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Smart Rehabilitation Devices: Part I – Force Tracking Control

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Abstract

Resistance exercise has been widely reported to have positive rehabilitation effects for patients with neuromuscular and orthopaedic conditions. This article presents prototypes of smart variable resistance exercise devices using magneto-rheological fluid dampers. An intelligent supervisory control for regulating the resistive force or torque of the device is developed, and is validated both numerically and experimentally. The device provides both isometric and isokinetic strength training for the human joints including knee, elbow, hip, and ankle.

Keywords

muscle strengthening; smart rehabilitation device; magnetorheological fluids; smart materials for rehabilitation; supervisory control; force tracking

INTRODUCTION

Muscle strengthening is critically important in maintaining physical function and overcoming muscle weakness from normal ageing as well as neurological or traumatic injury. Weakness can seriously impact the mobility and independence of individuals. A myriad of studies have shown the beneficial outcome of increased strength through resistance exercise. One benefit of muscle strengthening is to improve mobility (Madsen et al., 2000). The quadriceps femoris muscle strength gain from exercise could increase dynamic stability and avoid falls for functionally limited elderly individuals while rising from a chair or walking (Scarborough et al., 1999; Ryushi et al., 2000). Fisher et al. (1993) studied the effect of knee muscle strengthening in patients with knee osteoarthritis (OA) and they found that increased quadriceps femoris strength improved patients' ability to walk, rise from a chair, and climb stairs. Other studies report similar results that isometric and dynamic resistance exercise improves function and reduces knee joint pain of patients with OA knee (Topp et al., 2002; Bischoff and Roos, 2003). For people with neurologic conditions, strengthening is equally important. In children with cerebral palsy (CP) quadriceps strength is associated with overall motor function and walking speed (Damiano et al., 2000), and resistance exercise has been shown to increase muscle strength and improve motor activity (Dodd et al., 2002). Andrews and Bohannon (2003) found that resistance exercise after acute stroke resulted in an increase in limb muscle strength bilaterally. Teixeira-Salmela et al. (1999) found that quadriceps strengthening improved function in patients with hemiparesis following stroke.

While muscle strength is clearly important in improving walking ability and reducing impairments associated with many medical conditions, the question remains as to the most efficient and effective manner of strengthening in our cost-conscious health-care system. The

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delivery of rehabilitation services in the home represents one alternative to reduce the cost of rehabilitation. However, not many rehabilitation devices are available that provide appropriate resistance to achieve the desired outcome in the home. Typical home-based strengthening devices include elastic bands or sand-bag weights that strap around a limb. These are portable and low cost, but do not optimize strengthening because they do not take into account the length–tension relationship of muscle. The length–tension relationship states that a muscle can produce the highest force when its fibers are at an optimal length (Gordon et al., 1966). The optimal fiber length is often achieved when the joint that the muscle spans is in its mid-range of motion. This means that as a joint moves through the full range, less force is produced at the beginning and the end of the range. The elastic bands provide greater resistance as the tension increases. However, the highest resistance is often at a joint range of motion at which the muscle is weak. The resistance provided by elastic bands cannot be measured, making it difficult to precisely prescribe resistance and track progress. Sand-bag weights provide the same resistance throughout the joints range of motion and weights are commonly chosen so that a patient can lift the weight through the weakest ranges of motion. This results in less than optimal force generation through most of the range of motion. Low-cost rehabilitation devices that provide adequate resistance in an appropriate manner across the range of joint motion are needed to improve functional outcomes both in and out of the home.

In order to investigate the effect of precisely varied resistance training, we have developed a smart variable resistance exercise device (VRED) to facilitate patients' knee rehabilitation at home without the need of frequent visits to the clinic. We have fabricated several portable, energy efficient and multifunctional devices that have a user-friendly human–machine interface. The device can easily be re-configured for other joints such as elbow, hip, and ankle. The VRED is an intelligent device such that after a therapist programs the device, it provides resistance training for patients without intervention of the therapist. This article reports the design optimization and force control studies of the VRED.

