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Cerebral blood flow estimation from perfusion-weighted MRI using FT based MMSE filtering method

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Abstract

Introduction—Perfusion weighted MRI can be used for estimating blood flow parameters using bolus tracking technique based on DSC MRI. In order to extract flow parameters, several deconvolution techniques have been proposed, of which, the SVD and FT based techniques are more popular and widely used. In this work, an FT based method has been proposed that involves derivation of an optimal shaped filter (defined as a filter function) estimated using minimum mean-squared error (MMSE) technique in the frequency domain. The proposed technique has been compared with the well established SVD technique using simulation experiments.

Simulation Methods—Simulation was performed in multiple steps. An arterial input function was first defined based on a certain blood flow value. The $T2^*$ signal change was then derived from this AIF and noise was added to the signal. Then, a unique and optimal shaped filter function $\Phi(f)$ was derived in order to obtain the best estimate of scaled residue function. One way is by minimizing mean-squared error between the noiseless and noisy scaled residue function, i.e. using an MMSE method. Effect of low and moderate noise and distorted AIF on CBF estimates was obtained by using FT based MMSE method. Results were compared with the SVD technique. In this work SVD technique was assumed to be the standard reference deconvolution technique.

Results and Discussion—For low noise condition, the FT based technique was more stable than the SVD technique, while for moderate noise both techniques consistently underestimated CBF. SVD technique was found to be more stable in presence of AIF distortions. However, SVD technique was found to be unstable due to AIF delay compared to the FT based MMSE method. The shaped filter function was found to be sensitive to effect of AIF distortions.

Keywords

MRI; CBF; SVD; FT; perfusion

Introduction

Perfusion weighted (PW) MRI based on dynamic susceptibility contrast (DSC) MRI provides a powerful, relatively non-invasive way for assessing brain function. The PW-MRI technique allows estimation of three important parameters; cerebral blood flow (CBF), cerebral blood volume (CBV) and mean transit time (MTT). Due to the widespread availability of MR imaging modality as a diagnostic tool, PW-MRI method is routinely used for investigating neurological disorders such as stroke and guiding acute stroke therapy [1].

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The bolus tracking (BT) technique is the most popular PW DSC-MRI method used for estimating CBF, CBV and MTT. The BT technique uses a convolution model that establishes the input-output relationship between blood flow and the vascular tracer concentration. This relationship is expressed mathematically as a matrix equation and requires the use of deconvolution methods to extract blood flow. Furthermore, the BT technique relies on the indicator dilution method [2] and is based on first pass measurement of an instantaneous bolus of a vascular exogenous contrast agent such as Gd-DTPA.

A real bolus is never instantaneous since intravenous injection causes the bolus to be broadened during circulation through the lungs and heart prior to reaching the brain. As a result, the concentration-time curve of the injected contrast agent is a convolution of tissue concentration-time response to an idealized bolus and the arterial input function (AIF) time course. If the AIF is known, which is typically estimated in a major artery, a deconvolution procedure can be used to obtain the tissue concentration-time response. Unfortunately, AIF estimation is prone to errors due to image noise, bolus delay and dispersion before reaching the measured voxel, resulting in inaccurate results post deconvolution [3]. Despite these drawbacks, the deconvolution methods are useful for obtaining quantitative flow measurements.

A number of deconvolution methods have been proposed in the recent past [4-11]. Each of these techniques has advantages and disadvantages. Two model-independent deconvolution techniques that are popular and widely used for estimating PW-MRI parameters are the Singular Value Decomposition (SVD) and Fourier Transform (FT) based methods [9]. However, from the published literature it appears that SVD is more widely accepted than other techniques. SVD technique is an approach that applies a cutoff threshold to the singular values of the matrix that describes the convolution. This strategy makes the technique less sensitive to noise. However, SVD technique has some drawbacks; it is only an approximation of deconvolution, the CBF estimates are influenced by the choice of cutoff threshold for matrix diagonal, and as a result, induce distortion of high frequency components [12]. SVD technique is sensitive to both delay and dispersion [3] effects. The FT based deconvolution methods involve application of a shaping filter in the frequency domain and are very sensitive to noise and therefore requires filtering and/or fitting of the experimental data. On a positive note, these methods are insensitive to AIF delay. Additionally, this technique is sensitive to AIF dispersion effects.

