Mapping the Information Landscape: Discerning Peaks and Valleys for Ecological Monitoring

L. J. Moniz · J. D. Nichols · J. M. Nichols

Received: 13 July 2006 / Accepted: 4 February 2007 /
Published online: 28 February 2008
© Springer Science + Business Media B.V. 2007

Abstract We investigate previously unreported phenomena that have a potentially significant impact on the design of surveillance monitoring programs for ecological systems. Ecological monitoring practitioners have long recognized that different species are differentially informative of a system's dynamics, as codified in the well-known concepts of indicator or keystone species. Using a novel combination of analysis techniques from nonlinear dynamics, we describe marked variation among spatial sites in information content with respect to system dynamics in the entire region. We first observed these phenomena in a spatially extended predator–prey model, but we observed strikingly similar features in verified water-level data from a NOAA/NOS Great Lakes monitoring program. We suggest that these features may be widespread and the design of surveillance monitoring programs should reflect knowledge of their existence.

Keywords Ecological monitoring · Nonlinear systems · Transfer entropy · Spatial synchronization

1 Introduction

Some ecological monitoring programs are designed as components of larger science or management programs and, as such, are guided by a priori hypotheses [1]. However, many ecological monitoring programs are instead viewed as surveillance programs and are not

L. J. Moniz (✉)
Applied Physics Laboratory, John Hopkins University, Laurel, MD 20723-6099, USA
e-mail: lindamo@comcast.net

L. J. Moniz · J. D. Nichols
Patuxent Wildlife Research Center, US Geological Survey, Laurel, MD 20708, USA

J. M. Nichols
Naval Research Laboratory, 4555 Overlook Ave., Washington, DC 20375, USA

designed to inform a priori hypotheses. Most ecological monitoring programs target large areas over which monitoring cannot be carried out at all spatial locations. Most spatial sampling designs (e.g., random, stratified random) are developed to provide an unbiased estimate of some state variable (e.g., number of animals in a species of interest) accumulated over the entire area of interest (e.g., Thompson [2]). However, if an objective of monitoring is to develop an understanding of system dynamics (e.g., for the purpose of prediction), then other approaches to monitoring may be more useful. In particular, state variable dynamics at locations throughout a spatially extended system may be differentially informative about dynamics at other locations, and this possibility should be very relevant to monitoring.

Variation among locations in information content about system dynamics has received very little consideration in ecological monitoring until recently. Nichols et al. [3] investigated dynamics of a model predator–prey system and considered the ability to predict system dynamics at one location, based on observations of the dynamics at another location. They noted asymmetries in predictive abilities and speculated about information flow via animal movement in the system. Jonzen et al. [4] considered monitoring in a model source–sink system and noted that, for certain parameter regimes, monitoring the population in the sink provided more statistical power for trend detection than did monitoring the source. For other parameter regimes, the source provided more information about the two-location system.

We describe the use of information-theoretic metrics to investigate variation in information content and flow among locations of spatially extended systems. We apply these metrics first to a model predator–prey system and then to actual water level monitoring data from the Great Lakes. In both cases we identify substantial spatial variation in information content about system dynamics. In particular, we identify *information valleys*, locations that provide little information about dynamics of other system locations, and *information peaks*, those that provide significant information about dynamics of other system locations. We note that variation in information content of system components is a general phenomenon that applies not only to system locations but also to other system components (e.g., state variables such as different species in an ecological community). We conclude by emphasizing the relevance of variable information content for the selection of system components to be monitored as part of surveillance monitoring programs of natural systems.

2 Methods

Transfer entropy, recently introduced by Schreiber [5], is a powerful tool for analyzing information flow in nonlinear systems. Transfer entropy is based on information theoretic techniques developed in mathematical statistics by Fisher [6] and later Kullback [7], and in communications engineering by Hartley [8] and later Shannon and Weaver [9]. Information theoretic approaches are now widely used in nonlinear dynamics to analyze time series.