A key component of the VRED is a variable resistance damper. Several technological options are available to make the damper: hydraulic brake, piezo-electric friction brake, electric motor, and smart fluids such as magneto-rheological (MR) fluids. MR fluids offer the best choice due to their fast response and large controllable resistive force with input of only a few volts. MR fluids consist of stable suspensions of micro-iron particles in a liquid carrier such as oil or water. The magnetic particles respond to the applied magnetic field and change the yield stress of liquid in milliseconds. The magneto-rheological response can be actively controlled in real-time by changing the current input of the electromagnetic coil, thereby allowing the provision of variable resistance (Carlson and Jolly, 2000). The MR fluid dampers are widely used as energy dissipative devices to attenuate structural vibrations. Although MR fluids have fast response time and need a small excitation current, the dynamic response of an MR damper is generally difficult to predict because the response is a highly nonlinear function of displacement, velocity, and current input. Development of algorithms for the force control of the MR damper is one of the objectives of this work.

There have been many studies of MR fluids and devices. A popular constitutive equation of MR fluids is the Bingham model (Carlson and Jolly, 2000). The modified Bingham polynomial model and Bouc–Wen model to quantify hysteresis of MR devices are reported in Dyke et al. (1996), Spencer et al. (1997), Choi et al. (2001), and Yang et al. (2002). Wereley and associates have made significant progress in modeling MR fluids and dampers (Bitman et al., 2002; Dimock et al., 2002; Cho et al., 2003). Artificial neural networks (ANN) are nonparametric models employed to predict damper force and to compute the required voltage in control applications (Chang and Roschke, 1999; Chang and Zhou, 2002). Another important issue related to MR dampers is how to improve their dynamic performance by using proper amplifiers. The current driver technology based on pulse width modulation and integral

feedback algorithm can be used to shorten the transient time of the device (Milecki, 2002; Yang et al., 2002). More information about the characteristics of MR fluids and their applications can be found in Jolly et al. (1998) and Carlson and Jolly (2000).

In this article, we present an optimal design study of MR dampers under various constraints based on the Bingham model. We also develop control algorithms to regulate the force output of the MR damper according to the prescription of a therapist. The article is organized as follows. The following section describes the function of the VRED. The subsequent sections present an optimal design study of the MR damper, the control study to regulate the force output of the damper, and the simulations and experimental results of the control study. The final section concludes the article.

DESCRIPTION OF VARIABLE RESISTANCE EXERCISE DEVICE

Figures 1 and 2 show two prototypes of the VRED in the form of a knee brace and a chair. For the purpose of illustration, we will discuss the use of an MR damper that provides resistance to muscles around the knee joint. Ideally, the damper should be light, so that the patient can carry it with little burden. On the other hand, the damper should be strong enough to lock the knee at a given angle for isometric applications. A linear displacement sensor is used to measure the knee flexion and extension angle. The resistive force can be controlled to meet the exercise requirement as prescribed by the clinician. The thigh component of the VRED is attached to the upper leg, and the shank component to the lower leg. The rotation center of the VRED is aligned to the knee flexion/extension center. Two dampers are positioned on the medial and lateral sides of the leg to provide balanced torque, and prevent any internal/external torsion and varus/valgus bending.

The VRED works in two modes: the test mode and the exercise mode. In the test mode, the knee is set at a given angle; the dampers provide a large enough force to lock the knee. The patient generates a maximum voluntary isometric contraction (MVIC) during knee flexion and extension, and the largest torque in each direction is recorded. The MVIC is repeated at different ranges of motion to generate a torque profile. In the exercise mode, the physical therapist specifies the exercise effort as a certain percentage of the torque profile. Once programmed, the VRED will work with the patient to exercise automatically. It will produce the exact resistive force based on the prescribed exercise level with the help of an intelligent controller.