One of the drawbacks of existing FT deconvolution technique is that data filtering (for suppressing noise) is performed using empirically derived low pass filters. This may cause the filter to be unreliable and sub-optimal resulting in under or over estimation of CBF values. In this work, an FT based method has been proposed, which involves derivation of an optimal shaped filter (defined as a filter function) estimated using minimum mean-squared error (MMSE) technique in the Fourier domain. The proposed technique has been compared with the well established SVD technique using simulation experiments. The effect of AIF distortion (width), SNR (which is defined in terms of the noise standard deviation and the signal drop due to paramagnetic agent) and bolus delay on estimated CBF values using the SVD and proposed technique has been investigated using simulation experiments. Thus, there are two aims of this simulation study - first, to derive an optimal shaped filter function using FT based MMSE technique and second, to compare the performance of the proposed technique with SVD technique. The next section describes the theoretical basis of the simulation and the mathematical approach used in this simulation study to derive the shaped filter function using FT based MMSE technique.

Theory

In PW-MRI, pixel-wise concentration of the injected contrast agent, $c(t)$, at any time point during data acquisition is related to the measured pixel-wise signal intensity, $s(t)$, by the well known relation,

$$c(t) = -\frac{1}{(k \cdot T_E)} \cdot \ln\left(\frac{s(t)}{s_0}\right) \quad (1.1)$$

where, t denotes a unique time point during the time series, s_0 denotes the baseline signal intensity acquired prior to the injection of the contrast agent, T_E is the echo time, k is a proportionality constant that includes many factors such as the brain density and hematocrit values, which differs in arteries, veins and tissues, therefore effecting the amount of relaxivity change for different voxels. It is usually determined experimentally and for this study it is assumed to be equal to 0.2. In our study, the baseline signal, s_0 , has been assumed to be error-free. In practice, its estimate is sensitive to noise; however, it is relatively accurate since it is calculated as the mean value of 10-12 baseline time-points, and the noise standard deviation is low with respect to s_0 (practically less than 10%). The deconvolution model that has been wisely used to establish a relation between CBF and the tracer concentration [9] is given by

$$c(t) = F \cdot \left[\int_0^t a(\tau) \cdot r(t - \tau) d\tau \right] \quad (1.2)$$

Here, $a(\tau)$ denotes AIF, the time-distribution function of the blood contrast agent concentration, $r(t)$ denotes the residue function, which represents the fraction of the tracer in the vascular compartment and $F = k \cdot \text{CBF}$. By definition, $r(t=0)=1$. The goal is to estimate F (hence CBF), and this can be done by deconvolution of $c(t)$ with $a(t)$, which are measured and approximated experimentally. The result from deconvolution will provide us with an estimate of the product $F \cdot r(t)$.

From the basic properties of Fourier transform, equation (1.2) can be also written in the frequency domain as [13]

$$C(f) = F \cdot A(f) \cdot R(f) \quad (1.3)$$

where $C(f)$, $A(f)$ and $R(f)$ represents Fourier transforms of $c(t)$, $a(t)$ and $r(t)$ respectively. Then, the flow residue function product can be computed as,

$$F \cdot r(t) = \mathbb{F}^{-1} \left(\frac{C(f)}{A(f)} \right) \quad (1.4)$$

where $\mathbb{F}$, denotes Fourier transform, whereas $\mathbb{F}^{-1}$ denotes the inverse Fourier transform.

The acquired MR signal in practice is not pure and has noise that can be separated, based on its source, into physiological and measurement noise. In order to obtain accurate estimates of blood flow, it is important to have optimal image SNR. The source of physiological noise is heart rate and blood pressure, while measurement noise originates from the data acquisition hardware. In this simulation study, physiological noise has been ignored for simplicity. The measurement noise is modeled as an additive white Gaussian noise (AWGN) and it is defined as $n_m(t)$. Although the additive part of the noise in MR signal is actually Rician, for simplicity, it can be approximated with Gaussian for $\text{SNR} > 2$ [14]. In this case, the noisy measurement is

$$s_n(t) = s_0 \cdot e^{-(k \cdot c(t) \cdot T_E)} + n_m(t) \quad (1.5)$$

Hence, in practice, the measured concentration $c(t)$ that is derived from the measurement signal using equation (1.2) will also be corrupted with noise. Denoting the noise-corrupted concentration signal as $c_n(t)$ its relation to the measurement noise can be given by,

$$c_n(t) = -\frac{1}{(k \cdot T_E)} \ln \left(\frac{s(t) + n_m(t)}{s_0} \right) \quad (1.6)$$