Assume that we observe a random variable x_i , sampled from a random process X at discrete times $i = 1 \cdots M$. The transfer entropy statistic considers systems that can be characterized as Markov processes and quantifies the additional information about such a process provided by knowledge of a state variable of a second process. The transfer entropy statistic first considers transition probabilities of the focal system. For a first order Markov process, for example, transition probability between time steps i and $i + 1$ can be written as $p(x_{i+1}|x_i)$. The approach then considers another possible system state variable (process), Y , and asks whether knowledge of this variable provides additional information about the

dynamics of X . In the absence of information flow between Y and X , the following equality should hold: $p(x_{i+1} | x_i) = p(x_{i+1} | x_i, y_i)$. The extent to which extra information about the dynamics of X is provided by Y is quantified by the transfer entropy (TE):

$$TE_{Y \rightarrow X} = \sum_{x_{i+1}, x_i, y_i} p(x_{i+1}, x_i, y_i) \log_2 \left\{ \frac{p(x_{i+1} | x_i, y_i)}{p(x_{i+1} | x_i)} \right\}. \quad (1)$$

Near-zero transfer entropy is ambiguous in that two systems that are identical exhibit zero transfer entropy (in that case, $\frac{p(x_{i+1}|x_i, y_i)}{p(x_{i+1}|x_i)} = 1$, yielding zero for the log term), and two systems that are unrelated also exhibit zero transfer entropy. In a monitoring context, the first possibility reflects useful information, whereas the second does not. Some measure of shared information such as mutual information [10] or continuity statistics [3, 11] must be used to resolve the ambiguity.

In this study, we investigated pairwise information flow between locations or regions. Non-zero transfer entropy from region Y to region X translates to flow of dynamical information from Y to X ; knowledge of Y 's dynamics gives us some information about X 's dynamics.

Use of the transfer entropy statistic requires estimation of the probability densities comprising (1). For nonstationary data this would require an ensemble of measurements which are typically not available to the practitioner. However, if the data are assumed to be stationary and ergodic, the TE may be estimated from a single realization of each process, i.e., the time series x_i, y_i . Making this assumption, estimates may be obtained via kernel density estimation [12]. Here we use the “naive” Heaviside kernel applied to a fixed number of points near the point at which the estimate is being performed. This so-called “fixed-mass” approach has already been used to estimate the TE and is described in detail by Kaiser and Schreiber [13]. We also point the reader to Marschinski and Kantz [14] for another estimate of kernel density for transfer entropy.

As an example, to compute the density estimate $p(x_f, y_f)$ we use a near-neighbor search [15] to determine the $N = 10$ nearest neighbors to the data vector (x, y) with time index f . We then compute the volume of the smallest rectangle, centered at (x_f, y_f) , that exactly encompasses these nearest neighbors. An estimate for the probability density becomes simply $\hat{p}(x_f, y_f) = N / V(x(f), y(f))$ where $V(x(f), y(f))$ is the volume enclosed by the rectangle. For other densities e.g. $\hat{p}(x_{i+1}, x_i, y_i)$ the procedure remains the same, although the volume element becomes higher dimensional (for example, a cube).

Computation of densities for use in computing transfer entropy is straightforward when the time series are stationary. However, if this method were employed on field data, should a catastrophic event occur during collection of the data, causing availability of a prey species to change completely (or a predator to appear or disappear), we would have to consider alternate methods for estimating density in order to compute transfer entropy. For example, it may be possible to compute information statistics on stationary subsets of such data (i.e., locally stationary subsets), or to use several nearby measurements to form an ensemble. In short, while there are options available, accurate determination of probability densities using non-stationary time series will be more difficult and require more sophisticated techniques than those applied here.

In addition to one system's providing information about dynamics of another, as measured by transfer entropy, that two systems may exhibit very similar dynamics, such that knowledge of the state of one system (e.g. x_i) provides information about the state of the other system, y_i . Such generalized synchronization also reflects useful information for monitoring and can be assessed via computation of continuity statistics [11, 16].