OPTIMAL DESIGN OF MR DAMPER

In the two modes of the VRED, the MR damper operates in pre-yield and post-yield states. In the pre-yield state, MR fluids exhibit the elastic or viscoelastic property with the field-dependent yield stress (Weiss et al., 1994). In the post-yield state, the viscous flow of MR fluids is governed by the Bingham model (Jolly et al., 1998). According to Jolly et al. (1998), the on-state force f_τ and the off-state force f_η can be expressed as:

$$f_\tau = S_{\text{piston}} \Delta P_\tau, f_\eta = S_{\text{piston}} \Delta P_\eta, \quad (1)$$

where $\Delta P_\eta = 12\eta Ql(g^3w)$ is the pressure drop due to the viscous stress of the Newtonian steady laminar flow, $\Delta P_\tau = c\tau_y l/g$ is the pressure drop due to the applied magnetic field, Q is the volumetric flow rate, η is the dynamic viscosity, which is a function of the flow shear rate $\dot{\gamma}$ without the applied field, c is a yield stress correction factor (about 2.5), τ_y is the yield stress determined by the magnetic flux density, and S_{piston} is the cross area of the piston.

Neglecting the cross-sectional area of the shaft, we obtain the following geometric relationships:

$$w = \pi d, S_{\text{piston}} = \frac{\pi d^2}{4}, Q = \frac{U w^2}{4\pi}, \dot{\gamma} = \frac{\partial u}{\partial y} = \frac{Q}{g^2 w}, \quad (2)$$

where w is the piston circumference, d is the diameter of the piston, U is the piston-moving velocity, and g is the magnetic gap. $\lambda = f_{\tau}/f_{\eta}$ is known as the dynamic ratio of the MR damper. For the current design, we have

$$\lambda = \frac{c\tau_y g^2 w}{12\eta Q}, f_{\tau} = \frac{c\tau_y w^2 l}{4\pi g}. \quad (3)$$

The weight of the MR damper comes from the piston, the spool, the shaft, the cover, and the fluid. For a given specified force output f_{given} and dynamic ratio λ_{given} , we would like to minimize the weight of the damper. This leads to the following optimization problem: Minimize the weight of the MR damper with respect to the parameters w and g subject to the following constraints:

$$f_{\tau} + f_{\eta} \geq f_{\text{given}}, \lambda \geq \lambda_{\text{given}}. \quad (4)$$

Since the on-state force f_{τ} is often much larger than the off-state force f_{η} , the first constraint can be simplified to

$$f_{\tau} \geq f_{\text{given}}. \quad (5)$$

An explicit optimization solution is difficult to obtain for the general case when η is a function $\dot{\gamma}$, which is often unknown. In the present application, the shear rate $\dot{\gamma}$ is small, we assume that η is constant and the yield stress τ_y is taken to be the value when the MR fluid is saturated with the flux density. Under these assumptions, we obtain the optimal circumference and magnetic gap as:

$$w_{\text{opt}} = \left[\frac{3\lambda_{\text{given}} U \eta \left(\frac{4\pi f_{\text{given}}}{lc\tau_y} \right)^2}{\pi c\tau_y} \right]^{1/3}, \quad (6)$$

$$g_{\text{opt}} = \left[\left(\frac{3\lambda_{\text{given}} U \eta}{\pi c\tau_y} \right)^2 \frac{4\pi f_{\text{given}}}{lc\tau_y} \right]^{1/3}. \quad (7)$$

These are the parameters used in our damper design.

FORCE TRACKING CONTROL

Next, we consider the force tracking control of the VRED in the muscle-strengthening exercise. This is a protocol of exercise where a force or torque profile is prescribed by the therapist. The patient is instructed to exercise the joint at any speed, and will experience the prescribed resistive force provided by the VRED. The control is designed only to provide the resistive force for a given angle according to the prescription, regardless of the speed of joint motion. Joint motion during the exercise is actually treated as a disturbance to the control.

The Resistive Force

In the following study, we assume that the MR fluid and the iron core of the coil operate in the lower linear range of the $B-H$ curve with the applied magnetic intensity far away from saturation. This assumption agrees with the conditions, under which the rehabilitation exercise is carried out. In order to develop control algorithms to regulate the VRED force output, we need a dynamic model of the system. Note that the normal exercise with the VRED consists of the motion through the entire damper stroke back and forth. The total resistive force

magnitude of the MR damper is given by $f = f_\tau + f_\eta$. The off-state force magnitude is $f_\eta = C_1 + C_2|\dot{x}|$, where C_1 denotes the magnitude of the friction force, $C_2|\dot{x}|$ is the magnitude of the viscous force, and $\dot{x}$ is the joint angular speed. f_η does not contain the usual elastic component due to the volume compensator in the conventional design. The present damper is dual-end without the elastic element for volume compensation.