Let $C_n(f)$ denote the noise-corrupted concentration in the frequency domain. In the presence of the noise and from equation (1.4), $F \cdot r(t)$ will be degraded with noise, resulting in inaccurate estimates of blood flow. In order to reduce noise and improve signal, the acquired signal is filtered using a filter function $\Phi(f)$, defined in the Fourier domain, such that

$$\widehat{r}(t) = \mathbb{F}^{-1} \left(\frac{C_n(f) \cdot \Phi(f)}{A(f)} \right) \quad (1.7)$$

where, $\widehat{r}(t)$ denotes the estimate of the residue function scaled with the flow (for simplicity, flow has been incorporated into the residue function and henceforth will be called the scaled residue function). Residue function and flow can be easily extracted from the scaled residue function i.e. its maximum value.

In order to obtain a working relationship between $C(f)$ and $N_m(f)$ with $C_n(f)$ and since $C_n(f)$ is involved in the deconvolution step, Fourier transform is applied to both sides in equation (1.5). After some manipulation and since $|k \cdot c(t) \cdot T_E| \ll 1$, using the Taylor series expansion, $e^x \approx 1 + x$, we obtain

$$S_n(f) = s_0 \cdot (\delta(f) - k \cdot T_E \cdot C_n(f)) + N_m(f) \quad (1.8)$$

Hence, an estimate of concentration noise can be obtained as,

$$C_n(f) \approx \frac{s_0 \cdot \delta(f) + N_m(f) - S_n(f)}{s_0 \cdot k \cdot T_E} \quad (1.9)$$

Now, in the absence of any measurement noise, proceeding with the same steps above, a relation for noiseless signal can be obtained as

$$C(f) \approx \frac{s_0 \cdot \delta(f) - S_n(f)}{s_0 \cdot k \cdot T_E}. \quad (1.10)$$

From equations (1.9) and (1.10) we obtain,

$$C_n(f) \approx C(f) + \frac{N_m(f)}{s_0 \cdot k \cdot T_E} \quad (1.11)$$

or equivalently in the time domain,

$$C_n(t) \approx c(t) + \frac{n_m(t)}{s_0 \cdot k \cdot T_E} \quad (1.12)$$

As per equation (1.11) and (1.12) a deconvolution technique can be applied after computing

the measurement noise by treating the noise as scaled with $\frac{1}{(s_0 \cdot k \cdot T_E)}$. The new noise that is effectively additive on the concentration is defined as,

$$N_c(f) \triangleq \frac{N_m(f)}{s_0 \cdot k \cdot T_E} \quad (1.13)$$

If σ_{N_m} denotes the standard deviation (SD) of measurement noise, then the SD of the concentration noise, $N_c(f)$, is

$$\sigma_{N_c} = \frac{\sigma_{N_m}}{s_0 \cdot k \cdot T_E} \quad (1.14)$$

The signal drop Δs , due to bolus passage is defined as,

$$\Delta s = s_0 - s_{\min} \quad (1.15)$$

The measurement signal-drop-to-noise ratio (SDNR) can be then defined as,

$$\text{SDNR} \triangleq \frac{\Delta s}{\sigma_{N_m}} \quad (1.16)$$

In the simulation experiments, the noise SD was kept constant. Assuming that the AIF and the MTT are constant, the simulated CBF values increase the peak signal drop, Δs , and therefore, the simulated CBF values will directly affect the SDNR. Additionally, since AIF also depends on the shape and amplitude of signal change, a change in AIF would also be expected to directly influence SDNR. As a note, CBV increases with increasing CBF since MTT is held constant throughout this study.