Here we use the continuity statistic to measure generalized synchronization. Generalized synchronization provides both a separate measure of shared dynamics and resolution of the ambiguity present in transfer entropy.

3 A Predator–Prey Model

Here we consider a spatially extended predator–prey model with diffusive movement, first studied by Pascual [17]. The model follows the evolution of prey p and predator h populations over a varying spatial coordinate z . The parameters in the model are predator death rate m , diffusion coefficient d , predator–prey coupling a , prey carrying capacity b and the intrinsic growth rate of the prey r_z .

The intrinsic growth rate of the prey is a decreasing function of z . We impose reflective boundary conditions: $\partial p/\partial z = \partial h/\partial z = 0$. The equations for this model are then:

$$\begin{aligned}\partial p/\partial t &= r_z p(1-p) - \frac{ap}{1+bp}h + d\frac{\partial^2 p}{\partial z^2} \\ \partial h/\partial t &= \frac{ap}{1+bp}h - mh + d\frac{\partial^2 h}{\partial z^2} \\ r_z &= e - fz.\end{aligned}\tag{2}$$

The parameters used in this study were $m = 0.6$, $d = 10^{-4}$, $e = 5.0$, and $b = 2.0$. In order to obtain the time series, we integrated the model for $M = 10,000$ time steps numerically with a step size of 0.5. We compute the time series of abundances on a spatial grid of $j = 1, \dots, 20$ sites distributed evenly on $z \in [0, 1]$. The resource gradient described in the r_z equation by the parameter f provides the diversity of populations throughout the spatial extent. In this study we use three resource gradients, $f = 0.9$, $f = 1.4$ and $f = 1.9$.

In Nichols et al. [3] there is a comprehensive discussion of the behavior of the dynamics related to each grid site, as both resource gradient and spatial location are varied. This variation in dynamics illustrates the difficulties with simply describing system dynamics using the summed predator and prey abundances for the entire system (ignoring spatial variation).

4 Information Peaks and Valleys

Transfer entropy was computed between all possible pairs of spatial locations for all three resource gradients (Fig. 1). Computations were based on the two-dimensional system state defined by the abundance of both predator and prey ($\tilde{x}_i = (p_i, h_i)$ replaces x_i in (1)). To aid in the analysis of information flow in this model, we have compiled “total transfer entropy” in Fig. 2, the sum of transfer entropy from a focal site (the x -coordinate) to all other sites.

The horizontal and vertical axes on the plots in Fig. 1 are the (predator, prey) sites Y in (1). Positive transfer entropy from the site on the horizontal axis to the site on the vertical axis translates to the horizontal axis’ site providing dynamical information about dynamics at the vertical axis’ site. Thus the x -axis is the site *from which* information is transferred, the y -axis is the site *to which* information is transferred. A horizontal bright band of transfer entropy at site s corresponds to many other sites’ providing additional information about

Fig. 1 Transfer entropy for the Pascual model, with gradients 0.9 (*top left*), 1.4 (*top right*) and 1.9 (*bottom*). Horizontal and vertical axes are (predator, prey) sites

dynamics in s . A vertical band of transfer entropy at site s corresponds to site s 's providing additional dynamical information about other sites.

As the resource gradient becomes steeper, there is an increase in transfer entropy overall for all three examples. We also see an increase in the number of horizontal (bright) bands of transfer entropy, but transfer entropy in the vertical direction is relatively uniform in the middle of the region for the steeper gradients.

The vertical bands of transfer entropy are candidates for *information peaks*. These sites provide information about nearly all other sites. However, lower transfer entropy at other sites does not necessarily mean that the other sites are information valleys. Because transfer entropy may be zero for sites that are synchronized with other sites, a site may contain information about another without exhibiting positive transfer entropy to that site.