The on-state force magnitude is related to the current by $f_\tau = k(LI)^p$, where $k = cw\tau_y/4\pi gNB_y$ could be interpolated from the graph presented in Jolly et al. (1998) or calculated directly from the experimental data, N is the number of coil turns, and B_y is the magnetic flux density corresponding to the MR fluid yield stress τ_y . The power index p is reported to range from 1.5 to 1.75 in the low to intermediate magnetic fields (Jolly et al., 1998). Here, we let the coefficient C_3 absorb the factor kL^p and rewrite the force magnitude as

$$f = C_1 + C_2|\dot{x}| + C_3I^p, \quad (8)$$

where $1 \leq p \leq 2$. In this formulation, we use the current I as the control input. Note that the resistive force f is always opposite to the direction of the motion, and the current is assumed to be positive in the circuit, since the MR damper is insensitive to the sign of the current. We have designed a circuit such that the current is always positive.

We use an RL circuit to model the MR damper coil (Vaughan and Gamble, 1996; Elmer and Gentle, 2001).

$$u = RI + L \frac{dI}{dt} (I > 0), \quad (9)$$

where L is the self-inductance of the coil, R is the resistance of the coil, and u is the applied input voltage. Based on the experimental observations, we assume that the resistance R is constant because the damper temperature varies very little during exercise, and that the self-inductance L is also constant because the present application is well within the linear range of the B - H curve as discussed earlier.

The objective of the control is thus to find a current I such that the resistive force f will follow the prescribed profile at a given position x as the joint moves through a range of motion.

Supervisory Control

In general, C_1 , C_2 , C_3 , and p are all uncertain. However, C_1 , C_2 , and C_3 can be handled with adaptive controls (Slotine and Li, 1991). It is more difficult to study the effect of p . In the following study, we shall focus on p and simply use estimates of C_1 , C_2 , and C_3 in the control. As can be seen later, the real values of C_1 and C_2 will only affect the accuracy of resistive force tracking, and have no impact on the stability of the closed-loop system. The value of C_3 influences the stability of the closed-loop system. However, we can compensate for its effect by a feedback gain, provided that we know the lower and upper bounds of C_3 .

Because of the difficulty in applying adaptive controls to deal with the uncertain parameter p , we consider the supervisory control (Morse, 1996, 1997; Hespanha et al., 1999, 2003). The supervisory control proposes to use several estimates of uncertain parameters for the system model. For each estimate of parameters, a control is designed to achieve the desired performance. A supervisor monitors the real-time response of the system, selects a plant model according to a switching criterion and implements the corresponding control. In this case, we choose two estimates of the uncertain parameter p , i.e., $p = 1$ and $p = 2$, and design a control for each.

CONTROL FOR THE MODEL WITH $p=1$ —Let the lower case c_1 , c_2 , and c_3 denote the best estimated values of C_1 , C_2 , and C_3 from experiments. To design a digital control, we write the total force output as

$$f_{n+1} = c_1 + c_2 |\dot{x}_n| + c_3 I_n, (p = 1). \quad (10)$$

Let $f_{r,n}$ denote the prescribed resistive force profile as a function of x . Consider a current control,

$$I_{d,n} = \frac{f_{r,n} - c_1 - c_2 |\dot{x}_n|}{c_3} + k_f(f_{r,n} - f_n). \quad (11)$$

The first part of the current is the feedforward control based on the force model in Equation (10). This part of control intends to correct the influence due to the joint motion. The second part is the force tracking error feedback; $k_f > 0$ is the feedback gain. The feedback part is responsible for tracking the prescribed force profile $f_{r,n}$.