Estimating the optimal filter

The final goal of the proposed deconvolution method is to find an optimal filter that will provide the best estimate of the scaled residue function. In other words, a unique and optimal $\Phi(f)$ needs to be derived in order to obtain a best estimate for scaled residue function. One way to obtain best estimate is by minimizing the mean-squared error (MSE) between the noiseless and noisy scaled residue function. From the well known Parseval's Theorem [13], the MSE can be written as,

$$\mathbf{E} \int |r(t) - \widehat{r}(t)|^2 dt = \mathbf{E} \int |R(f) - \widehat{R}(f)|^2 df \quad (1.17)$$

where, $\mathbf{E}$ denotes the expectation operator. Using equation (1.4) and (1.7), we have

$$\text{MSE} = \mathbf{E} \int \left| \frac{C(f)}{A(f)} - \frac{\Phi(f) \cdot C_n(f)}{A(f)} \right|^2 df \quad (1.18)$$

Using the linearity of Fourier Transform operator, and assuming that the noise is uncorrelated with the measurement signal, we can write

$$\text{MSE} = \mathbf{E} \int \frac{|C(f)|^2 \cdot |1 - \Phi(f)|^2 + |\Phi(f)|^2 \cdot |N_c(f)|^2}{|A(f)|^2} df \quad (1.19)$$

Now, minimizing equation (1.20) with respect to $\Phi(f)$ by differentiating MSE with respect to $\Phi(f)$ and setting it equal to zero, an optimal $\Phi(f)$ is obtained, which is denoted as $\Phi^*(f)$,

$$\Phi^*(f) = \frac{|C_n(f)|^2 - |N_c(f)|^2}{|C_n(f)|^2} \quad (1.20)$$

Using filter $\Phi^*(f)$, derived in the above equation, would provide the optimal MMSE solution for the scaled residue function as,

$$\widehat{r}^*(t) = \mathbb{F}^{-1} \left(\frac{C_n(f) \cdot \Phi^*(f)}{A(f)} \right) \quad (1.21)$$

From the inherent property of Fourier transform [13], the flow can be determined without taking the inverse transform, but by computing the integral

$$\widehat{F}^* = \widehat{r}^*(0) = \int \frac{C_n(f) \cdot \Phi^*(f)}{A(f)} df \quad (1.22)$$

However, in practice the estimation of $\widehat{r}^*(t)$ would not usually result in a function that has maximum at $t=0$. The maximum value of $\widehat{r}^*(t)$ is usually taken as an estimate of flow, $\widehat{F}^*$.

Methods

Computer Simulation

In this study, simulation experiments were performed based on the equations described in the theory section. Computer programming for the simulation experiments was implemented in MATLAB (7.0.4 R14 SP2, 32bit) (Mathworks, Natick, MA) environment on a PC x86 workstation running Windows XP. Computer routines used in this simulation experiments were developed as follows:

- First, the observed signal $s(t)$ was simulated as shown in Figure 1a that represented a change in pixel signal intensity in a medium sized artery due to T2* effect from a bolus of Gd-DTPA injected 10s after the start of imaging. On a 1.5T clinical scanner, a bolus injection of 0.1 mM/kg Gd-DTPA typically produces ~40-45% drop in signal intensity (represented by $s(t)$) in healthy subjects for the highest flow, a fact used in this simulation study. Furthermore, $s(t)$ was derived from an gamma-variate function fitted AIF (Figure 1b) with the following analytical expression,

$$a(t) = \max \{0, \alpha \cdot (t - t_0) \cdot e^{-\beta \cdot (t - t_0)}\} \quad (1.23)$$

where, parameters $\alpha = 0.0816$, $\beta = 0.2857$ and $t_0 = 20.5$ s. The parameter t_0 determines the time at which the bolus arrives at any given region. The simulated signal corresponded to a blood flow value of 80 ml 100g⁻¹ min⁻¹ (Figure 1b). SDNR of 30 was assumed for this simulated signal, a value obtained from clinical experience.

- Uncorrelated measurement noise with desired noise SDNR was then added to $s(t)$ in order to obtain $s_n(t)$. The noise was assumed to be additive white Gaussian noise (AWGN). The noisy signal was synthesized at a sampling rate of 0.625 Hz corresponding to data point interval of 1.6s, a number derived from our clinical imaging protocol.
- $S_n(f)$ was computed from $s_n(t)$.
- $C_n(f)$ and $N_c(f)$ were computed using equations (1.11) and (1.13).
- Optimal estimates of F and $r(t)$ were obtained using equations (1.21) and (1.22).