To resolve the ambiguity in transfer entropy, we look at the continuity statistic, which measures generalized synchronization [11, 16] of one site to another (Fig. 3). To interpret the continuity plots, we note that *small* continuity statistics (dark on the plots) correspond to *good* synchronization, and *large* continuity statistics (brightest on the plot) correspond to *poor* synchronization; intermediate colors correspond to some synchronization (grey) or to little evidence for synchronization (light grey). Details regarding the computation and interpretation of the continuity statistic can be found in [3].

The continuity result clarifies the transfer entropy results in the following way. If transfer entropy from site A to sites B, C and D is comparatively low, but site A synchronizes well with sites B, C and D, site A is *not* an information valley. It provides information about other sites that is not detected by transfer entropy. However, if site A exhibits *low* transfer

Fig. 2 Total transfer entropy for the Pascual model, with gradients 0.9 (*top left*), 1.4 (*top right*) and 1.9 (*bottom*)

entropy to other sites and does *not* synchronize well with them, it *is* an information valley. In this case, monitoring site A provides information about dynamics at site A, but says little about the dynamics of the system at other sites.

We now turn to the plots of transfer entropy, total transfer entropy, and continuity (synchronization) from the predator–prey model. These are found in Figs. 1–3. We begin with the shallowest gradient, 0.9. These are the top left plots in all three figures.

Gradient 0.9 shows some distinct features in both the spatial transfer entropy plot (Fig. 1, left) and in the total transfer entropy, (Fig. 2, left). There is a horizontal band showing significant transfer entropy to site 0.85. We also see a wide vertical band of transfer entropy from sites near 0.65. The plot of total transfer entropy reinforces this observation: sites 0.60, 0.65 and 0.70 transfer more entropy to other sites overall than sites 0.05–0.55 or sites 0.75–1.0.

Now we consider the continuity plot, to see if sites with lower total transfer entropy actually synchronize with other sites and thus share dynamics. The continuity plot is shown in Fig. 3, top left. Sites 0–0.6 synchronize with other sites – the transfer entropy is low in these regions because of shared dynamics. These sites also transfer some entropy to the high-resource end sites. In contrast, sites 0.85–1.0 show very little synchronization with sites at the high-resource end of the region, and do not transfer much entropy to sites other than 0.85. For this reason, we characterize the sites from 0.85–1.0 as information valleys – they do not provide as much information about total system dynamics as other monitoring choices.

Fig. 3 Continuity statistics for the Pascual model, with gradients 0.9 (*top left*), 1.4 (*top right*) and 1.9 (*bottom*). Horizontal and vertical axes are (predator, prey) sites

We characterize sites 0.6–0.75 as information peaks – they exhibit significant dynamical information about most sites in the entire region. Sites 0.10–0.5 do not transfer much entropy to sites 0.6–0.7 and, although they do exhibit some synchronization with those sites, we cannot justify characterizing them as information peaks.

There is much more transfer entropy overall in the gradient 1.4 case compared to the gradient 0.9 case (Fig. 1, top right). There is also less synchronization between sites in the entire region (Fig. 3, top right). In the transfer entropy plot we also see a “downhill” flow of information from the high-resource end of the region to the low-resource end. The slightly increasing total transfer entropy plot also reflects the “downhill” trend.

One notable feature of the plot for transfer entropy in gradient 1.4 (Fig. 1, top right) is that there are few sites in the middle of the region that either transfer entropy to the high resource end or synchronize to it. We see this reflected in the increasing but still relatively flat total transfer entropy plot, Fig. 2, top right. We see that sites 0.75 and 0.8 have high total transfer entropy, including some to the high-resource end, and they exhibit at least some transfer entropy to all other sites. Thus, we would characterize these as information peaks. Sites at the high-resource end (0.1–0.3) of the region have lower total transfer entropy, and they do not synchronize with some sites at the low resource end of the region. These sites could be characterized as information valleys; if they were monitored, they would provide little information about dynamics at the low resource end of the region. If they were the *only* sites monitored, the dynamical description of the region would be misleading.

