined to the surfaces of the body, for it has been demonstrated that no organ is absolutely sterile, but only potentially so.

The flora of the mouth and tonsils is restricted; it consists chiefly of the haemolytic and non-haemolytic streptococci and the staphylococci. The pneumococcus, which is closely related to *Streptococcus viridans*, is a fairly constant inhabitant of the mouth and tonsils. Occasionally more virulent organisms are found, such as the *Streptococcus mucosa* and the meningococcus. Special pathogenic organisms such as the diphtheria bacillus do not form a part of the normal colonies of the mouth and tonsils. Nor does the colon bacillus, for this bacterium is not adapted to growth in the tonsils or mouth; if implanted experimentally in the tonsil it apparently dies out. In addition to the bacterial flora the endamoeba appears to be a constant denizen of the tonsils and alveoli. All human tonsils from a few hours after birth to the end of life are infected with these bacteria; they are not distributed through the lymphoid tissue, but are confined to the crypts and follicles.

The benign physiological action of the tonsillar and alveolar bacteria is digestive and, probably, in no way essentially differs from bacterial digestion in the colon. This will become more apparent if we study the normal relation of the tonsils. After partaking of food, some undigested food particles, often in a state of fine division owing to the soft nature of the modern human diet, find access to the dental troughs and the tonsillar crypts. Milk, cheese, vegetable debris mingled with saliva, salivary corpuscles and mucus collect in the follicles, forming the so-called "cheesy" matter familiar to all laryngologists. In this way tubercle bacilli are introduced; indeed milk may be considered the customary vehicle for this organism. These are attacked by the oral and tonsillar bacteria, and their complex proteins are reduced by

* Paper read at the first meeting of the British Association of Dermatology and Syphilology.

ACCORDING to the *Castilla Médica* a State regulation is now in force in Spain by which vaccination against typhoid fever is compulsory, when typhoid or paratyphoid fever are prevailing in epidemic form, for all persons attending cases or in relation with the sick, directly or indirectly.