Results

The proposed deconvolution technique was simulated for a range of CBF values (0-80 ml 100g⁻¹ min⁻¹) that represented both normal and abnormal blood flow in an adult human brain. An analytical expression for $\Phi^*(f)$ was obtained by fitting an exponential function to $\Phi^*(f)$. The effect of SDNR change and AIF on CBF values and $\Phi^*(f)$ was investigated in these experiments. A maximum SDNR of 30 and 10 (for the maximum flow) were used in these simulation experiments to investigate the effect of low and moderate noise on CBF values and $\Phi^*(f)$. A plot of SDNR versus actual flow is shown in Figure 2a. Figure 2b shows the relation between signal drop and actual blood flow values. The effect of AIF on CBF values and $\Phi^*(f)$ was evaluated by using distorted AIF (narrow and wide AIF) that reflected the efficiency of a gamma-variate fit to the observed signal. Additionally, the effect of bolus delay on estimated CBF values was also investigated in this study. Finally, results obtained from these experiments were compared with those obtained from SVD technique. The simulated and approximated residue function used in this simulation for FT based MMSE and SVD techniques are shown in Figure 3a and 3b respectively. The residue function was simulated using an exponential decay as a general model that assumed the vasculature to be a single well-mixed compartment [3] and can be mathematically represented as

$$r(t, MTT) = e^{-\left(\frac{t}{MTT}\right)} \quad (1.24)$$

where, MTT is the mean transit time. The ratio of measured-to-actual CBF values was used as a metric for comparison of SVD versus the proposed technique.

A plot of simulated versus actual CBF values for FT based MMSE technique for low and moderate noise is shown in Figure 4a and 4b respectively. Figures 4c and 4d show similar plots obtained using SVD technique. For low noise, FT based MMSE and SVD technique perform similar to SVD technique for blood flow values in the range 0-40 ml 100g⁻¹ min⁻¹ while for flow values > 40 ml 100g⁻¹ min⁻¹ the estimated and actual flow values diverge with the SVD technique. For moderate noise levels, the proposed technique estimated the blood flow consistently lower than the actual values, while with the SVD technique there was agreement between estimated and actual flow values in the range 10-40 ml 100g⁻¹ min⁻¹.

The effect of inefficient AIF fit and bolus delay on estimated CBF values using the proposed and SVD technique is shown in Figure 5. The proposed technique consistently under estimated the flow for wide (Figure 5a) and narrow (Figure 5b) AIF compared to the SVD (Figure 5d and 5e) technique. The effect of AIF delay on flow estimates was more pronounced with SVD technique (Figure 5f) compared to the FT based MMSE technique.

Figure 6 shows a plot of $\Phi(f)$, $|C(f)|^2$, $|A(f)|^2$, $\frac{|C(f)|^2}{|A(f)|^2}$ and $\Phi(f) \frac{|C(f)|^2}{|A(f)|^2}$. An exponential fit to the filter function was performed in order to characterize the filter. The full-width at half-maximum (FWHM) of the filter was estimated to be $23 \cdot \left(\frac{2\pi}{TR}\right)$, where TR is the repetition time.

The plots in Figure 7a, b compare the effect of distorted AIF on the shaped filter function. The FWHM for the shaped filter function was approximated as $25 \cdot \left(\frac{2\pi}{TR}\right)$ for the wide AIF and $28 \cdot \left(\frac{2\pi}{TR}\right)$ for the narrow AIF.

Discussion

Deconvolution methods such as FT and SVD are useful for estimating blood flow values from data acquired using bolus tracking DSC technique. Both SVD and FT based techniques have their advantages and disadvantages as described in the introduction. SVD is less sensitive to noise because cutoff threshold applied to the singular values of the matrix causes the noise to be suppressed. However, the choice of cutoff threshold has significant influence on the CBF values. On the other hand, the FT based methods are sensitive to noise and require application of shaped filters in the frequency domain to experimental data in order to obtain reliable estimates of CBF. The low-pass shaped filters are applied to remove high frequency noise components of the residue function in order to ensure the stability of the FT deconvolution process. The frequency domain shaped filters are empirically derived based on some *a priori* knowledge about the experimental data. The application of shaped filter in FT methods is equivalent to removal of smaller singular values in the SVD method. Both the approaches are aimed towards removal of high frequency components that are assumed to be dominated principally by noise. One drawback of this approach is that the CBF values are consistently underestimated due to loss of some useful information during filtering (FT) or removal of singular values (SVD). A comparison of the proposed MMSE versus SVD technique was done with the assumption that SVD is the standard reference technique. The purpose of this study was compare FT based MMSE technique with the standard SVD technique.