The gradient 1.9 case exhibits interesting contrasts to both the 0.9 gradient case and the 1.4 gradient case. There is an additional increase in overall transfer entropy over the 1.4

case, but we see some other features emerging in Fig. 1, bottom. The most striking feature is the absence of transfer entropy from sites 0.90–1.0 to most other sites, including sites that are adjacent to them. The synchronization plot (Fig. 3, bottom) shows little synchronization of these sites with other sites, indicating that the lack of transfer entropy is not due to shared dynamics. Therefore, we characterize sites 0.9–1.0 as information valleys. Monitoring these sites would give little information about dynamics in the rest of the region. However, this phenomenon at the edge of the region may be due to the reflective boundary conditions in the model. In contrast, sites in the middle of the region showed high total transfer entropy (see Fig. 2, bottom), and moderately high transfer entropy to most other sites. Although monitoring sites 0.9–1.0 would not yield much information about dynamics in other sites, we see significant transfer entropy from sites 0.4–0.8 to sites 0.9–1.0. Monitoring these sites in the middle of the region would still describe dynamics at the very low end of the resource gradient relatively well. These sites, from 0.4 to 0.8 are information peaks. Site 0.10, in contrast, does not provide much transfer entropy to sites at the low-resource end of the region and only synchronizes well to sites 0.05–0.3. This site, in addition to sites 0.9–1.0, could be characterized as an information valley.

The characterization of sites 0.9–1.0 as information valleys initially appears to contradict statements made in Nichols et al. [3], in which sites 0.9–1.0 were found to be more suitable for monitoring than sites at the high resource end. We note that although there is better synchronization from sites 0.9–1.0 to sites 0.1–0.4 than from 0.1–0.4 to 0.9–1.0, and more transfer entropy *from* sites 0.9–1.0 to sites 0.1–0.4 than from site 0.1 to any other site, it is low in comparison to the information flow and synchronization of sites 0.4–0.8. Thus, the information valley distinction is not in comparison to the high resource sites, but to the information peaks.

In summary, all three resource gradients exhibited information peaks and information valleys. The steepest and the shallowest gradients had the most strikingly obvious information valleys.

In all cases presented here, we see that other sites transfer entropy *to* the information valley sites. This is important in the monitoring context; if we avoid monitoring a site it is advantageous to have that site's dynamics described by other sites that *are* monitored. Otherwise, important information about system dynamics could be lost. Further study is needed to verify that sites in a spatially extended system which neither transfer information to other sites nor have information transferred to them, are unlikely to exist in ecological systems with diffusive movement or diffusion.

5 Results with Field Data

We stress that these information peaks and valleys exist not necessarily because the model in question is a nonlinear population model, but because it is a nonlinear spatially extended system with diffusion. We downloaded water level data from the Great Lakes at one site on each lake and used transfer entropy to see if there is a similar monitoring phenomenon present in a completely different (and relatively arbitrarily chosen) set of ecological data. The four sites were Buffalo, Lake Erie, Lakeport, Lake Huron, Ludington, Lake Michigan, and Point Iroquois, Lake Superior between 7/01/2005 and 7/31/2005, taken at 6-min intervals (7,390 data points) from NOAA/NOS [18]. To justify using transfer entropy to analyze these data, we performed a reverse-arrangement test [19] for stationarity using $N = 40$, $N = 60$ and $N = 120$. The water level time series for these four lake sites

were within one or two standard deviations for both mean and variance using these tests. We tried two locations from Lake Ontario, Rochester and Oswego, but this data appeared to be nonstationary; the means were more than 5 standard deviations away from the mean value for every N in all of our reverse-arrangement tests. Other locations on Lake Ontario did not have complete data for this time period.