Next, we design a current driver to deliver the desired current I_d . First, we use the zero order holder (ZOH) to convert the continuous transfer function of the electric circuit to the discrete transfer function $G_p(z)$ leading to

$$G_p(z) = \frac{I(z)}{u(z)} = \frac{1 - \alpha}{R(z - \alpha)}, \alpha = e^{-(R/L)T_s} \quad (12)$$

where T_s is the sample period. Second, we use the proportional-integral feedback control (PI) to update the input voltage u as

$$u(z) = \left(k_p + \frac{k_i}{z - 1} \right) [I_d(z) - I(z)], \quad (13)$$

where k_p and $k_i > 0$ are the proportional and integral gains, respectively. The PI control has an ability to lead to the zero steady state tracking error for the step reference. The closed-loop transfer function of the current loop reads

$$G(z) = \frac{I(z)}{I_d(z)} = \frac{(1 - \alpha)(k_p z - k_p + k_i)}{R(z - 1)(z - \alpha) + (1 - \alpha)(k_p z - k_p + k_i)}. \quad (14)$$

Define two shorthands $H_{1,n}$ and $H_{2,n}$ as

$$\begin{aligned} H_{1,n} &= c_1 + c_2 |\dot{x}_n|, \\ H_{2,n} &= \frac{f_{r,n} - c_1 - c_2 |\dot{x}_n|}{c_3} + k_f f_{r,n}. \end{aligned} \quad (15)$$

Equations (10) and (11) can be written as

$$\begin{aligned} f_{n+1} &= c_3 I_n + H_{1,n}, \\ I_{d,n} &= -k_f f_n + H_{2,n}. \end{aligned} \quad (16)$$

Applying z transformation to these equations, we obtain the closed-loop force and current output,

$$\begin{aligned} F(z) &= \frac{H_1(z) + c_3 G(z) H_2(z)}{z + k_f c_3 G(z)}, \\ I(z) &= G(z) \frac{-k_f H_1(z) + z H_2(z)}{z + k_f c_3 G(z)}. \end{aligned} \quad (17)$$

The characteristic equation of the closed-loop system reads

$$1 + k_f c_3 \frac{G(z)}{z} = 0, \quad (18)$$

or explicitly,

$$\begin{aligned} 0 = & R z^3 + [k_p(1 - \alpha) - R(1 + \alpha)] z^2 \\ & + [R\alpha + (1 - \alpha)(k_i - k_p + k_f k_p c_3)] z \\ & + k_f c_3 (1 - \alpha)(k_i - k_p). \end{aligned} \quad (19)$$

Jury's test is carried out to determine the gains k_f , k_p , and k_i to guarantee the stability of the current I . The stability of the system is guaranteed when

$$\begin{aligned} |\xi_3| &< 1, \\ \left| \frac{\xi_2 - \xi_1 \xi_3}{1 - \xi_3^2} \right| &< 1, \\ \left| \frac{\xi_1 - \xi_2 \xi_3}{1 - \xi_3^2 + \xi_2 - \xi_1 \xi_3} \right| &< 1, \end{aligned} \quad (20)$$

where

$$\begin{aligned} \xi_1 &= (1 - \alpha) \frac{k_p}{R} - (1 + \alpha), \\ \xi_2 &= \alpha + (1 - \alpha) \frac{k_i - k_p + k_f c_3 k_p}{R}, \\ \xi_3 &= k_f c_3 (1 - \alpha) \frac{k_i - k_p}{R}. \end{aligned} \quad (21)$$

As can be seen in Equation (18), the factor $k_f c_3$ will influence the stability of the system. If we know the bounds of C_3 , k_f can be adjusted to account for the effect of C_3 in the stability consideration.

When C_1 , C_2 , and C_3 are not exactly estimated, the parameter errors will affect $H_{1,n}$ and $H_{2,n}$. These two terms, however, also include the effect of human motion and act like a disturbance to the control system. This disturbance is compensated by the feedback term in the control, and does not affect the stability of the closed-loop system.

CONTROL FOR THE MODEL WITH $p=2$ —The force output for $p = 2$ is

$$f_{n+1} = c_1 + c_2 |\dot{x}_n| + c_3 I_n^2 (p = 2). \quad (22)$$

The square of the desired current can be obtained as,

$$I_{d,n}^2 = \frac{f_{r,n} - c_1 - c_2 |\dot{x}_n|}{c_3} + k_f (f_{r,n} - f_n), \quad (23)$$

and the current loop transfer function remains unchanged. From Equations (22) and (23), we have

$$\begin{aligned} f_{n+1} &= c_3 I_n^2 + H_{1,n}, \\ I_{d,n}^2 &= -k_f f_n + H_{2,n}. \end{aligned} \quad (24)$$

Recall that the circuit is designed so that $I_n > 0$. When the computed value $I_{d,n} \leq 0$, we set it to zero. Note that when $I_{d,n} = 0$, the system is passive and therefore stable.