In this study, an attempt has been made to derive and characterize the shaped filter function using an FT based MMSE method rather than derive it empirically by assuming an arbitrary filter function. The filter function would then be used to obtain a residue function from which flow can be estimated while in the frequency domain. The potential advantages of such a filter function would be the ability to optimally filter high frequency components, improve SDNR and the reliability of estimated CBF values. Thus, the characteristics of this explicitly derived filter function will determine which spectral components of the residue function and noise are retained in the deconvolution solution. Furthermore, it may be possible to optimize the filter function by directly examining the residue function in the time domain.

The simulation experiments aim at obtaining an optimal estimate of the shaped filter function by minimizing the error between the noisy and noiseless scaled residue function. The MMSE method is an optimal approach to solve such problems and in this study it has been applied in the frequency domain in order to derive the filter function. Additionally, the filter function characteristics were investigated due to distorted AIF, the presence of low and moderate noise and AIF delay.

During the simulation experiments, it was assumed that SDNR, as per definition, has a relationship with actual CBF. Furthermore, the percent decrease in signal due to T2* effect from injected Gd-DTPA bolus is also related to actual flow. This definition of SDNR allowed us to perform simulation experiments to investigate the effect of SNR on the entire range of CBF values. For low noise condition (max. SDNR=30), there was good agreement between the actual and estimated flow values for MMSE (Figure 4a) and SVD (Figure 4c) method, except for higher flow values SVD appears to be underestimating flow and FT based MMSE method appears to be more stable than SVD. The instability in SVD procedure may arise from the fact that the behavior of SVD algorithm changes for each deconvolution performed, resulting in being equivalent to FT based MMSE at times, while sometimes generating high frequency noise leakage and low frequency signal cut-off[15]. This behavior of SVD technique has been attributed to the fact that the SVD algorithm does not behave as a low pass filter but as a series of band pass filters that depend on the characteristics of noisy arterial concentration curve used during deconvolution process. For moderate noise (max. SDNR=10), both MMSE (Figure 4b) and SVD (Figure 4d) techniques underestimated CBF. One of the possible reasons for this finding is that the application of filter function in FT based MMSE technique and sharp eigenvalue cutoff thresholds in SVD procedure may accentuate noise in conditions with low SDNR.

The effect of distorted AIF on CBF estimates demonstrated that SVD (Figure 5a, b) was more stable compared to MMSE technique (Figure 5d, e). AIF distortions in this study represented inefficient gamma-variate function fits to the AIF. The significance of this finding is that FT based MMSE technique is sensitive to the performance of function fit to AIF. On the other hand, AIF delay did not have any effect on CBF estimates obtained using FT based MMSE technique suggesting that MMSE technique is more stable and less sensitive to AIF delays compared to the SVD algorithm. This observation can be explained based on Fourier transform property of delay in the time domain being equivalent to a phase shift in the frequency domain. As a result it does not affect the CBF estimates obtained using the FT based MMSE technique.

The shape of the FT window has been traditionally considered to be essentially equivalent to a rectangular low-pass filter. This filter shape has been obtained empirically based on some *a priori* knowledge of the MR signal and system. In this study, an attempt has been made to explicitly derive and characterize an optimal filter function as shown in Figure 6. The filter function ($E|\Phi|$) is a low-pass filter as is restricted to lower half of the spectrum since the upper half has been assumed to predominantly noise with majority of the signal spectrum localized in the lower end of the spectrum. An exponential function was fitted to this filter in order to

smooth the filter edges. This is beneficial since sharp filter edges induce oscillations in the time domain residue function. An exponential function was fitted to this filter and the FWHM was

estimated to be approximately $23 \cdot \left(\frac{2\pi}{TR}\right)$. A plot of $|C(f)|^2$ and $|A(f)|^2$ provided useful information about the frequency domain characteristics of the residue function. Moreover, the

function $\Phi(f) \frac{|C(f)|^2}{|A(f)|^2}$ was found to be sensitive to the effect of AIF distortion as seen in Figure

7a and 7b. The FWHM of the filter was approximated as $25 \cdot \left(\frac{2\pi}{TR}\right)$ and $28 \cdot \left(\frac{2\pi}{TR}\right)$ for wider

and narrower AIF, respectively. Thus, the modified filter function, $\Phi(f) \frac{|C(f)|^2}{|A(f)|^2}$ can be used as measure of inefficiencies in computation and distortions in the signal and maybe used for correction of these abnormalities.