We embedded the data sets (see, e.g. Pecora et al. [20] and Sauer et al. [21]) in dimension 5 and calculated transfer entropy for the grid of sites. The results appear in Fig. 4, alphabetically by lake name. Although this was a completely different system from the population model presented in the previous section, we see comparable phenomena. In this data set, the Lake Erie site transferred more entropy to other sites than was transferred to it. The Lake Huron site and the Lake Michigan site did not transfer much entropy to other sites, and the Lake Superior site transferred entropy moderately well in both directions. Looking at the total transfer entropy in Fig. 4, we see that the Lake Erie site had the highest total transfer entropy, the Lake Superior site had moderately high total transfer entropy and the Lake Huron and Lake Michigan sites had very little total transfer entropy. Continuity tests between the embedded time series showed little synchronization between any of the sites. From these results we can characterize the site on Lake Erie as an information peak, and sites on Lake Michigan and Lake Huron as information valleys. We must stress that this was water level data for one month and does not indicate that the same conclusions can be made for other months or even other locations on the same lakes. However, these results are compelling evidence that the information peaks and valleys are not an artifact of the particular population model that we used and could be, in fact, quite common.

Fig. 4 Transfer entropy, total transfer entropy and continuity for Great Lakes data

6 Conclusions and Recommendations

We express a caveat with these results: an information valley still provides information about system dynamics, but this information is limited to that site only. Thus, an information valley should not be the *only* site that is monitored, if a limitation of resources dictates that only a few sites can be chosen. The information peaks, on the other hand, would provide information about system dynamics in the whole region. If resources were severely limited, they would be better choices when only a few sites or even one site could be monitored.

It is interesting that monitoring programs for ecological communities have recognized variation in information content of different species for some time, as captured by the concept of *indicator species* (e.g. Noon [22]). However, variation in information content of different locations has received little attention. The transfer entropy and continuity statistics methods used here should be useful for selecting informative species, locations, and other system components. We believe that information peaks and valleys are likely to occur across spatial landscapes in many ecological communities with varying resources and should be considered when surveillance monitoring is planned.

The water level data from the Great Lakes provide evidence that the phenomenon of variable information content is not limited to populations. This work suggests that design of any ecological surveillance program should be augmented by some consideration of the information flow and transport in the suggested monitoring sites.

We believe that our inferences about spatial variation in information content across ecological systems may be useful at several different levels in the development of surveillance monitoring programs. In cases such as our Great Lakes example, where long time series of relevant ecological state variables already exist, use of the methods that we describe should be useful in assessing information content and thus in selecting high-information locations as monitoring foci. In other cases where existing time series are much shorter, our general methods may not be applicable. In such cases, more traditional approaches involving linear cross-correlation (e.g., Bjornstad et al. [23] and Koenig [24]) may be useful in identifying locations at which ecological state variables vary more and less closely with those of other locations. In situations where no existing time series are available, but where investigators have developed models representing hypotheses about system dynamics, the approaches described in this paper can be used with simulated time series data to develop informed approaches to spatial monitoring. In fact, work with such models may even lead to generalizations about information content as a function of such system characteristics as asymmetric movement patterns [3, 4]. Such generalizations may prove useful in guiding monitoring efforts even in the absence of fully specified models of the systems of interest.

Finally, we believe that the concept of spatial variation in information content is itself useful. By analogy, we note that the concept of “indicator species” has received substantial discussion with respect to design of surveillance monitoring programs, even in the absence of good approaches for identifying such species (e.g., Noon [22]). The concept we introduce of a spatially variable information landscape with peaks and valleys should be useful in generating thought and discussion about the kinds of properties that should distinguish information-rich locations from those that carry relatively little information. Such conjectures about spatial variation in information can then be investigated using model systems in conjunction with the methods described in this paper, with results ultimately providing guidance to the establishment of surveillance monitoring programs.

Acknowledgements We thank Paul Dresler and the US Geological Survey Inventory and Monitoring Program for support of this project. We also thank Evan Cooch for useful discussions of these ideas.