When $I_{d,n} > 0$, it is difficult to prove the global stability of the system. We can, however, show the local stability of the system near the equilibrium. Let $H_{1,\infty}$ denote the steady state value of $H_{1,n}$ and $H_{2,\infty}$ of $H_{2,n}$. The equilibrium of the system (24) can be found as

$$I_{\infty} = \sqrt{\frac{-k_f H_{1,\infty} + H_{2,\infty}}{1 + k_f c_3}}. \quad (25)$$

Let $I_n = I_{\infty} + \delta I_n$, where δI_n is a small perturbation. Substituting I_n to Equation (24), making use of Equation (14), and neglecting the higher-order terms of δI_n , we have

$$\begin{aligned} 0 = & R\delta I_{n+2} + [(1 - \alpha)k_p - R(1 + \alpha)]\delta I_{n+1} \\ & + [R\alpha + (1 - \alpha)(k_i - k_p + k_f k_p c_3)]\delta I_n \\ & + k_f c_3(1 - \alpha)(k_i - k_p)\delta I_{n-1}. \end{aligned} \quad (26)$$

The characteristic equation for the stability of δI_n reads

$$\begin{aligned} 0 = & Rz^3 + [k_p(1 - \alpha) - R(1 + \alpha)]z^2 \\ & + [R\alpha + (1 - \alpha)(k_i - k_p + k_f k_p c_3)]z \\ & + k_f c_3(1 - \alpha)(k_i - k_p), \end{aligned} \quad (27)$$

which is the same as Equation (19) for the case $p=1$. Therefore, the control gains k_f , k_p , and k_i satisfying Equation (20) also stabilize the equilibrium state of the system with $p=2$. Note that the characteristic equation (27) can also be obtained by linearizing Equation (24) at about the equilibrium.

SWITCHING LOGIC—Let us define the prediction errors of the damper force as

$$\begin{aligned} e_{1,n} &= f_{m,n} - (c_1 + c_2 \left| \dot{x}_{n-1} \right| + c_3 I_{n-1})(p = 1), \\ e_{2,n} &= f_{m,n} - (c_1 + c_2 \left| \dot{x}_{n-1} \right| + c_3 I_{n-1}^2)(p = 2), \end{aligned} \quad (28)$$

where $f_{m,n}$ is the measured damper force at the n th sample period. Consider a performance index θ governed by

$$\theta_{j,n} = \kappa \theta_{j,n-1} + e_{j,n}^2, \quad (29)$$

where $j=1, 2$ and $0 < \kappa < 1$.

Let p_n denote the value of p at the n th time step. The digital version of the hysteresis switching logic for selecting the next value of p can be stated as (Hespanha et al., 1999)

- Case 1, $p_{n-1} = 1$: if $\theta_{1,n} > (1 + h) \theta_{2,n}$, $p_n = 2$; else $p_n = 1$.
- Case 2, $p_{n-1} = 2$: if $\theta_{2,n} > (1 + h) \theta_{1,n}$, $p_n = 1$; else $p_n = 2$.

Note that h is a small positive number to avoid chattering, which behaves like the boundary layer parameter in sliding controls (Slotine and Li, 1991). Since both the controls for $p=1$ and $p=2$ are stable, we shall only need to prove that the switching logic will only result in a finite number of switching in a given time period. The proof presented below follows the same line as in Hespanha et al. (1999).

Define a variable $\theta_{p,n} = \theta_{p,n}/\kappa^n$. Let the initial conditions of the performance index be $\theta_{p,0} > 0$ for both $p=1$ and $p=2$. Then, $\theta_{p,0} = \theta_{p,0} > 0$. From Equation (29), we obtain an equation for $\theta_{p,n}$ as

$$\bar{\theta}_{p,n} = \bar{\theta}_{p,n-1} + \frac{e_{p,n}^2}{k^n} = \bar{\theta}_{p,0} + \sum_{i=1}^n \frac{e_{p,i}^2}{k^i}. \quad (30)$$

Hence, $\bar{\theta}_{p,n}$ is monotonically increasing and positive for all $n > 0$. Therefore, $\bar{\theta}_{p,n}$ will not be oscillatory and will converge as the tracking error $e_{p,n}$ decreases. This implies that in any finite time interval, there will be a finite number of switchings.