The proposed deconvolution technique shares some common limitations with the FT based technique without MMSE. However, the use of MMSE filter method does not require empirical determination of the filter function and can be uniquely and optimally derived using the proposed technique.

Conclusion

Perfusion weighted (PW) MRI based on dynamic susceptibility contrast (DSC) MRI provides a powerful non-invasive way for assessing brain function. The bolus tracking (BT) technique is the most popular DSC-MRI method used for estimating CBF, CBV and MTT. The BT technique uses a convolution model that establishes the input-output relationship between blood flow and the vascular tracer concentration. Two deconvolution techniques that are popular and widely used for estimating PW-MRI parameters are the SVD and FT based methods. Both these techniques have their respective drawbacks. In this study, an FT based MMSE technique has been proposed for estimating CBF and its application has been demonstrated using computer simulation method. The proposed technique in this work has been used to derive filter function characteristics and has been shown to be sensitive to effect of AIF distortion. Future work would involve using the filter function characteristics for correction of other distortion effects on CBF estimates.

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Figure 1.

(a) A simulated signal, $s(t)$, with $\text{SDNR}=30$ & $\text{CBF}=80$. It has a signal drop of $\sim 48.1\%$ based on a 1.5T clinical scanner, a bolus injection of 0.1 mM/kg Gd-DTPA, and healthy subject. Small noise with $\text{SDNR}=30$ is added on top of the signal. **(b)** Simulated gamma-fitted AIF ($a(t)$), which, signal $s(t)$ in Fig.1a is obtained from.

Here $a(t)=\max\{0, \alpha(t-t_0) \exp\{-\beta(t-t_0)\}\}$, where $\alpha=0.0816$, $\beta=0.286$, $t_0=20.5$ sec.

Figure 2.

(a) SDNR vs. actual flow (CBF). We have used two SDNR groups, namely 'moderate' and 'low' SDNR groups. Flow proportionately effects the amount of signal drop, therefore SDNR is proportionally effected. Low SNR group represents a case in which we have high noise level (standard deviation) in the scanner. Moderate SNR group represents an example case in which we have moderate level of noise SD.

(b) percentage signal-drop vs. actual flow (CBF) used in the simulations. For our simulations, signal drop vs. CBF figure is fixed, based on certain assumptions that only CBF is effecting the signal drop.

Figure 3.

A simulated and approximated effective residue function **(a)** by the FT based MMSE technique **(b)** by the SVD technique. SDNR=30, CBF=80, $r(t)=\exp(-t/MTT)$, where $MTT=2$. The greatest 20 singular values (out of 50) were used for the SVD technique.

Figure 4.

(a) Simulated actual and estimated CBF values with the FT based MMSE technique, averaged over 100 realizations, with moderate SDNR depicted in Fig.2 (a). Bars denote the standard deviation of results for 100 realizations. (b) Results with low SDNR values for the FT based MMSE technique. (c) Simulated actual and estimated CBF values with the SVD technique, with moderate SDNR. (d) Results with low SDNR values for the SVD technique. Reference lines (Estimated Flow = Actual Flow) are also shown.

Figure 5.

CBF estimation results for MMSE-based technique with (a) (twice) wider AIF, (b) (twice) narrower AIF, (c) AIF with 3.2 sec. delay. Estimation results for SVD technique with (d) (twice) wider AIF, (e) (twice) narrower AIF, (f) AIF estimated with an error of 3.2 sec. delay. Reference lines (Estimated Flow = Actual Flow) are also shown.

Figure 6. Function $\Phi(f)$, and important ratios of $C(f)$ and $A(f)$ involved in the deconvolution process are illustrated. A special exponential fit for $\Phi(f)$, as presented with the dashed curve in the top plot. The FWHM of the filter can be approximated as $23(2\pi/TR)$, where $TR=1.6$ sec.

Figure 7.

(a) Function $\Phi(f)$ computed by the MMSE algorithm, in the case of wider AIF. The FWHM of the filter can be approximated as $25 (2\pi/TR)$, where $TR=1.6$ sec. **(b)** $\Phi(f)$ in the case of narrower AIF. The FWHM of the filter can be approximated as $28(2\pi/TR)$.