References

1. Yoccoz, N., Nichols, J., Boulinier, T.: Monitoring of biological diversity in space and time. *Trends Ecol. Evol.* **16**, 446–453 (2001)
2. Thompson, S.: *Sampling*. Wiley, New York (2002)
3. Nichols, J., Moniz, L., Nichols, J., Pecora, L., Cooch, E.: Assessing spatial coupling in complex population dynamics using mutual prediction and continuity statistics. *Theor. Popul. Biol.* **67**, 9–21 (2005)
4. Jonzen, N., Rhodes, J.B., Possingham, H.P.: Trend detection in source-sink systems: when should sink habitats be monitored? *Ecol. Appl.* **15**(1), 326–334 (2005)
5. Schreiber, T.: Measuring information transfer. *Phys. Rev. Lett.* **85**, 462–464 (2000)
6. Fisher, R.: Theory of statistical estimation. *Trans. Camb. Philos. Soc.* **22**, 700–725 (1925)
7. Kullback, S.: *Information Theory and Statistics*. Dover, Mineola, New York (1997)
8. Hartley, R.: Transmission of information. *Bell Syst. Tech. J.* **7**, 535–563 (1928)
9. Shannon, C., Weaver, W.: *The Mathematical Theory of Communication*. University of Illinois Press, Chicago, IL (1949)
10. Fraser, A., Swinney, H.: Independent coordinates for strange attractors from mutual information. *Phys. Rev., A* **33**, 1134–1140 (1986)
11. Pecora, L., Carroll, T., Heagy, J.: Statistics for mathematical properties of maps between time series embeddings. *Phys. Rev., E* **52**(4), 3420–3439 (1995)
12. Silverman, B.: *Density estimation for Statistics and Data Analysis*, Monographs on Statistics and Applied Probability. Chapman and Hall, London. Available online at <http://www.doc.ic.ac.uk/~xh1/Reference/density-estimation/density-estimation-for-statistics-and-data-analysis.pdf> (1986)
13. Kaiser, A., Schreiber, T.: Information transfer in continuous processes. *Physica, D* **166**, 43–62 (2002)
14. Marschinski, R., Kantz, H.: Analysing the information flow between financial time series: an improved estimator for transfer entropy. *Eur. Phys. J., B* **30**, 275–281 (2002)
15. Hjaltason, G.R., Samet, H.: Ranking in spatial databases. In: Engenhoffer, M.J., Herring J.R. (eds.) *Proceedings of the 4th Symposium on Large Spatial Databases*, vol. 951 of *Lecture Notes in Computer Science*, pp. 83–95. Springer, Berlin (1995)
16. Hunt, B., Ott, E., Yorke, J.A.: Generalized differentiable synchronization of chaos. *Phys. Rev., E* **44**, 4029–4034 (1997)
17. Pascual, M.: Diffusion-induced chaos in a spatial predator–prey system. *Proc. R. Soc. B* **25**, 1–7 (1993)
18. NOAA/NOS: Verified Lake Level Data for the Great Lakes. <http://www.glerl.noaa.gov/data/now/wlevels/levels.html> (2005)
19. Bendat, J., Piersol, A.: *Random Data: Analysis and Measurement Procedures*. Wiley, New York (1986)
20. Pecora, L., Moniz, L., Nichols, J., Carroll, T.: A unified approach to attractor reconstruction. *Chaos* **17**, 013110 (2007)
21. Sauer, T., Yorke, J.A., Casdagli, M.: Embedology. *J. Stat. Phys.* **65**(3/4), 579–616 (1991)
22. Noon, B.: Conceptual issues in monitoring ecological resources. In: Busch, D., Trexler, J. (eds.) *Monitoring Ecosystems*, pp. 27–71. Island Press, Washington, DC (2003)
23. Bjornstad, O., Ims, R., Lambin, X.: Spatial population dynamics: analyzing patterns and processes of population synchrony. *Trends Ecol.* **14**, 427–432 (1999)
24. Koenig, W.: Spatial autocorrelation of ecological phenomena. *Trends Ecol.* **14**, 427–432 (1999)