SIMULATION AND EXPERIMENTAL RESULTS

Before we present the simulation and experimental results, we will discuss the hardware and its limitations. The DC voltage applied to the coil is within the range of 0–12 V. We have used anti-integral windup to prevent overshoot in simulations and experiments. The voltage amplifier for the digital control signal also has a dead zone and a non-proportional gain. We linearize this path by using $u = 5.56(u_c - 0.2)$ based on the experimental data. u is the applied voltage, and u_c is the corresponding digital command. The resistance R of the coil is measured by a multimeter. The inductance L is identified from the coil transient response to a step voltage. The current I in the coil is measured through the voltage across a 1Ω resistor in serial with the coil. Prior to the experiments, the resistive force and flexion angle of the VRED are calibrated.

The angular velocity of the knee is obtained by differentiating a low-pass filtered angle measurement. The expression for the filtered velocity is given by

$$\dot{x}_n = 0.6\dot{x}_{n-1} + 100(x_n - x_{n-1}). \quad (31)$$

The sample frequency of the digital controller is 250 Hz both in simulations and experiments.

The parameters for the control simulations and experiments are as follows: $c_1 = 8$, $c_2 = 0.05$, $c_3 = 90$, $R = 10$, $L = 0.195$, $\alpha = 0.815$, $k_f = 0.008$, $k_p = 10$, and $k_i = 2$. The performance index filter constant = 0.5, and the switching boundary factor $h = 0.05$. The poles of the current loop are at $0.815 \pm 0.053i$, and the poles of the closed-loop plant are 0.77, 0.64, and 0.21 for the system with $p = 1$ and for the linearized system with $p = 2$. Both model systems are stable.

In the simulation, we assume that the actual plant parameters are $C_1 = 7$, $C_2 = 0.05$, $C_3 = 88$, and $p = 1.3$. The response of the actual plant is simulated for a range of flexion velocities from 0 to 400°/s. The angle is in the range from 15 to 90°.

Figure 3 shows simulated step force tracking, and Figure 4 shows sinusoidal force tracking. The figures show that the supervisory control properly switches the plant model and the corresponding control. The tracking performance is improved after each switching without overshoot. In addition, the control output is very smooth and compensates for variation in flexion velocity. Note that we have converted the time signal to the corresponding angular position in the figures, as is usually done in the rehabilitation community.

We implement the simulated control in the experiment without any modification. Extensive experiments have been conducted. The experimental control results of a step force tracking are shown in Figure 5, and of a variable reference tracking in Figure 6. The results from extensive experiments indicate that the force response of the system is bounded and the force tracking is generally accurate with a fast response sufficient for rehabilitation application. The irregular curve in the experimental plot at the beginning comes from the measurement noise when the angular reading is small and the signal-to-noise ratio is also small. After about two sample periods, the irregularity goes away.

CONCLUSIONS

We have presented prototypes of smart and variable resistance exercise devices for rehabilitation, and discussed the optimal design of MR dampers for the device. A supervisory control for regulating the resistive force or torque of the dampers has also been developed. The supervisory control effectively deals with the parameter uncertainty of the nonlinear MR damper force, and achieves excellent force tracking performance both numerically and experimentally. The VRED will be tried in a clinic setting in the near future. Data from the clinic trials will be used to further study the human-machine interaction in order to develop an intelligent rehabilitation device.

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Figure 1.
A variable resistance knee brace with MR damper.

Figure 2.
A versatile rehabilitation device with MR dampers.

Figure 3. Simulation of the step response. Solid line: the desired force. Dashed-line: the actual force.

Figure 4. Simulation of the response to a sinusoidal reference. Solid line: the desired force. Dashed-line: the actual force.

Figure 5. Experimental results of the step response. Solid line: the desired force. Dashed-line: the actual force.

Figure 6.

Experimental results of the response to a variable reference. Solid line: the desired force. Dashed-line: the actual force.